

STATISTICAL ANALYSIS OF THE MECHANICAL PROPERTIES AND WEIGHT OF REINFORCING BARS

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ABSTRACT

The variability of the mechanical properties and weight of steel reinforcing bars produced in the United States and Canada under ASTM A 615, A 616, and A 706 in 1997 is evaluated and expressions are developed to represent the probability distribution functions for yield and tensile strength. Thirty-four mills were invited to submit data on yield strength, tensile strength, elongation, and percent of nominal weight. Of these, 29 mills submitted data on a heat-by-heat basis, three mills provided average values (no data on a heat-by-heat basis), one mill provided data on “No Grade” bars (these were not used in this analysis), and one mill did not respond to the request for data.

A statistical analysis of bar properties is conducted. Trends in the data are evaluated based on grade, bar size, and production mill. Beta functions are developed to represent the probability distribution functions for yield and tensile strength for each bar size, grade, and steel type, as well as for all bars for each grade and steel type.

The analyses show that less than 0.1% of the steel heats failed to meet minimum ASTM standards for yield strength, and less than 0.1% of the steel heats failed to meet minimum ASTM standards for tensile strength. Approximately 1.2% of the steel heats failed to meet minimum ASTM standards for elongation, but no heats failed to meet the minimum ASTM standard for weight. The beta distributions for yield strength covering all A 615 Grade 40 and all A 615 Grade 60 bars provide good representations for the distributions for individual bar sizes within each of these grades, with the exception of A 615 No. 14 and No. 18 bars, which exhibit significantly different distribution functions. Both normal and beta distribution functions (for the individual bars and all bars) can be used to represent the distributions of yield strength for A 615

Grade 75, A 616, and A 706 bars. For tensile strength, the distribution for all bar sizes is recommended for A 615 Grade 40 bars. The beta functions developed for the individual bar sizes for A 615 Grade 60 bars provide a good match with the actual tensile strength distributions, with the exception of No. 3 through No. 5 and No. 7 bars. Both normal and beta distribution functions can be used to represent the distributions of tensile strength for A 615 Grade 75, A 616, and A 706 bars for both individual bar sizes and all bars.

Keywords: concrete, reinforcement, deformed reinforcement, elongation, probability distributions, reliability, statistics, strength, weight.

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INTRODUCTION

In structural design, uncertainties in loadings, design, and construction make it possible for load effects to be higher than computed and resistances to be lower than computed. Potential sources of these uncertainties include unforeseen loading conditions, changes in structure use, varying construction tolerances and loadings, and material property variations. As a result, building codes and design specifications require the use of load factors greater than 1.0 and strength reduction factors less than 1.0 in the design of structures. The purpose of these factors is to limit the probability of failure to an acceptably low level.

Under the category of material property variations, the variability of the physical and mechanical properties of reinforcing steel affects the performance of reinforced concrete structures. In North America, these properties have minimum requirements, as detailed by ASTM International standards A 615, A 706, and A 996 (a combination of the former A 616 and A 617) and by the Canadian Standards Association (CSA) standard G30.18-M92 (R1998). [Note: At the time data for this study were collected, ASTM A 616 and A 617 were the governing specifications for rail and axle steel, respectively. Hence, the earlier designations will be used in this report.]

This study is undertaken to evaluate the variability of the physical and mechanical properties of reinforcing steel produced throughout the United States and Canada and to analyze the degree to which manufacturers satisfy the minimum requirements established by ASTM International. This study is conducted by statistically analyzing data provided by steel mills in the United States and Canada.

Analysis of Problem

ASTM standards establish minimum criteria for the mechanical properties of all reinforcing steel produced in the United States. With the exception of ASTM A 706, however, they do not set maximum limits nor do they address instances where properties may significantly exceed the standards. With the exception of structures designed for seismic applications, designers use the minimum values in design without considering the true strength of the reinforcing steel. This may be of concern because member behavior can differ from the planned response if material properties are significantly higher than those used in the design. For instance, if the reinforcement is too strong in a reinforced concrete flexural member, it is said to be overreinforced. If the member is overloaded, this can result in brittle failure, with the concrete crushing before the steel yields. For members subjected to severe lateral loads, an increase in flexure strength can increase the induced shear forces on the member, also resulting in a brittle failure.

An understanding of the variability of steel properties is also useful in the development of statistically-based expressions for member strength, which are used in the development of reliability-based strength-reduction factors in design codes. For these reasons, it is worth examining the actual values of the mechanical properties of the reinforcing steel as compared to those used in design.

The mechanical properties of reinforcing steel are controllable in the manufacturing process, but variations between manufacturers and between heats for the same manufacturer do exist. Results may be influenced by several factors including, but not limited to the rate of loading, bar cross-sectional area, and variations in the composition of the steel. All reinforcing steel manufacturers must perform tests on their products to verify that they meet the ASTM

standards. These tests measure the yield strength, tensile strength, elongation, and weight per foot (or percent of nominal weight) of the reinforcing steel.

The statistical analyses of the manufacturers data presented here are conducted to evaluate the quality of the reinforcing steel in terms of these minimum ASTM standards. As part of the analyses, mathematical expressions are developed that closely correspond to the actual strength (yield and tensile) distributions of each bar size for possible use in the development of reliability-based strength-reduction factors.

Review of Previous Work

Three earlier studies deal specifically with the variability of the mechanical properties of reinforcing steel (Allen 1972, Mirza and MacGregor 1979, Nowak and Szerszen 2003).

In 1972, the National Research Council (NRC) of Canada published the results of a study analyzing the mechanical properties of reinforcing steel (Allen 1972). Two data samples were used, one consisting of 132 bars from a Canadian manufacturing plant and the other consisting of tests by the NRC on 102 bars obtained from five separate heats. The NRC tests, performed using greater control than those specified by ASTM or CSA, provided information about the variations along a bar and from one bar to another within a heat. The manufacturing plant data provided information on the overall variability of reinforcing steel from one manufacturer. The results showed that the coefficient of variation increased as one moved from one bar to a group of bars from the same heat and then to the entire plant. Additionally, a significant difference in the stress-strain curves was found to exist between No. 3 and larger bars. The curves for No. 3 bars had no yield plateau, while that of the larger bars did. This difference was explained based on the possibility that the No. 3 bars had undergone cold working due to their smaller size.

Several properties were measured under two loading conditions: static and dynamic (standard). Allen (1972) felt that the static loading condition better reflected actual loading conditions, and he concluded that subtracting an empirically derived value from mill test results would provide the static yield stresses at which failure would occur in practice. He also acknowledged that the deviations could be reduced if CSA specifications were modified in the following ways: (1) the nominal bar area was used instead of the actual bar area in calculations of stress and (2) the maximum rate of loading was reduced. Allen concluded, however, that the current control methods for testing were adequate.

A second study was subsequently published that addressed the variability of mechanical properties of reinforcing steel (Mirza and MacGregor 1979). In this study, variations in yield and tensile strengths and in the modulus of elasticity were examined. These variations were believed to be caused by varying rolling practices and quality control measures used by different manufacturers, as well as possible variations in cross-sectional area, steel strength, and rate of loading. The study was based on a sample that included 3,947 bars taken from 13 sources, some published and some unpublished. Mirza and MacGregor (1979) found that the beta distribution could be used to represent the probability distributions for both yield and tensile strength. They also found that the data they used for each grade of steel could be closely represented with a normal distribution between about the 5th and 95th percentile. At the lower end of the tail, their data dropped well below the normal distribution line for all grades evaluated. Conversely, data at the upper tail curved above the normal distribution line for Grade 40 yield strength and below the normal distribution line for Grade 60 yield strength and tensile strength.

Equations (1) through (4) are the beta probability density functions (PDFs) that Mirza and MacGregor (1979) found provided the best fit of their data. Equation (1) is the PDF for

yield strength for Grade 40 bars. Equation (2) is the PDF for yield strength for Grade 60 bars. Similarly, Eqs. (3) and (4) represent the PDFs for tensile strength for Grades 40 and 60 bars, respectively. Because they did not have data on the tensile strength of Grade 40 steel, Mirza and MacGregor (1979) “arbitrarily” selected Eq. (3) based on data correlation between Grade 60 reinforcement and the corresponding beta distribution.

$$\text{PDF} = 3.721 \times \left(\frac{f_y - 36}{32} \right)^{2.21} \times \left(\frac{68 - f_y}{32} \right)^{3.82}, \text{ (where } 36 \leq f_y \text{ (ksi)} \leq 68 \text{)} \quad (1)$$

$$\text{PDF} = 7.141 \times \left(\frac{f_y - 57}{51} \right)^{2.02} \times \left(\frac{108 - f_y}{51} \right)^{6.95}, \text{ (where } 57 \leq f_y \text{ (ksi)} \leq 108 \text{)} \quad (2)$$

$$\text{PDF} = 2.381 \times \left(\frac{f_u - 55}{50} \right)^{2.21} \times \left(\frac{105 - f_u}{50} \right)^{3.82}, \text{ (where } 55 \leq f_u \text{ (ksi)} \leq 105 \text{)} \quad (3)$$

$$\text{PDF} = 4.610 \times \left(\frac{f_u - 88}{79} \right)^{2.02} \times \left(\frac{167 - f_u}{79} \right)^{6.95}, \text{ (where } 88 \leq f_u \text{ (ksi)} \leq 167 \text{)} \quad (4)$$

A recent study, involving the analysis of material properties of concrete and steel for use in developing resistance models for the 2002 ACI Building Code (ACI 318-02), partially focused on the strength distribution of steel reinforcing bars (Nowak and Szerszen 2003). The authors plotted the yield strengths of 416 samples of No. 3 through No. 11 bars of Grade 60 reinforcement in terms of a cumulative density function using normal probability paper. The distributions were analyzed for each bar size. Regardless of the bar size, a normal distribution was found to provide a good representation of the data.

Objective and Scope

These analyses are conducted using Microsoft Excel® and are based on data obtained in 1997 on 23,768 heats of steel from 29 steel mills in the United States and Canada. Three other mills did not provide data on a heat-by-heat basis but did provide averages, which are considered in this report. One mill provided data on “No Grade” bars only. These data are not considered. The 33 mills represent all but one of the mills then producing reinforcing steel under ASTM standards A 615, A 616, and A 706 (no data under ASTM standard A 617 was reported). Each of the 29 mills providing data on a heat-by-heat basis, to a varying degree, provided information on yield strength, tensile strength, elongation, and percent of nominal weight. Data for yield and tensile strengths were provided for all 23,768 heats. Elongation was also provided for most, but not all heats, as can be seen in Tables 1-3. Percent of nominal weight was provided by less than one-half of the mills. More specifically, just four out of the 29 mills provided information on weight for A 706 Grade 60 bars. Only one mill provided information on weight for A 615 Grade 75 bars and only one other mill for A 616 Grade 60 bars. Seven mills provided data for A 615 Grade 40 bars and just 10 out of 29 mills provided information on weight for A 615 Grade 60 bars. Overall, the analyses include No. 3 through No. 6 A 615 Grade 40 bars, No. 3 through No. 18 A 615 Grade 60 bars, No. 4 through No. 14 A 615 Grade 75 bars, A 616 Grade 60 bars of No. 8 and No. 10, and No. 3 through No. 18 A 706 Grade 60 bars. It is important to note that No. 4 and No. 5 bars are not permitted under ASTM standard A 615 for Grade 75 reinforcement. The number of heats provided by each mill is included in Tables 1-3.

The statistical analyses are conducted to determine how the heats compare with the ASTM requirements and how the product differs between mills. General data descriptors are used, including mean, median, standard deviation, coefficient of variation, minimum, maximum,

5 percent fractile, skewness, and kurtosis for each steel type, grade, and bar size. In addition, representations of the statistical distribution of the yield strength and tensile strength are developed for all bar sizes for each grade and steel type. The representations are expressed using a beta distribution and presented as a cumulative density function (CDF) using normal probability paper plots. As will be demonstrated, the beta function provides a reasonably accurate description of the yield and tensile strength distributions.

DESCRIPTION OF WORK

Data Collection

In 1997, all 34 reinforcing steel manufacturers in the United States and Canada were invited to submit data for this study regarding the mechanical and physical properties of each bar size of every grade produced for each type of steel on a heat by heat basis. Each mill was requested to provide data for the last 100 heats or the last full year's production, whichever was smaller. Thirty-three of the 34 mills responded to the request [A. B. Steel, Ameristeel Charlotte, Ameristeel Jacksonville, Ameristeel Knoxville, Ameristeel West Tennessee, Auburn Steel, Austeel Lemont, Birmingham Birmingham, Birmingham Joliet, Birmingham Kankakee, Birmingham Jackson, Birmingham Seattle, Border Steel, Cascade Steel, CF & I Steel, Co-Steel Sayreville, Connecticut Steel, Marion Steel, N. S. Beaumont, N. S. Kingman, N. S. Monroe, N. S. St. Paul, N. S. Wilton, Nucor South Carolina, Nucor Texas, Nucor Utah, Riverview Steel, Sheffield Steel, Silver, S. M. I. Arkansas, S. M. I. South Carolina, S. M. I. Texas, Tamco]. Twenty-nine mills provided at least some data on a heat-by-heat basis, as requested. Three mills provided only average values for mechanical and physical properties (they no longer had

individual heat data on record), and one mill provided data for “No Grade” bars only (these are not considered in this report). One mill did not respond to the request for data. Of the 29 mills that provided data on a heat-by-heat basis, in many cases, significantly more than 100 heats of data were received.

All manufacturers who submitted data on a heat-by-heat basis provided data on the yield strength and tensile strength of the reinforcing steel. All but one manufacturer provided data on specimen elongation for at least some grades, and fewer than one-half of the manufacturers provided at least some data on the percent light or weight per unit length of the bars. It is unclear as to why so many manufacturers did not keep a record of this property. Many of those who did not provide this data provided a general range of bar weights but no data to back up their claims.

These data provide information concerning the general variability of reinforcing steel produced within the United States and Canada. The analyses that follow analyze yield strength, tensile strength, elongation, and percent of nominal weight. The analyses are based solely on the data provided by the manufacturers. Therefore, variations in reported mechanical properties that result because of differences in test methods are not considered.

To provide confidentiality to all manufacturers submitting data for this study, a random number is assigned to each mill. Upon publication of this report, each manufacturer has been informed of their identification number allowing them to compare their data with that of the rest of the industry. The raw data are available from the third author at daved@ku.edu.

Statistical Analysis

The data provided are statistically analyzed to evaluate the variability in the properties of steel reinforcement. Strength (yield and tensile), elongation, and percent of nominal weight are

assessed in terms of the requirements as set forth by ASTM. Different bar sizes are compared and a mill-by-mill comparison is made to evaluate the variability between mills. The statistical analyses are conducted using Microsoft Excel. For yield strength and tensile strength, the following parameters are evaluated for each bar size, grade, and steel type: mean, median, standard deviation, coefficient of variation, minimum, maximum, 5 percent fractile, skewness, and kurtosis. The 5 percent fractile represents the strength exceeded by 95 percent of the data. The skewness is a measure of symmetry. Negative values indicate data that is skewed to the left and positive values indicate data that is skewed to the right. The kurtosis is a measure of whether the data is peaked or flat relative to a normal distribution. An increased kurtosis indicates an increased peak near the mean of the data. Elongation and percent of nominal weight are evaluated based on the mean, minimum, maximum, standard deviation, and coefficient of variation. Summary statistics for bars of each size, grade, and type are included in Tables 4-13. Additionally, Figures 1-10 depict the range of data for each of the four mechanical and physical properties (these figures are discussed further in the Results and Analysis section).

Representing strength distributions with the beta function

Mirza and MacGregor (1979) showed that a beta function could be used to develop a representation of the distribution of yield (or tensile) strength of reinforcing bars. In their study, the cumulative probability of the actual data and a beta CDF were plotted on normal probability paper. The beta function was manipulated to fit the shape of the cumulative probability as closely as possible. The cumulative probability ranged from 0 to 100 percent. This suggests the expression for the beta distribution, which is developed to match the actual cumulative probability, should begin and end at the corresponding points in the actual distribution.

Therefore, their study inferred that there is a 0 probability of obtaining a strength that is outside values between the lower and upper bounds of the actual distribution.

The work by Mirza and MacGregor (1979) serves as the basis for the current analyses. Their work was based on raw data from both published and unpublished studies covering various bar sizes for Grades 40 and 60, respectively. This study, in contrast, is based on data obtained directly from steel manufacturers and represents a considerably larger sample size. The paragraphs that follow explain the procedure that was used to represent strength data with a beta function. The strength data is first shown in the form of a CDF (Figure 11), where it forms an S-shaped curve. It is also depicted on a normal probability plot, along with a beta CDF and a normal probability distribution, as shown in Figures 12a-95a.

To develop a beta function to emulate the strength distributions of a given bar, it is first necessary to look at the general form of the beta probability density function (PDF), shown in Eq. (5).

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^{\alpha} \times \left(\frac{UB - f}{D} \right)^{\beta} \quad (5)$$

The variable f represents an individual strength value (either yield or tensile), and the variable D represents the difference between the values for the upper bound UB and the lower bound LB . The lower bound is taken as the minimum strength of all steel heats for each particular bar size. The values for the upper bound, coefficient C , and the exponents α and β are derived empirically as described below.

Several steps are required to derive a unique beta PDF to closely represent an actual strength distribution. It is necessary to transform the actual data into a CDF, integrate the beta PDF in question to obtain a beta CDF, and then match the two CDF plots as closely as possible

using the method of least squares. The process is the same for both yield strength and tensile strength for every bar size of each grade and steel type.

The first step is to transform the actual data into a CDF. To do this, the strengths for the heats are arranged in ascending order on a spreadsheet. Each strength is represented by the cumulative probability, equal to $i/(N+1)$, where i is the count and N is the total number of heats. This value is plotted on a graph of strength versus cumulative probability, forming an S-shaped curve, as shown in Figure 11.

The next step is to select values for use in the generalized form of the beta PDF [Eq. (5)] as a starting point to develop the final PDF expression. The value for the lower bound LB is taken as the lowest reported strength for that particular bar size. In the case of yield strength, if one or more data points are below the minimum specified value of yield strength, then LB is taken as the minimum specified yield strength for that grade of steel. It will later be shown that the upper bound UB is found by trial and error, but a good starting point is to set it equal to the largest reported value of strength for the particular bar size. As previously explained, the difference D is simply the difference between LB and UB . A good starting place for the coefficient C is about 19,000 for Grade 40 reinforcement and about 37,000 for all other types of reinforcement in this study. For the exponents, α and β , useful initial values include any single digit whole number for α and any higher, three-digit whole number for β . It will be shown that these figures can be significantly refined using Microsoft Excel, ensuring a better fit of the beta function to the actual distributions.

The intervals between adjacent data points are separated into five equal divisions to facilitate the numerical integration of the beta PDF to obtain the beta CDF using the trapezoidal rule. The values of strength f between data points are obtained by interpolation. Each interval

value for strength f is inserted into the beta PDF [Eq. (5)] resulting in six values of the beta PDF for each interval between two ascending, actual data points. The PDF values at the six points are summed after multiplying the four interval values by two. Each sum is multiplied by one-half of the interval to yield the area under the beta curve between each two actual data points. The process is repeated for each interval until the entire data set has been integrated. The procedure for using the trapezoidal rule, as presented here, can be found in most mathematical texts. Numerically integrating the beta PDF yields the CDF of the beta distribution.

There is a fundamental difference between the beta CDF and the actual cumulative probability. The beta CDF, like all beta distributions, ranges in value from 0 to 1. Conversely, the cumulative probability, $i/(N+1)$, ranges from a value that is slightly greater than 0 to a value that is slightly less than 1; the difference, $1/(N+1)$, depends on the value of N . The difference between the beta and actual CDFs has to be accounted for to maintain consistency between the two plots. Since $i/(N+1)$ yields values that are greater than 0 and less than 1, it is considered to be a more realistic estimate of probability than $(i-1)/(N-1)$, used by Mirza and MacGregor (1979), to give cumulative probabilities of exactly 0 and 1, respectfully, for the lower and upper bound strengths. Thus, the probability that a heat of reinforcement will have a value of yield strength lower than the lowest value of the actual data in this study is, more realistically, a very small number rather than 0. Although a beta cumulative distribution function varies from 0 to 1, to get the best match in this curve, it is necessary, in most cases, to set the first and last points of the fitted beta CDF equal to $1/(N+1)$ and $N/(N+1)$, respectively. In this way, the beta CDF will coincide with the end points of the actual data. [In a few cases, the end points are not used because adjacent data points provide a better match between the beta and actual CDFs.]

The next step is to modify the shape of the beta CDF to represent the shape of the actual cumulative probability as closely as possible. This is done by using a weighting procedure and then minimizing the sum of the squared differences between data points from the beta CDF and the corresponding individual data points from the cumulative probability (the weighting procedure is described in the next paragraph). The Solver function in Microsoft Excel is used to minimize the sum by adjusting the coefficient C and exponents α and β in Eq. (5). The lower bound LB remains equal to the minimum strength for each bar size (excluding values below the specified minimum yield strength). The upper bound UB is manually adjusted through trial and error to provide the best possible fit of the beta CDF to the cumulative probability. As previously mentioned, a good starting value is the largest value of strength. However, it is found that larger values of UB produce beta functions that more closely represent the shape of the actual distribution. In many cases in this study, UB was increased significantly (up to seven figures) to obtain a close match. Thus, the UB may be much larger than the largest value of strength. A PDF equation, however, remains valid only between CDF values of 0 and 1. In other words, the beta PDF equations are valid only between the minimum and maximum values for each bar size and grade shown in Tables 4, 6, 8, 10, and 12.

To provide the best possible fit of the beta function to the actual distribution, a weighting system is used. Without a weighting system, the Solver function weights all data points equally and, as a result, works to minimize differences in the middle of the distributions (the most common strength range) more than at the lower and upper tails, where the data are sparse. This yields a better fit in the middle portion of the distribution than at the tails. Therefore, to produce the best possible fit throughout, the data points are weighted to compensate for the uneven distribution of the actual data. To do this, each squared difference between the beta CDF and the

actual cumulative probability is multiplied by a constant that is obtained by taking the inverse of the number of data points in each 5 ksi interval of strength over the entire data range. This process equally distributes the weights of each portion of the entire distribution. Once the squared differences are multiplied by this constant, minimizing the sum of the squared differences between the beta CDF and the actual cumulative probability distribution yields an improved match between the two curves. The final beta PDF provides a close match with the actual distribution. As an example, the beta PDF of yield strength for A 615 Grade 40 No. 3 bars is

$$\text{PDF} = 19,629 \times \left(\frac{f_y - 41,672}{88,328} \right)^{6.89} \times \left(\frac{130,000 - f_y}{88,328} \right)^{38.11} \quad (6)$$

The beta CDF that results from Eq. (6) is compared to the actual CDF and the CDF for a normal distribution in Figure 12a.

The beta CDF for yield or tensile strength, along with the actual cumulative probability and the normal distribution CDF, for each bar size of each grade and type of steel is plotted on normal probability paper in Figures 12a-95a.

On normal probability paper, the variable (yield or tensile strength in this case) is plotted on the horizontal axis and the standard normal variable Z ($Z = (X - \bar{x})/s$, where X = a value of strength, \bar{x} = the sample mean, and s = the sample standard deviation), which represents the deviation from the mean expressed as the number of standard deviations, is plotted on the vertical axis. The construction of normal probability paper used in this report follows the procedure outlined by Nowak and Collins (2000). Normal probability paper has three important characteristics: (1) a straight line represents a CDF for a normal distribution with the same mean

and standard deviation as the data, (2) the value of the variable at the point at which the standard normal variable equals 0 is approximately the mean of the data, and (3) the slope of the CDF can be used to determine the standard deviation of the data if the data is normally distributed (a straight line).

In this report, the actual and beta functions are compared with a straight line, which represents data that is normally distributed. The addition of a normal distribution serves to gauge the similarity between the normal distribution and the actual distribution. The figures also include a probability scale as an alternate y-axis. From these plots it is evident that some of the distributions can be approximated with a normal distribution, such as shown in Figures 29-53 and 71-95. Others, however, possess unique shapes that are best described using the beta function. Although Mirza and MacGregor (1979) found that their data for Grades 40 and 60 exhibited a close agreement with a normal distribution from about the 5th to the 95th percentile, results from this study indicate noteworthy deviations from a normal distribution.

A tangent line to the beta CDF at a particular point may be a good match with a normal distribution in that vicinity. This is often used when approximating a distribution at the lower end of the tail, which is done in calculations to determine strength reduction factors. A further discussion of the distributions is contained later in this report.

The process of plotting the beta CDFs, actual probability distributions, and normal distributions on normal probability paper is conducted for the yield and tensile strengths for every bar size, grade, and steel type, as well as for the combined distributions for each grade and steel type. These are included in Figures 12a-95a. Additionally, histograms of actual data with the beta and normal PDFs are presented in Figures 12b-95b. The equations for the individual beta PDF distributions are included in Tables 14-23.

RESULTS AND ANALYSIS

Overview of Tables and Figures

In this report a statistical analysis is conducted to evaluate the physical and mechanical properties of reinforcing bars and to compare these properties with minimum ASTM standards. Additionally, a beta function is formed to represent the actual distribution of yield and tensile strength for each bar size, grade, and steel type. A summary of the number of heats reported by each mill is included in Tables 1-3. Tables 4-13 contain a summary of the statistics for all bars (these tables only consider actual data received and not the averages provided by three of the mills), and Tables 14-23 present the beta PDF equations used to represent the actual distributions of each bar size, grade, and steel type, as well as the PDF equations for the combined distributions for each grade and steel type. A mill-by-mill comparison of the physical and mechanical properties of reinforcement is shown in Tables 24-35 (these tables also include the average values from the three mills that provided only average values [mills 435, 687, 845]).

Figures 1-10 depict the range of values for the mechanical and physical properties. In Figure 11, an example of a CDF plotted on standard paper (not normal probability paper) is presented. Figures 12a-95a contain the beta CDF, along with the CDF of the actual data and the normal probability line – all plotted on normal probability paper for each bar size, grade and steel type. Figures 12b-95b include histograms of the actual data, the beta PDF, and normal PDF associated with each bar size of all grades and steel types. Figures 12-53 cover yield strength, while Figures 54-95 cover tensile strength. In addition to the plots for individual bar sizes, grades, and steel types, Figures 16, 28, 38, 41, and 53 represent all of the heats for the specified yield strength for A 615 Grade 40, A 615 Grade 60, A 615 Grade 75, A 616 Grade 60, and A 706

Grade 60, respectively. Similarly, for tensile strength, Figures 58, 70, 80, 83, and 95 represent all of the heats for A 615 Grade 40, A 615 Grade 60, A 615 Grade 75, A 616 Grade 60, and A 706 Grade 60, respectively. (Because No. 4 and No. 5 bars are not permitted under ASTM standard A 615, this data is not included in Figures 38a and 80a.) Histograms describing the distribution of elongation and percent of nominal weight are included in Figures 96a-137a and 96b-137b, respectively. An analysis of the results is presented next.

Summary Figures

The range of values for yield and tensile strength is summarized in Figures 1-5. Similar summaries for elongation and weight are presented in Figures 6-10. In both cases, the figures consist of graphical representations of bar size versus yield strength (or tensile strength, etc...), providing a visual image of the range of data that was received for this study. It is evident that, for most bars sizes of A 615 Grades 40 and 60 (for both yield strength and tensile strength), the mean is not situated at the midpoint of the data range, indicating non-normal distributions. Conversely, most bar sizes of A 615 Grade 75 and A 706 reinforcement (for both yield strength and tensile strength) show that the mean is located relatively close to the midpoint of the data range (see Figure 3). The graphical representations of the two bar sizes for A 616 reinforcement (Figure 4) show that the means for tensile strength are closer to the midpoint of the range than the distributions for yield strength.

Figures 6a-10a show the distributions for elongation. In each case, the mean is located very close to the midpoint of the range. In the case of percent of nominal weight, however, most bars have a mean that is closer to the lower end of the distribution (see Figures 6b-10b). In some cases, such as with A 615 Grade 40 and 60 reinforcement, the maximum nominal weight reaches

values of nearly 106%. Overall, however, the heats average 98 to 99% of the nominal weight for Grades 40 and 60, respectively (see Tables 5 and 7).

Comparing Data with Minimum ASTM Standards

In comparing the data with the minimum standards set forth by ASTM, the following observations are made:

Yield Strength:

A value of yield strength was recorded for all 23,768 heats analyzed for this study. Of those, 19 (0.079%) heats did not meet the minimum ASTM standard for their respective grade.

Tensile Strength:

A value of tensile strength was recorded for all 23,768 heats analyzed for this study. In this case, 15 (0.063%) heats did not meet the minimum ASTM standard for their respective grade.

Elongation:

Of the 23,768 total heats, a value of elongation was recorded for 22,954 (96.6%) heats. Of these, 284 (1.2%) heats did not meet the minimum ASTM standard for their respective bar size, grade, and steel type.

Weight:

Of the 23,768 heats recorded, a value of percent of nominal weight was recorded for 8,019 (33.7%) heats. No heat failed to meet the minimum ASTM standard of 94%.

Interestingly, most of the heats that failed to meet the minimum standards for elongation were produced by only one or two mills, rather than being evenly spread over all of the manufacturers. For example, of the A 615 Grade 60 bars, 246 out of 19,378 (1.3%) heats did not meet minimum requirements for elongation. Of these, 242 heats (98.4%) were produced by just

two mills. Mill 41 had 178 (73.6%) elongation-deficient heats and mill 227 had 64 (26.4%) elongation-deficient heats. One heat did not meet minimum elongation requirements for each of the following mills: 184, 437, 575, and 973. Of the A 706 Grade 60 bars, 35 out of 1,334 (2.6%) heats did not meet minimum requirements for elongation. Of these, 31 heats (88.6%) were produced by mill 227. In addition, mills 639 and 770 each had one heat, respectively, that did not meet minimum elongation requirements. There were only three deficient heats in terms of elongation for A 615 Grade 40 bars. Two were produced by mill 575 and one was produced by mill 739. There were no elongation-deficient heats for A 616 Grade 60 bars or A 615 Grade 75 bars.

There were a limited number of deficient heats in terms of yield strength. For A 615 Grade 60 bars, only five out of 19,886 (0.025%) heats were deficient. Three of these were produced by mill 437, and two were produced by mill 874. Of the A 706 Grade 60 bars, just one heat out of 1,568 (0.064%) had a low yield strength. That was from mill 874. Additionally, one A 706 heat from mill 874 exceeded the maximum allowable yield strength of 78 ksi, with a value of 85.4 ksi. For A 615 Grade 75, 12 out of 211 heats (5.7%) had a low yield strength. All 12 of these were produced by mill 639. There were no low yield strength heats for A 615 Grade 40 or A 616 Grade 60.

The results for tensile strength were similar. For A 615 Grade 60, 15 out of 19,886 heats (0.075%) were deficient. Nine of the heats were produced by mill 227, four were produced at mill 437, and one heat each were produced by mills 184 and 676, respectively. There were no other low tensile strength heats.

Data Trends: Bar Sizes and Grades**Yield Strength:**

There is a significant trend observed in the data for yield strength of A 615 Grade 60 bars. Here, the average yield strength first decreases from 71.9 ksi for No. 3 bars to 68.9 ksi for No. 5 bars and then increases steadily with bar size to a maximum average strength of 72.2 ksi for No. 18 bars (Table 6). Overall, the average yield strength is 69.6 ksi for all A 615 Grade 60 bars. For the other bars, the average yield strengths were 55.9 ksi for all A 615 Grade 40 bars, 81.5 ksi for all A 615 Grade 75 bars (note that No. 4 and No. 5 bars, while provided by two mills, were not considered in this average because they are not permitted for ASTM A 615 Grade 75 bars), 66.4 ksi for all A 616 Grade 60 bars, and 69.1 ksi for all A 706 Grade 60 bars.

The study conducted by Mirza and MacGregor (1979) was based on a sample that included 3,947 bars taken from 13 sources. In their analysis they found that the coefficient of variation (COV) of yield strength for reinforcing bars taken from various sources ranged from 5 to 8% for individual bar sizes. Their data consisted of Grade 40 and Grade 60 steel. Similarly, Allen (1972), whose study was based on two data samples, one consisting of 132 bars and the other consisting of 102 bars obtained from five heats, found the COV for yield strength to be 7 to 8% for Grade 40 bars. As can be seen in Tables 4, 6, 8, 10, and 12, the COV varies for different grades and types of steel. A 615 Grade 40 reinforcement had values of COV between 7.0 and 9.9% for yield strength. A 615 Grade 60 bars were slightly less variable, with the values of COV for yield strength between 3.6 and 9.0%, and exhibited a general decrease in COV with an increase in bar size. A 615 Grade 75 bars exhibited COVs between 2.5 and 5.4%, A 616 Grade 60 bars exhibited COVs between 2.3 and 2.6%, and A 706 Grade 60 bars exhibited COVs between 3.6 and 6.1% for yield strength.

The 5% fractile for yield strength for individual bar sizes is relatively consistent for A 615 Grade 40 bars, ranging from a minimum of 47.1 ksi to a maximum of 49.7 ksi. The range widens for the other standards and grades: 63.1 to 67.2 ksi for A 615 Grade 60 bars, 62.5 to 68.6 ksi for A 706 Grade 60 bars, and 72.9 to 81.2 ksi for A 615 Grade 75 bars. The 5% fractile for A 616 Grade 60 bars ranged from 62.9 ksi to 65.5 ksi. Values for the 5% fractile, along with other statistical parameters, are shown in Tables 4-13 for the individual bar sizes, as well as for all bars for each of the grades and steel types.

Tensile Strength:

Unlike yield strength, the tensile strength of A 615 Grade 60 bars generally decreases with an increase in bar size (109.2 to 100.7 ksi) (Table 6). As shown in Table 4, A 615 Grade 40 bars show a average tensile strength that varies very little as a function of bar size (81.3 ksi to 82.7 ksi). Ranges in tensile strength for other standards and grades are 110.3 to 117.4 ksi for A 615 Grade 75 bars (Table 8), 100.2 to 104.4 ksi for A 616 Grade 60 bars (Table 10), and 94.0 to 99.4 ksi for A 706 Grade 60 bars (Table 12).

A 615 Grade 60 bars exhibit the greatest variation in tensile strength with COVs ranging from 4.6 to 11.8%. The most discernable trend in the COV for tensile strength occurs for the A 706 Grade 60 bars. Here, the COV decreases as the bar size increases (5.7 to 3.0%). Other ranges for COV include 7.1 to 9.4% for A 615 Grade 40 bars, 1.7 to 2.4% for A 616 Grade 60 bars, and 2.7 to 4.3% for A 615 Grade 75 bars.

The 5% fractile ranges are 73.5 to 75.0 ksi for A 615 Grade 40 bars, 93.5 to 98.2 ksi for A 615 Grade 60 bars, 105.0 to 110.5 ksi for A 615 Grade 75 bars, 97.8 to 102.3 ksi for A 616 Grade 60 bars, and 85.2 to 95.8 ksi for A 706 Grade 60 bars.

Weight:

Ranges for average percent of nominal weight, by bar size, vary from 96.3 to 97.6% for A 615 Grade 40 bars, 96.4 to 98.2% for A 615 Grade 60 bars, 96.1 to 98.5% for A 615 Grade 75 bars, 96.2 to 96.6% for A 616 Grade 60 bars, and 96.6 to 97.2% for A 706 Grade 60 bars. Additionally, a trend is observed in A 615 Grade 40 bars. Here, the bars are consistently lighter relative to the nominal weight as the bar size increases.

Data Trends: Mills

A mill-by-mill comparison of average yield strength is presented in Tables 24-26. Similar information is presented for tensile strength, elongation, and nominal weight in Tables 27-28, 30-32, and 33-35, respectively. These tables include the average values provided by mills 435, 687, and 845 (these mills did not provide raw data). It is evident that mill 638 has the lowest average strength of all mills for both A 615 Grade 60 bars and A 706 Grade 60 bars. Conversely, mill 874 had the highest average for A 706 Grade 60 bars and the lowest average for A 615 Grade 40 bars.

In comparing yield strength and percent of nominal weight, it is interesting to see that for A 615 Grade 40 bars mill 463 has the lowest average percent of nominal weight, but has the highest average yield strength. Alternatively, mill 638 is tied for the lowest average value of weight and stands alone with the lowest average yield strength for both A 615 Grade 60 bars and A 706 Grade 60 bars. From Tables 33 and 35, mill 681 has the highest average percent of nominal weight for both A 615 Grade 40 bars (99.0) and A 706 Grade 60 bars (97.1). Additionally, mill 638 has the lowest average percent of nominal weight for A 706 Grade 60 bars (95.3) and is tied for the lowest average with mill 463 for A 615 Grade 60 bars (95.7).

Yield and Tensile Strength Distributions

The portion of this study dealing with the development of a relationship between the beta CDF and the cumulative probability of the strength data demonstrates that the beta function can be used to represent the actual strength distribution of steel reinforcement. The advantage of using a relatively large sample size of data is that, in theory, it provides a better indication of the actual distribution. In this study, some of the strength distributions closely follow a straight line on normal probability paper for the entire range of the data, indicating the data can be accurately represented by a normal distribution, as well as a beta function. The majority of the comparisons, however, show that the data exhibits distributions that are best represented by the beta, rather than the normal, function. A unique beta function is used to represent the distribution of each bar size of each grade of each steel type. These equations are included in Tables 14-23.

Mirza and MacGregor (1979) found that the data they used for each grade of steel could be closely represented with a normal distribution between about the 5th and the 95th percentile. At the lower end of the tail their data dropped well below the normal distribution line for all grades evaluated. Conversely, data at the upper tail curved above the normal distribution line for Grade 40 yield strength and below the normal distribution line for Grade 60 yield strength and tensile strength. Mirza and MacGregor (1979) did not have data in their study for tensile strength of Grade 40 steel, so no distribution is available [they selected Eq. (3) based on data correlation].

Yield Strength:

The beta distributions of yield strength for the different A 615 Grade 40 bar sizes are relatively consistent in shape with one another and each forms a good fit with the actual data for

that particular bar size (see Figures 12a-16a). In each case, the beta function drops slightly below the actual distribution at the lower tail and rises slightly above the actual distribution at the upper tail. An exception is the upper tail of Figure 14a, where the beta function fits the data quite well.

The beta distributions of yield strength for the A 615 Grade 60 bars are also relatively consistent in shape and each forms a good fit with the actual data (Figures 17a-28a). Beta functions representing the distributions of No. 7 and No. 8 bars are excellent fits throughout their respective distributions (Figures 21a and 22a). The beta functions for No. 4 through No. 8 bars and No. 18 bars form excellent fits with their respective distributions at the lower tail (Figures 18a-22a, 27a). Additionally, the upper tail of the beta function representing No. 3 through No. 6 bars rises slightly over of the actual distribution at this portion of the plot (Figures 17a-20a).

Overall, the distributions of yield strength for A 615 Grades 40 and 60 bars in this report are represented relatively accurately by the beta function. Some of the plots, however, show that the beta function fits the strength distribution very well at the lower half and less accurately at the upper half (see Figures 18a-20a). Unlike the data used by Mirza and MacGregor (1979), most of the distributions in this report do not form a straight line, which would indicate a close match with a normal distribution, from the 5th to the 95th percentile. Most of the distributions, including those covering all A 615 Grade 40 and 60 bars, form a curved shape and drop significantly below the normal distribution at both the lower and upper tails (see Figures 16a and 28a). A 615 Grade 40 No. 4 bars (Figure 13a) and A 615 Grade 60 No. 14 (Figure 26a), and No. 18 bars (Figure 27a) are exceptions. These three distributions can be closely approximated by a normal distribution or the given beta function from the 5th to the 95th percentile. The beta distribution representing all A 615 Grade 60 bars (Figure 28a) is a good fit at the lower tail and

slightly above the actual distribution at the upper tail. For a general yield strength representation, however, the two beta functions describing all bars for A 615 Grade 40 and Grade 60 reinforcement provide relatively good representations for those two grades, with the exception of A 615 Grade 60 No. 14 and No. 18 bars (Figures 26a and 27a), which have significantly different distribution functions from those exhibited by the other A 615 Grade 60 bars.

In contrast to A 615 Grades 40 and 60, a normal distribution, as well as the given beta function, can be used to describe the yield strength of all bar sizes for A 615 Grade 75, A 616, and A 706 reinforcement, as shown in Figures 38a, 41a, and 53a, respectively. In each case the data plot close to a straight line.

The beta distribution for A 615 Grade 75 No. 5 bars is the most inaccurate fit of the beta functions describing yield strength (Figure 30a). There are just 11 data points in this distribution. Three of these are below 75 ksi and are, therefore, not considered as the lower bound of the data. The remaining eight data points allow for a good fit in this case. However, the data plot close to the line indicating that a normal distribution would be appropriate.

Tensile Strength:

The beta distributions of tensile strength for each of the A 615 Grade 40 bars are relatively consistent in shape with one another and each forms a good fit with the actual data for that particular bar size (Figures 54a-57a). Therefore, the beta function presented for tensile strength of all A 615 Grade 40 bars (Figure 58a) provides an accurate representation and is recommended. The tensile strength beta distribution for all A 615 Grade 60 bars (Figure 70a), provides a poor match and is not recommended. The data for No. 3 through No. 5 and No. 7 bars follow very erratic shapes when plotted. The other bar sizes, however, exhibit a fairly good fit

between the actual data and the beta distribution. As a result, the beta functions for each individual bar size (not including No. 3 through 5 or No. 7) are recommended as more accurate representations than the beta function for all of the Grade 60 bars.

As with yield strength, a normal distribution can be used to represent the tensile strength of A 615 Grade 75 bars, A 616, and A 706 bars because their CDF plots are nearly straight lines (Figures 71a-95a). Among the tensile strength distributions, the most inaccurate fitting beta functions were those for A 615 Grade 60 No. 3 through No. 5 and No. 7 bars (Figures 59a-61a, 63a).

Elongation and Weight Distributions

Elongation:

The distributions for elongation and weight for the individual bar sizes for each grade and steel type are shown as histograms in Figures 96-137. Most of the histograms for elongation exhibit a shape that is relatively close to what would be expected for a normal distribution (bell-shaped curve). This, however, is not true for all distributions. For instance, the distribution of A 615 Grade 60 No. 14 bars (Figure 110a) is skewed considerably to the right, as is the distribution for No. 18 bars (Figure 111a), though to a lesser degree. In contrast, the distributions for No. 4 and No. 9 bars of A 615 Grade 75 steel and No. 3 bars of A 706 Grade 60 steel (Figures 113, 118, 126) are generally block-shaped.

Weight:

The majority of the histograms describing the weight distributions of the bars also exhibit a shape that resembles a normal distribution (some are slightly skewed right and others are slightly skewed left). One exception is the distribution of A 706 Grade 60 No. 8 bars (Figure

131b). This distribution is significantly skewed to the right. Additionally, the distributions for some of the bars, such as A 616 Grade 60 bars (Figures 123b and 124b), are based on very little data. In this case, information was provided for only 13 heats of No. 8 bars and only 6 heats of No. 10 bars.

SUMMARY AND CONCLUSIONS

Summary

This study was undertaken to gain an understanding of the variability of the properties of steel reinforcing bars. The data used in this report were provided by steel mills from across the United States and Canada. The mills, to varying degrees, provided information on yield strength, tensile strength, elongation, and percent of nominal weight. Thirty-four mills were invited to submit data. Of these, 29 mills submitted data on a heat-by-heat basis, three mills provided average values (no data on a heat-by-heat basis), one mill provided data on “No Grade” bars (these were not used in this analysis), and one mill did not respond to the request for data.

The mechanical and physical properties were evaluated in terms of standard statistical parameters, such as mean, standard deviation, and coefficient of variation. Trends in the data were evaluated based on grade and bar size, as well as producing mill. Building from the work of Mirza and MacGregor (1979), beta functions were developed to represent the yield and tensile strength distributions of each bar size, grade, and steel type. Comparisons were made with the findings of previous studies.

Conclusions

Based on these analyses, the following conclusions are made:

1. Less than 0.1% of the heats failed to meet minimum ASTM standards for yield strength.

2. Less than 0.1% of the heats failed to meet minimum ASTM standards for tensile strength.
3. Approximately 1.2% of the heats failed to meet minimum ASTM standards for elongation.
4. No heats failed to meet minimum ASTM standards for weight.
5. The beta distributions for yield strength covering all A 615 Grade 40 and all A 615 Grade 60 bars provide good representations for the distributions for individual bar sizes within each of these grades, with the exception of A 615 No. 14 and No. 18 bars, which exhibit significantly different distribution functions.
6. Both the normal and beta distribution functions (for the individual bars and all bars) can be used to represent the distributions of yield strength for A 615 Grade 75, A 616, and A 706 bars.
7. The beta distribution for tensile strength covering all A 615 Grade 40 bars provides a good representation for the distribution for individual bar sizes within this grade.
8. The beta functions representing tensile strength for the individual bar sizes for A 615 Grade 60 bars provide a good match with the actual distributions, with the exception of these four: No. 3 through No. 5 and No. 7 bars.
9. Both the normal and beta distribution functions (for the individual bars and all bars) can be used to represent the distributions of tensile strength for A 615 Grade 75, A 616, and A 706 bars.
10. The distributions for elongation of each bar size, grade, and steel type appear to be relatively close to normal with the exception of A 615 Grade 60 No. 14 and 18 bars, A 615 Grade 75 No. 4 and 9 bars, and A 706 No. 3 bars, which exhibit either significantly skewed or block-shaped distributions.
11. The distributions for percent of nominal weight for most bar sizes of each grade and steel type appear close to normal (some slightly left-skewed and others slightly right-skewed).

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Table 1: Number of Heats, ASTM A 615 Grade 40 and 60 Bars

	A 615 Grade 40				A 615 Grade 60										
	Bar Size				Bar Size										
Mill ID	3	4	5	6	3	4	5	6	7	8	9	10	11	14	18
41					100	100	100								
45	19	73	18		24	26	19								
80						100	100								
184					13	100	100	100	56	79	55	24	53		
227		100	85			104	106	118	120	110	115	105	117	26	41
241		40	48			224	192	198	102	134	61	64	36		
252						12									
323						96	89	67	67	77	51	40	71		
437						325	282	193	95	135	186	105	133		
463	29				64	222	150	96	12	20					
539		162	32	69		134	253	157	151	220	119	130	123		
541	46	100	100		101	101	103	102	107	102	103	102	102		
561						291	271	154	68	73	341	258	259		
574					100	100	100								
575	68					90									
581					31	100	100	100	100	100	100	100	100		
638						120	86	85	67	83					
639	175	63	56	11	200	200	199	200	200	200	200	200	200	58	85
676		38	3			56	116	69	52	33	34	32	18	3	6
681	41				60	100	105	102	98	87	104	93	101		
690					56	124									
739		265	81	7		315	472	166	75	156	88	74	64		
770		68	14	73	4	100	100	100	100	100	100	100	100	30	
808	20	8			90	78									
829														77	90
874	54	64	12	12	194	240	46	28							
896						100	100	100	100	100	100	100	100		
947						100	100	100	100	100	100	100	100		
973		26	3			79	100	145	35	37	41	33	41		
Total	452	1007	452	172	1037	3737	3389	2380	1705	1946	1898	1660	1718	194	222

*To provide confidentiality, mills are identified by ID number.

Table 2: Number of Heats, ASTM A 615 Grade 75 and A 616 Grade 60 Bars

	A 615 Grade 75									A 616 Grade 60		
	Bar Size									Bar Size		
Mill ID	4	5	6	7	8	9	10	11	14		8	10
639		11	74	9	13	20	15	14				
681											13	6
770	38							2	1			
829									14			
Total	38	11	74	9	13	20	15	16	15		13	6

*To provide confidentiality, mills are identified by ID number.

Table 3: Number of Heats, ASTM A 706 Grade 60 Bars

	A 706 Grade 60										
	Bar Size										
Mill ID	3	4	5	6	7	8	9	10	11	14	18
227		108	106	109	71	101	91	82	105	105	12
541		90	125	70							
638		10									
639	15	26	27	20	21	15	8	3	7		
681				3	2	1	1				
770	1	17	17	9	6	9	6	2	3		
874	20	98	34	12							
Total	36	349	309	223	100	126	106	87	115	105	12

*To provide confidentiality, mills are identified by ID number.

Table 4: Yield Strength and Tensile Strength of A 615 Grade 40 Reinforcement

A 615 Grade 40	Bar Size							
	3		4		5		6	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	56798	82115	56113	81302	55782	82731	53376	81396
Median (psi)	56000	80000	55650	80000	55000	80650	53250	80500
Minimum (psi)	41672	69900	41050	70600	47097	70000	43227	72727
Maximum (psi)	80479	118000	76800	117000	78100	115800	72954	103659
Std. Deviation (psi)	5644	7717	4532	6717	4833	7649	4239	5765
Coeff. of Variation	0.0994	0.0940	0.0808	0.0826	0.0866	0.0925	0.0794	0.0708
5% Fractile (psi)	49255	73706	49500	73500	49700	74179	47113	75000
Kurtosis	1.997	2.953	1.017	3.108	0.877	1.177	3.995	3.782
Skewness	1.075	1.603	0.578	1.523	0.888	1.167	1.218	1.731
Number of Heats	452	452	1007	1007	452	452	172	172

A 615 Grade 40	Bar Size	
	All	
	Yield	Tensile
Mean (psi)	55968	81806
Median (psi)	55239	80000
Minimum (psi)	41050	69900
Maximum (psi)	80479	118000
Std. Deviation (psi)	4910	7116
Coeff. of Variation	0.0877	0.0870
5% Fractile (psi)	49338	73600
Kurtosis	1.644	2.665
Skewness	0.871	1.489
Number of Heats	2084	2084

Table 5: Elongation and Weight of A 615 Grade 40 Reinforcement

A 615 Grade 40	Bar Size							
	3		4		5		6	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	18.1%	-2.2%	19.0%	-3.2%	18.2%	-3.5%	19.6%	-3.7%
Minimum	9.0%	-5.5%	11.0%	-5.8%	12.0%	-5.7%	12.0%	-5.5%
Maximum	25.5%	5.5%	28.5%	5.3%	28.0%	0.1%	26.5%	-0.8%
Std. Deviation	3.1%	1.7%	2.8%	1.4%	3.0%	0.9%	2.9%	0.8%
Coeff. of Variation	0.1708	0.8078	0.1453	0.4407	0.1630	0.2497	0.1505	0.2229
Number of Heats	398	136	943	408	440	220	160	69

A 615 Grade 40	Bar Size
	All
	% Light
Mean	-3.2%
Minimum	-5.8%
Maximum	5.5%
Std. Deviation	1.4%
Coeff. of Variation	-0.4423
Number of Heats	833

Table 6: Yield Strength and Tensile Strength for A 615 Grade 60 Reinforcement

A 615 Grade 60	Bar Size							
	3		4		5		6	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	71941	109267	69677	105446	68891	105629	68933	106177
Median (psi)	70900	106400	68400	104500	67500	104951	67700	106000
Minimum (psi)	60190	90454	55370	80730	54258	87500	60000	90227
Maximum (psi)	100900	159800	100000	152000	97742	147700	96930	129000
Std. Deviation (psi)	6510	12935	5548	7117	5051	7104	4966	4984
Coeff. of Variation	0.0905	0.1184	0.0796	0.0675	0.0733	0.0673	0.0720	0.0469
5% Fractile (psi)	63298	95280	63153	96900	63234	97000	63182	98182
Kurtosis	1.572	3.184	2.779	7.916	2.749	9.575	2.444	0.671
Skewness	1.058	1.826	1.475	2.121	1.553	2.348	1.423	0.304
Number of Heats	1037	1037	3737	3737	3389	3389	2380	2380

A 615 Grade 60	Bar Size							
	7		8		9		10	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	69086	105344	69723	105147	69747	105598	69821	104873
Median (psi)	68200	105600	68500	105000	69011	105000	69152	104300
Minimum (psi)	60000	74000	60100	88500	60400	87500	60200	89000
Maximum (psi)	98300	125000	97468	131000	92846	130000	98500	126000
Std. Deviation (psi)	4489	5094	5155	5248	4307	5730	4127	5114
Coeff. of Variation	0.0650	0.0484	0.0739	0.0500	0.0618	0.0543	0.0591	0.0488
5% Fractile (psi)	63400	97174	63290	97100	64037	97265	64416	97254
Kurtosis	2.722	1.817	1.404	0.841	1.649	0.202	2.767	0.830
Skewness	1.215	-0.272	1.113	0.479	1.054	0.467	1.180	0.484
Number of Heats	1705	1705	1946	1946	1898	1898	1660	1660

A 615 Grade 60	Bar Size							
	11		14		18		All	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	70028	105155	70327	101052	72204	100738	69610	105572
Median (psi)	69500	104710	70503	99967	72400	99964	68600	105000
Minimum (psi)	60800	90064	63200	90300	65500	90112	54258	74000
Maximum (psi)	86979	124820	78000	119000	81500	115000	100900	159800
Std. Deviation (psi)	3786	5144	2532	5503	3105	4653	4976	6645
Coeff. of Variation	0.0541	0.0489	0.0360	0.0545	0.0430	0.0462	0.0715	0.0629
5% Fractile (psi)	64500	97585	66266	93564	67719	94000	63500	97000
Kurtosis	0.972	0.300	0.742	0.547	-0.555	-0.257	2.648	9.867
Skewness	0.738	0.420	0.244	0.815	0.204	0.281	1.335	2.051
Number of Heats	1718	1718	194	194	222	222	19886	19886

Table 7: Elongation and Weight of A 615 Grade 60 Reinforcement

A615 Grade 60	Bar Size							
	3		4		5		6	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	12.6%	-2.1%	13.0%	-3.2%	13.3%	-3.7%	13.0%	-3.6%
Minimum	6.0%	-5.5%	6.0%	-5.9%	2.0%	-5.9%	3.0%	-5.7%
Maximum	17.5%	4.6%	25.0%	6.7%	22.0%	2.5%	27.5%	3.8%
Std. Deviation	2.0%	1.5%	2.0%	1.3%	2.0%	1.0%	2.1%	1.0%
Coeff. of Variation	0.1552	0.7103	0.1544	0.4050	0.1524	0.2859	0.1628	0.2847
Number of Heats	843	371	3497	1039	3343	919	2352	729

A615 Grade 60	Bar Size							
	7		8		9		10	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	13.1%	-3.5%	13.2%	-3.6%	12.6%	-3.3%	12.5%	-3.6%
Minimum	2.0%	-5.8%	2.0%	-5.8%	2.0%	-5.8%	1.0%	-5.8%
Maximum	21.0%	3.3%	23.0%	3.3%	20.5%	0.0%	20.0%	-0.3%
Std. Deviation	2.4%	1.2%	2.4%	1.1%	2.3%	0.9%	2.5%	1.0%
Coeff. of Variation	0.1826	0.3373	0.1787	0.3081	0.1825	0.2883	0.2035	0.2725
Number of Heats	1705	607	1946	655	1898	475	1660	462

A615 Grade 60	Bar Size						
	11		14		18		All
	Elongation	% Light	Elongation	% Light	Elongation	% Light	% Light
Mean	12.3%	-3.4%	11.5%	-2.4%	12.1%	-2.8%	-3.4%
Minimum	3.0%	-5.6%	2.0%	-4.9%	3.0%	-5.1%	-5.9%
Maximum	27.5%	0.4%	21.0%	1.5%	23.0%	0.0%	6.7%
Std. Deviation	2.8%	0.9%	3.5%	1.6%	3.1%	0.9%	1.2%
Coeff. of Variation	0.2292	0.2739	0.3066	0.6561	0.2567	0.3210	0.3575
Number of Heats	1718	461	194	106	222	137	5961

Table 8: Yield Strength and Tensile Strength of A 615 Grade 75 Reinforcement

A 615 Grade 75	Bar Size							
	3		4		5		6	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	No Data		79458	110347	78640	111645	78518	112157
Median (psi)			79500	110750	78500	111200	78550	111400
Minimum (psi)			75500	104500	72300	107700	71900	104000
Maximum (psi)			85000	117000	86500	117000	87000	121800
Std. Deviation (psi)			2137	3002	4264	2985	3259	3883
Coeff. of Variation			0.0269	0.0272	0.0542	0.0267	0.0415	0.0346
5% Fractile (psi)			76410	105000	72950	108350	73600	106500
Kurtosis			0.038	0.057	-0.115	-0.599	0.093	0.480
Skewness			0.287	0.094	0.372	0.648	0.341	0.707
Number of Heats			38	38	11	11	74	74

A 615 Grade 75	Bar Size							
	7		8		9		10	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	85789	117100	83892	116262	84895	117420	82607	113027
Median (psi)	85800	117200	84400	117500	85400	118450	83500	113900
Minimum (psi)	81200	108700	76000	108700	78300	109500	75600	104100
Maximum (psi)	91700	124300	89800	122000	92700	127600	88200	119800
Std. Deviation (psi)	3946	5058	4259	4209	4337	4693	4106	4208
Coeff. of Variation	0.0460	0.0432	0.0508	0.0362	0.0511	0.0400	0.0497	0.0372
5% Fractile (psi)	81280	110140	76180	109300	78395	110070	75670	105990
Kurtosis	-1.080	-0.467	0.076	-0.525	-0.906	-0.014	-0.545	0.162
Skewness	0.333	-0.118	-0.711	-0.785	0.037	-0.115	-0.674	-0.625
Number of Heats	9	9	13	13	20	20	15	15

A 615 Grade 75	Bar Size							
	11		14		18		*All	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	84756	116275	82145	112792	No Data		81471	113957
Median (psi)	85650	116250	82315	113943			81100	113700
Minimum (psi)	79300	109700	78591	107662			71900	104000
Maximum (psi)	88700	123600	85777	117204			92700	127600
Std. Deviation (psi)	2712	4310	2045	3079			4460	4563
Coeff. of Variation	0.0320	0.0371	0.0250	0.0273			0.0547	0.0400
5% Fractile (psi)	79525	110525	79325	108390			74810	107664
Kurtosis	0.237	-0.922	-0.657	-1.249			-0.564	-0.381
Skewness	-0.983	0.145	0.060	-0.254			0.145	0.340
Number of Heats	16	16	15	15			162	162

***No. 4 and No. 5 bars are not permitted under ASTM A 615 and were not considered for values of all bars**

Table 9: Elongation and Weight of A 615 Grade 75 Reinforcement

A 615 Grade 75	Bar Size							
	3		4		5		6	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	No Data		11.9%	NA	12.2%	NA	12.3%	NA
Minimum			9.0%	NA	10.0%	NA	10.0%	NA
Maximum			16.0%	NA	14.5%	NA	16.0%	NA
Std. Deviation			1.6%	NA	1.5%	NA	1.6%	NA
Coeff. of Variation			0.1333	NA	0.1212	NA	0.1286	NA
Number of Heats			38	NA	11	NA	74	NA

A 615 Grade 75	Bar Size							
	7		8		9		10	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	12.8%	NA	12.2%	NA	12.0%	NA	11.0%	NA
Minimum	10.0%	NA	9.5%	NA	8.0%	NA	9.9%	NA
Maximum	15.1%	NA	16.0%	NA	15.0%	NA	14.0%	NA
Std. Deviation	1.8%	NA	1.7%	NA	2.4%	NA	1.4%	NA
Coeff. of Variation	0.1428	NA	0.1397	NA	0.2018	NA	0.1255	NA
Number of Heats	9	NA	13	NA	20	NA	15	NA

A 615 Grade 75	Bar Size							
	11		14		18		All	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	% Light	
Mean	10.4%	NA	13.6%	-1.5%	No Data		-1.5%	
Minimum	9.0%	NA	6.0%	-2.6%			-2.6%	
Maximum	12.9%	NA	16.6%	-0.5%			-0.5%	
Std. Deviation	0.9%	NA	2.6%	0.7%			0.7%	
Coeff. of Variation	0.0826	NA	0.1873	0.4416			0.4416	
Number of Heats	16	NA	15	14			14	

Table 10: Yield Strength and Tensile Strength of A 616 Grade 60 Reinforcement

A 616 Grade 60	Bar Size					
	8		10		All	
	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	67509	104367	64177	100204	66457	103052
Median (psi)	67478	104017	63608	99864	66560	103124
Minimum (psi)	65263	101587	62917	97553	62917	97553
Maximum (psi)	71900	108064	66930	104564	71900	108064
Std. Deviation (psi)	1763	1767	1503	2439	2287	2772
Coeff. of Variation	0.0261	0.0169	0.0234	0.0243	0.0344	0.0269
5% Fractile (psi)	65551	102330	62984	97787	63159	98396
Kurtosis	2.128	0.023	2.176	2.054	0.373	-0.3471
Skewness	1.233	0.556	1.555	1.228	0.367	-0.3639
Number of Heats	13	13	6	6	19	19

Table 11: Elongation and Weight of A 616 Grade 60 Reinforcement

A 616 Grade 60	Bar Size				
	8		10		All
	Elongation	% Light	Elongation	% Light	% Light
Mean	12.5%	-3.4%	14.5%	-3.8%	-3.5%
Minimum	8.5%	-4.0%	12.6%	-4.0%	-4.0%
Maximum	14.4%	-2.5%	15.5%	-3.5%	-2.5%
Std. Deviation	1.4%	0.5%	1.0%	0.2%	0.4%
Coeff. of Variation	0.1157	0.1340	0.0694	0.0574	0.1202
Number of Heats	13	13	6	6	19

Table 12: Yield Strength and Tensile Strength of A 706 Grade 60 Reinforcement

A 706 Grade 60	Bar Size							
	3		4		5		6	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	74021	99434	68440	94020	68590	93977	69289	95126
Median (psi)	74577	99227	67800	94350	68193	94000	69000	95000
Minimum (psi)	67300	91000	60000	80000	60300	81613	61545	83600
Maximum (psi)	78000	109363	85400	108000	78000	112935	77000	108000
Std. Deviation (psi)	3112	4364	4196	5340	3697	5129	3227	4783
Coeff. of Variation	0.0420	0.0439	0.0613	0.0568	0.0539	0.0546	0.0466	0.0503
5% Fractile (psi)	68575	93181	62540	85240	63080	86440	63910	86840
Kurtosis	-0.736	-0.802	-0.062	-0.505	-0.444	0.101	-0.301	-0.220
Skewness	-0.564	0.185	0.539	-0.060	0.265	0.408	-0.019	0.096
Number of Heats	36	36	349	349	309	309	223	223

A 706 Grade 60	Bar Size							
	7		8		9		10	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	69622	96319	68891	95662	69010	96180	70232	96868
Median (psi)	69750	96500	68750	95158	68500	96000	70000	97000
Minimum (psi)	62500	85300	62000	85600	63500	85200	63100	84900
Maximum (psi)	78000	109000	77000	116000	78000	114000	78000	109000
Std. Deviation (psi)	3031	4400	3318	4425	3473	4659	3121	4255
Coeff. of Variation	0.0435	0.0458	0.0482	0.0463	0.0503	0.0484	0.0444	0.0439
5% Fractile (psi)	64760	89500	64000	89000	64000	89625	64500	91500
Kurtosis	0.011	0.263	-0.472	2.609	0.075	1.840	0.194	0.650
Skewness	0.154	0.220	0.286	0.885	0.639	0.790	0.088	0.281
Number of Heats	100	100	126	126	106	106	87	87

A 706 Grade 60	Bar Size							
	11		14		18		All	
	Yield	Tensile	Yield	Tensile	Yield	Tensile	Yield	Tensile
Mean (psi)	69521	96161	69462	95848	72167	99458	69144	95196.9
Median (psi)	69000	95500	69000	95500	71750	99000	69000	95000
Minimum (psi)	63500	86300	63000	87500	67500	95500	60000	80000
Maximum (psi)	78000	107000	78000	109000	76500	105000	85400	116000
Std. Deviation (psi)	2984	3725	3575	4281	2614	2872	3678	4925
Coeff. of Variation	0.0429	0.0387	0.0515	0.0447	0.0362	0.0289	0.0532	0.0517
5% Fractile (psi)	65000	91440	64500	89600	68600	95775	63500	87000
Kurtosis	-0.055	0.637	-0.865	0.095	-0.374	-0.394	-0.276	0.211
Skewness	0.356	0.677	0.190	0.480	0.080	0.518	0.255	0.137
Number of Heats	115	115	105	105	12	12	1568	1568

Table 13: Elongation and Weight of A 706 Grade 60 Reinforcement

A 706 Grade 60	Bar Size							
	3		4		5		6	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	14.8%	NA	15.4%	-3.4%	15.5%	-3.2%	15.2%	-3.3%
Minimum	14.0%	NA	10.0%	-5.7%	11.0%	-5.7%	6.0%	-5.2%
Maximum	17.0%	NA	19.0%	1.3%	19.0%	1.4%	21.0%	1.3%
Std. Deviation	0.9%	NA	1.4%	1.1%	1.6%	1.0%	1.9%	0.9%
Coeff. of Variation	0.0624	NA	0.0883	0.3387	0.1010	0.3191	0.1281	0.2809
Number of Heats	16	NA	251	208	275	231	211	182

A 706 Grade 60	Bar Size							
	7		8		9		10	
	Elongation	% Light	Elongation	% Light	Elongation	% Light	Elongation	% Light
Mean	15.2%	-3.1%	15.6%	-3.2%	15.8%	-3.6%	15.3%	-3.5%
Minimum	4.0%	-5.9%	10.0%	-5.1%	12.0%	-5.3%	9.0%	-5.3%
Maximum	20.0%	0.8%	20.0%	3.6%	20.0%	-0.4%	20.0%	0.4%
Std. Deviation	2.3%	1.5%	2.0%	1.5%	2.0%	1.0%	2.1%	1.0%
Coeff. of Variation	0.1519	0.4867	0.1310	0.4654	0.1273	0.2829	0.1339	0.2828
Number of Heats	100	73	126	102	106	92	87	82

A 706 Grade 60	Bar Size						
	11		14		18		All
	Elongation	% Light	Elongation	% Light	Elongation	% Light	% Light
Mean	14.9%	-3.2%	12.8%	-2.8%	12.2%	-2.7%	-3.3%
Minimum	10.0%	-4.9%	6.0%	-5.1%	10.0%	-5.2%	-5.9%
Maximum	19.3%	-0.3%	18.0%	-0.8%	15.0%	-1.2%	3.6%
Std. Deviation	1.9%	1.0%	2.4%	0.9%	1.5%	1.0%	1.1%
Coeff. of Variation	0.1250	0.3141	0.1874	0.3292	0.1206	0.3605	0.3432
Number of Heats	115	105	105	105	12	12	1192

Table 14: Beta Probability Density Functions for Yield Strength of A 615 Grade 40 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 615 Grade 40		
Bar Size		
No. 3	<i>C</i>	19629
	<i>α</i>	6.89
	<i>β</i>	38.11
	<i>LB</i>	41672
	<i>UB</i>	130000
	<i>D</i>	88328
No. 4	<i>C</i>	19629
	<i>α</i>	7.73
	<i>β</i>	30.32
	<i>LB</i>	41050
	<i>UB</i>	110000
	<i>D</i>	68950
No. 5	<i>C</i>	19616
	<i>α</i>	2.28
	<i>β</i>	3733.04
	<i>LB</i>	47097
	<i>UB</i>	10000000
	<i>D</i>	9952903
No. 6	<i>C</i>	19633
	<i>α</i>	5.49
	<i>β</i>	61.60
	<i>LB</i>	43227
	<i>UB</i>	150000
	<i>D</i>	106773
All	<i>C</i>	19629
	<i>α</i>	6.86
	<i>β</i>	38.39
	<i>LB</i>	41050
	<i>UB</i>	130000
	<i>D</i>	88950

Table 15: Beta Probability Density Functions for Yield Strength of A 615 Grade 60 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 615 Grade 60								
Bar Size			Bar Size			Bar Size		
No. 3	<i>C</i>	37336	No. 8	<i>C</i>	37336	No. 18	<i>C</i>	37185
	α	2.81		α	2.82		α	2.66
	β	1278.46		β	1176.67		β	1511.02
	<i>LB</i>	60190		<i>LB</i>	60100		<i>LB</i>	65500
	<i>UB</i>	4000000		<i>UB</i>	3000000		<i>UB</i>	3000000
	<i>D</i>	3939810		<i>D</i>	2939900		<i>D</i>	2934500
No. 4	<i>C</i>	37336	No. 9	<i>C</i>	37337	All	<i>C</i>	37337
	α	2.68		α	3.73		α	3.00
	β	1609.53		β	269.29		β	813.21
	<i>LB</i>	60000		<i>LB</i>	60400		<i>LB</i>	60000
	<i>UB</i>	4200000		<i>UB</i>	600000		<i>UB</i>	2000000
	<i>D</i>	4140000		<i>D</i>	539600		<i>D</i>	1940000
No. 5	<i>C</i>	37337	No. 10	<i>C</i>	37090			
	α	3.30		α	4.47			
	β	468.28		β	131.50			
	<i>LB</i>	60000		<i>LB</i>	60200			
	<i>UB</i>	1000000		<i>UB</i>	300000			
	<i>D</i>	940000		<i>D</i>	239800			
No. 6	<i>C</i>	37336	No. 11	<i>C</i>	37186			
	α	2.84		α	5.09			
	β	1073.24		β	84.94			
	<i>LB</i>	60000		<i>LB</i>	60800			
	<i>UB</i>	2500000		<i>UB</i>	200000			
	<i>D</i>	2440000		<i>D</i>	139200			
No. 7	<i>C</i>	37337	No. 14	<i>C</i>	37186			
	α	3.34		α	5.42			
	β	447.83		β	67.80			
	<i>LB</i>	60000		<i>LB</i>	63200			
	<i>UB</i>	1000000		<i>UB</i>	150000			
	<i>D</i>	940000		<i>D</i>	86800			

Table 16: Beta Probability Density Functions for Yield Strength of A 615 Grade 75 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 615 Grade 75					
Bar Size			Bar Size		
No. 4	<i>C</i>	37332	No. 9	<i>C</i>	37333
	α	2.34		α	2.60
	β	3015.56		β	1815.87
	<i>LB</i>	75500		<i>LB</i>	78300
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3924500		<i>D</i>	3921700
No. 5	<i>C</i>	37314	No. 10	<i>C</i>	37336
	α	1.99		α	2.72
	β	8131.26		β	1397.70
	<i>LB</i>	77700		<i>LB</i>	75600
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3922300		<i>D</i>	3924400
No. 6	<i>C</i>	37332	No. 11	<i>C</i>	37334
	α	2.37		α	2.61
	β	2979.74		β	1645.29
	<i>LB</i>	75000		<i>LB</i>	79300
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3925000		<i>D</i>	3920700
No. 7	<i>C</i>	37334	No. 14	<i>C</i>	37331
	α	2.75		α	2.32
	β	1348.80		β	3132.38
	<i>LB</i>	75500		<i>LB</i>	78591
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3924500		<i>D</i>	3921409
No. 8	<i>C</i>	37336	All	<i>C</i>	37335
	α	2.77		α	2.59
	β	1299.74		β	1882.86
	<i>LB</i>	76000		<i>LB</i>	75000
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3924000		<i>D</i>	3925000

Table 17: Beta Probability Density Functions for Yield Strength of A 616 Grade 60 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^{\alpha} \times \left(\frac{UB - f}{D} \right)^{\beta}$$

A 616 Grade 60		
Bar Size		
No. 8	<i>C</i>	37186
	<i>α</i>	3.41
	<i>β</i>	275.26
	<i>LB</i>	65263
	<i>UB</i>	200000
	<i>D</i>	134737
No. 10	<i>C</i>	37178
	<i>α</i>	2.07
	<i>β</i>	3844.18
	<i>LB</i>	62917
	<i>UB</i>	1000000
	<i>D</i>	937083
All	<i>C</i>	37186
	<i>α</i>	3.93
	<i>β</i>	174.22
	<i>LB</i>	62917
	<i>UB</i>	200000
	<i>D</i>	137083

Table 18: Beta Probability Density Functions for Yield Strength of A 706 Grade 60 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 706 Grade 60								
Bar Size			Bar Size			Bar Size		
No. 3	<i>C</i>	37186	No. 8	<i>C</i>	37184	No. 18	<i>C</i>	37181
	α	2.63		α	2.50		α	2.37
	β	1567.61		β	2298.32		β	3079.87
	<i>LB</i>	67300		<i>LB</i>	62000		<i>LB</i>	67500
	<i>UB</i>	5000000		<i>UB</i>	5000000		<i>UB</i>	5000000
	<i>D</i>	4932700		<i>D</i>	4938000		<i>D</i>	4932500
No. 4	<i>C</i>	37184	No. 9	<i>C</i>	37182	All	<i>C</i>	37185
	α	2.57		α	2.38		α	2.63
	β	2043.13		β	2935.64		β	1793.25
	<i>LB</i>	60000		<i>LB</i>	63500		<i>LB</i>	60000
	<i>UB</i>	5000000		<i>UB</i>	5000000		<i>UB</i>	5000000
	<i>D</i>	4940000		<i>D</i>	4936500		<i>D</i>	4940000
No. 5	<i>C</i>	37185	No. 10	<i>C</i>	37184			
	α	2.59		α	2.53			
	β	1956.71		β	2151.87			
	<i>LB</i>	60300		<i>LB</i>	63100			
	<i>UB</i>	5000000		<i>UB</i>	5000000			
	<i>D</i>	4939700		<i>D</i>	4936900			
No. 6	<i>C</i>	37185	No. 11	<i>C</i>	37183			
	α	2.57		α	2.44			
	β	1985.67		β	2623.11			
	<i>LB</i>	61545		<i>LB</i>	63500			
	<i>UB</i>	5000000		<i>UB</i>	5000000			
	<i>D</i>	4938455		<i>D</i>	4936500			
No. 7	<i>C</i>	37184	No. 14	<i>C</i>	37183			
	α	2.52		α	2.47			
	β	2211.44		β	2451.52			
	<i>LB</i>	62500		<i>LB</i>	63000			
	<i>UB</i>	5000000		<i>UB</i>	5000000			
	<i>D</i>	4937500		<i>D</i>	4937000			

Table 19: Beta Probability Density Functions for Tensile Strength of A 615 Grade 40 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 615 Grade 40		
Bar Size		
No. 3	<i>C</i>	19628
	<i>α</i>	2.72
	<i>β</i>	1247.95
	<i>LB</i>	69900
	<i>UB</i>	4000000
	<i>D</i>	3930100
No. 4	<i>C</i>	19628
	<i>α</i>	2.77
	<i>β</i>	1062.89
	<i>LB</i>	70600
	<i>UB</i>	3000000
	<i>D</i>	2929400
No. 5	<i>C</i>	19623
	<i>α</i>	2.43
	<i>β</i>	2728.52
	<i>LB</i>	70000
	<i>UB</i>	10000000
	<i>D</i>	9930000
No. 6	<i>C</i>	19633
	<i>α</i>	3.18
	<i>β</i>	462.79
	<i>LB</i>	72727
	<i>UB</i>	1000000
	<i>D</i>	927273
All	<i>C</i>	19628
	<i>α</i>	2.71
	<i>β</i>	1262.11
	<i>LB</i>	69900
	<i>UB</i>	4000000
	<i>D</i>	3930100

Table 20: Beta Probability Density Functions for Tensile Strength of A 615 Grade 60 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 615 Grade 60								
Bar Size			Bar Size			Bar Size		
No. 3	<i>C</i>	37337	No. 8	<i>C</i>	37337	No. 18	<i>C</i>	37186
	α	2.87		α	6.85		α	2.91
	β	1308.19		β	43.52		β	1014.23
	<i>LB</i>	90454		<i>LB</i>	88500		<i>LB</i>	90112
	<i>UB</i>	6000000		<i>UB</i>	200000		<i>UB</i>	3000000
	<i>D</i>	5909546		<i>D</i>	111500		<i>D</i>	2909888
No. 4	<i>C</i>	37337	No. 9	<i>C</i>	37337	All	<i>C</i>	37338
	α	10.34		α	7.07		α	11.09
	β	21.62		β	41.07		β	20.17
	<i>LB</i>	80730		<i>LB</i>	87500		<i>LB</i>	74000
	<i>UB</i>	153000		<i>UB</i>	200000		<i>UB</i>	160000
	<i>D</i>	72270		<i>D</i>	112500		<i>D</i>	86000
No. 5	<i>C</i>	37337	No. 10	<i>C</i>	37090			
	α	9.18		α	6.72			
	β	25.01		β	45.11			
	<i>LB</i>	87500		<i>LB</i>	89000			
	<i>UB</i>	150000		<i>UB</i>	200000			
	<i>D</i>	62500		<i>D</i>	111000			
No. 6	<i>C</i>	37337	No. 11	<i>C</i>	37186			
	α	6.78		α	6.60			
	β	44.28		β	46.62			
	<i>LB</i>	90227		<i>LB</i>	90064			
	<i>UB</i>	200000		<i>UB</i>	200000			
	<i>D</i>	109773		<i>D</i>	109936			
No. 7	<i>C</i>	37337	No. 14	<i>C</i>	37186			
	α	18.12		α	4.09			
	β	11.59		β	189.20			
	<i>LB</i>	74000		<i>LB</i>	90300			
	<i>UB</i>	126000		<i>UB</i>	500000			
	<i>D</i>	52000		<i>D</i>	409700			

Table 21: Beta Probability Density Functions for Tensile Strength of A 615 Grade 75 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 615 Grade 75					
Bar Size			Bar Size		
No. 4	<i>C</i>	37335	No. 9	<i>C</i>	37334
	α	2.53		α	2.70
	β	2060.81		β	1548.34
	<i>LB</i>	104500		<i>LB</i>	109500
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3895500		<i>D</i>	3890500
No. 5	<i>C</i>	37296	No. 10	<i>C</i>	37336
	α	2.31		α	2.81
	β	3375.73		β	1227.86
	<i>LB</i>	107700		<i>LB</i>	104100
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3892300		<i>D</i>	3895900
No. 6	<i>C</i>	37335	No. 11	<i>C</i>	37334
	α	2.65		α	2.60
	β	1645.55		β	1840.15
	<i>LB</i>	104000		<i>LB</i>	109700
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3896000		<i>D</i>	3890300
No. 7	<i>C</i>	37334	No. 14	<i>C</i>	37335
	α	2.75		α	2.55
	β	1442.03		β	1913.87
	<i>LB</i>	108700		<i>LB</i>	107662
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3891300		<i>D</i>	3892338
No. 8	<i>C</i>	37336	All	<i>C</i>	37336
	α	2.79		α	2.75
	β	1232.33		β	1391.93
	<i>LB</i>	108700		<i>LB</i>	104000
	<i>UB</i>	4000000		<i>UB</i>	4000000
	<i>D</i>	3891300		<i>D</i>	3896000

Table 22: Beta Probability Density Functions for Tensile Strength of A 616 Grade 60 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 616 Grade 60		
Bar Size		
No. 8	<i>C</i>	37186
	<i>α</i>	3.85
	<i>β</i>	174.75
	<i>LB</i>	101587
	<i>UB</i>	200000
	<i>D</i>	98413
No. 10	<i>C</i>	37186
	<i>α</i>	2.60
	<i>β</i>	1323.20
	<i>LB</i>	97553
	<i>UB</i>	1000000
	<i>D</i>	902447
All	<i>C</i>	37186
	<i>α</i>	4.88
	<i>β</i>	90.45
	<i>LB</i>	97553
	<i>UB</i>	200000
	<i>D</i>	102447

Table 23: Beta Probability Density Functions for Tensile Strength of A 706 Grade 60 Reinforcement

$$\text{PDF} = C \times \left(\frac{f - LB}{D} \right)^\alpha \times \left(\frac{UB - f}{D} \right)^\beta$$

A 706 Grade 60								
Bar Size			Bar Size			Bar Size		
No. 3	<i>C</i>	37185	No. 8	<i>C</i>	37185	No. 18	<i>C</i>	37177
	α	2.60		α	2.66		α	2.26
	β	1948.94		β	1749.49		β	3994.01
	<i>LB</i>	91000		<i>LB</i>	85600		<i>LB</i>	95500
	<i>UB</i>	5000000		<i>UB</i>	5000000		<i>UB</i>	5000000
	<i>D</i>	4909000		<i>D</i>	4914400		<i>D</i>	4904500
No. 4	<i>C</i>	37186	No. 9	<i>C</i>	37186	All	<i>C</i>	37187
	α	2.87		α	2.71		α	6.44
	β	1207.68		β	1594.56		β	49.50
	<i>LB</i>	80000		<i>LB</i>	85200		<i>LB</i>	80000
	<i>UB</i>	5000000		<i>UB</i>	5000000		<i>UB</i>	200000
	<i>D</i>	4920000		<i>D</i>	4914800		<i>D</i>	120000
No. 5	<i>C</i>	37186	No. 10	<i>C</i>	37186			
	α	2.77		α	2.78			
	β	1437.47		β	1406.06			
	<i>LB</i>	81613		<i>LB</i>	84900			
	<i>UB</i>	5000000		<i>UB</i>	5000000			
	<i>D</i>	4918387		<i>D</i>	4915100			
No. 6	<i>C</i>	37186	No. 11	<i>C</i>	37185			
	α	2.76		α	2.67			
	β	1460.97		β	1701.15			
	<i>LB</i>	83600		<i>LB</i>	86300			
	<i>UB</i>	5000000		<i>UB</i>	5000000			
	<i>D</i>	4916400		<i>D</i>	4913700			
No. 7	<i>C</i>	37186	No. 14	<i>C</i>	37184			
	α	2.73		α	2.57			
	β	1524.90		β	2030.04			
	<i>LB</i>	85300		<i>LB</i>	87500			
	<i>UB</i>	5000000		<i>UB</i>	5000000			
	<i>D</i>	4914700		<i>D</i>	4912500			

Table 24: Average Yield Strengths for Individual Mills, A 615 Grade 40 and 60 Bars, ksi

	A 615 Grade 40					A 615 Grade 60											
	Bar Size					Bar Size											
Mill ID	3	4	5	6	Avg.	3	4	5	6	7	8	9	10	11	14	18	Avg.
41						84.0	79.5	79.9									81.1
45	56.2	53.5	54.7		54.8	72.7	66.3	65.8									68.3
80							71.1	69.6									70.3
184						69.3	65.3	66.1	66.4	66.4	65.2	69.4	67.6	68.2			67.1
227		58.6	59.1		58.8		70.7	72.5	73.0	72.2	70.2	72.0	70.7	71.2	70.4	73.0	71.6
241		61.9	58.9		60.4		72.0	69.9	69.5	68.1	68.6	66.2	69.4	67.7			68.9
252							74.7										74.7
323							70.7	67.3	67.9	65.8	68.0	66.7	66.8	67.1			67.5
435						69.3	69.1	67.3	68.6	68.6	68.1	68.7	68.9	67.7	68.2		68.5
437							67.6	69.4	70.4	69.2	72.4	72.5	74.9	74.6			71.4
463	66.6				66.6	71.1	70.1	68.8	69.3	66.9	67.8						69.0
539		59.3	51.0	53.9	54.7		84.9	78.6	80.6	77.3	79.4	78.7	77.1	75.3			79.0
541	55.5	51.2	52.1		52.9	66.2	64.5	67.3	66.7	65.8	67.2	67.4	67.6	67.3			66.7
561							65.4	65.1	64.6	69.1	68.1	67.7	69.2	71.0			67.5
574						71.9	68.8	66.5									69.1
575	53.9				53.9		67.8										67.8
581						70.9	69.1	67.8	68.9	66.9	70.6	69.6	69.3	68.5			69.1
638							67.6	66.3	65.1	65.9	66.2						66.2
639	57.6	56.5	57.4	56.3	57.0	69.6	68.1	68.1	67.7	67.7	69.0	68.0	68.9	68.9	70.3	73.9	69.1
676		55.7	52.3		54.0		70.5	68.9	69.4	68.8	66.0	69.3	67.0	68.2	71.2	70.3	69.0
681	58.7				58.7	72.4	67.7	66.4	67.2	70.7	69.6	69.2	67.5	68.7			68.8
687	52.6	53.4	54.0		53.3	68.0	67.2	68.7	66.9	70.8	68.6	71.2	71.4	69.9	73.1	73.2	69.9
690						74.9	70.9										72.9
739		55.3	55.5	54.9	55.2		65.8	66.1	65.7	66.0	66.3	71.8	70.5	69.8			67.7
770		57.3	60.3	52.8	56.8	68.1	69.7	68.5	67.1	72.2	70.4	70.4	69.7	68.6	71.1		69.6
808	59.6	54.9			57.3	75.7	71.3										73.5
829															70.0	70.4	70.2
845	57.2	54.2	55.7		55.7	80.9	68.0	65.4	65.8	66.7	66.6	66.6	66.7	65.6			68.1
874	51.4	50.7	51.5	50.2	51.0	69.1	74.9	69.8	66.9								70.2
896							66.7	67.3	68.0	67.9	67.1	69.0	68.8	69.2			68.0
947							68.0	67.1	68.0	67.4	66.5	66.5	66.6	68.5			67.3
973		63.3	65.9		64.6		71.0	68.9	70.0	69.0	70.7	71.5	67.9	69.1			69.8
Avg.	56.9	56.1	56.0	53.6		72.1	69.8	68.6	68.4	68.6	68.8	69.6	69.3	69.3	70.6	72.2	

Table 25: Average Yield Strengths for Individual Mills, A 615 Grade 75 and A 616 Grade 60 Bars, ksi

	A 615 Grade 75										A 616 Grade 60		
	Bar Size										Bar Size		
Mill ID	4	5	6	7	8	9	10	11	14	*Avg.	8	10	Avg.
639		78.6	78.5	85.8	83.9	84.9	82.6	85.4		83.5			
681											67.5	64.2	65.8
687					82.7	83.7	81.7			82.7			
770	79.5							80.3	85.8	83.0			
829									81.9	81.9			
Avg.	79.5	78.6	78.5	85.8	83.3	84.3	82.1	82.8	83.8		67.5	64.2	

*No. 4 and No. 5 bars are not permitted under ASTM A 615 and are not considered in these average values

Table 26: Average Yield Strengths for Individual Mills, A 706 Grade 60 Bars, ksi

	A 706 Grade 60											
	Bar Size											
Mill ID	3	4	5	6	7	8	9	10	11	14	18	Avg.
227		67.3	69.1	70.3	69.3	68.7	69.1	70.3	69.5	69.5	72.2	69.5
435		70.4	70.0	71.7	69.2	70.3	70.8	71.8	69.8	72.6		70.7
541		65.5	66.5	67.8								66.6
638		63.9										63.9
639	73.4	71.3	71.1	71.3	70.7	70.5	67.7	67.8	70.7			70.5
681				71.7	69.6	68.2	68.4					69.5
687		66.9	69.2	71.4	66.0	70.3	69.8	68.6	67.9	66.3	65.3	68.2
770	67.3	68.5	72.0	69.0	70.0	68.3	68.9	69.7	67.4			69.0
845		68.9	67.9									68.4
874	74.8	72.1	70.8	65.4								70.8
Avg.	71.8	68.3	69.6	69.8	69.1	69.4	69.1	69.6	69.0	69.5	68.7	

Table 27: Average Tensile Strengths for Individual Mills, A 615 Grade 40 and 60 Bars, ksi

	A 615 Grade 40					A 615 Grade 60												
	Bar Size					Bar Size												
Mill ID	3	4	5	6	Avg.	3	4	5	6	7	8	9	10	11	14	18	Avg.	
41						143.4	135.1	134.8									137.8	
45	82.8	78.9	81.6		81.1	107.9	101.8	101.5									103.7	
80							109.7	108.3									109.0	
184						103.0	99.9	104.2	105.8	106.3	104.8	110.2	101.5	102.5			104.3	
227		87.6	88.8		88.2		108.5	110.0	111.0	108.0	108.8	111.2	110.3	110.3	108.3	104.4	109.1	
241		87.2	84.6		85.9		107.7	107.4	108.0	106.9	108.9	109.0	106.6	106.9			107.7	
252							116.4										116.4	
323							104.0	101.1	103.0	98.9	99.5	100.9	100.3	101.0			101.1	
435						106.7	103.1	104.5	105.9	107.3	105.8	108.6	108.4	106.9	104.1		106.1	
437							103.5	106.1	108.8	107.9	109.4	110.7	111.3	112.0			108.7	
463	96.1				96.1	100.9	102.7	100.7	101.6	100.2	101.5						101.3	
539		79.5	78.4	82.7	80.2		100.2	99.4	100.6	100.6	102.1	102.9	101.2	100.6			101.0	
541	79.4	76.1	77.1		77.5	97.9	99.6	103.9	102.3	102.0	101.6	102.1	103.5	103.2			101.8	
561							105.5	103.8	106.6	101.6	101.6	99.8	102.8	103.5			103.1	
574						108.5	104.4	102.3									105.0	
575	78.3				78.3		99.4										99.4	
581						108.9	106.6	106.0	107.9	107.0	106.3	106.4	106.6	106.6			106.9	
638							103.6	102.8	101.4	103.7	101.2						102.5	
639	82.1	84.0	84.7	85.0	84.0	106.4	106.3	107.1	107.4	107.7	107.9	107.8	104.4	103.6	100.9	101.5	105.6	
676		79.1	75.9		77.5		108.6	107.2	109.5	108.9	106.4	112.1	108.9	109.1	105.9	101.7	107.8	
681	87.2				87.2	111.1	106.4	106.1	108.8	107.6	105.9	106.0	103.2	106.4			106.9	
687	77.2	81.4	81.5		80.0	103.4	106.2	107.3	105.7	108.8	108.5	107.2	108.6	105.0	101.2	104.4	106.0	
690						107.6	102.2										104.9	
739		80.1	81.9	80.6	80.9		103.0	104.2	104.4	106.2	105.6	110.8	110.5	109.3			106.7	
770		85.8	91.9	79.9	85.9	104.2	108.9	106.2	107.7	106.4	105.1	105.1	102.6	102.8	102.4		105.1	
808	84.0	79.9			81.9	112.2	110.7										111.4	
829															98.0	98.3	98.1	
845	83.0	79.8	81.2		81.4	114.1	103.5	102.6	104.9	106.2	107.0	106.6	108.5	108.0			106.8	
874	77.0	76.0	75.0	79.7	76.9	103.0	103.7	102.8	103.9								103.3	
896							104.2	106.3	104.5	105.4	102.9	103.4	103.0	103.4			104.2	
947							104.8	104.5	105.7	106.4	106.6	105.6	106.6	108.8			106.1	
973		97.2	102.7		100.0		109.0	108.6	108.7	100.3	101.9	102.2	99.6	99.9			103.8	
Avg.	82.7	82.3	83.5	81.6		108.7	106.1	105.9	105.8	105.2	105.0	106.4	105.4	105.5	103.0	102.1		

Table 28: Average Tensile Strengths for Individual Mills, A 615 Grade 75 and A 616 Grade 60 Bars, ksi

	A 615 Grade 75										A 616 Grade 60		
	Bar Size										Bar Size		
Mill ID	4	5	6	7	8	9	10	11	14	*Avg.	8	10	Avg.
639		111.6	112.2	117.1	116.3	117.4	113.0	116.6		115.4			
681											104.4	100.2	102.3
687					119.9	121.3	119.1			120.1			
770	110.3							114.1	117.2	115.7			
829									112.5	112.5			
Avg.	110.3	111.6	112.2	117.1	118.1	119.3	116.1	115.3	114.8		104.4	100.2	

*No. 4 and No. 5 bars are not permitted under ASTM A 615 and are not considered in these average values

Table 29: Average Tensile Strengths for Individual Mills, A 706 Grade 60 Bars, ksi

	A 706 Grade 60											
	Bar Size											
Mill ID	3	4	5	6	7	8	9	10	11	14	18	Avg.
227		96.2	96.2	97.7	97.3	95.9	96.6	97.1	96.5	95.8	99.5	96.9
435		92.5	92.5	95.4	91.6	94.7	96.0	96.5	95.0	100.1		94.9
541		88.0	90.5	91.2								89.9
638		89.8										89.8
639	98.1	94.0	92.2	93.2	91.9	94.6	90.6	90.3	91.6			92.9
681				98.9	97.8	96.1	97.0					97.5
687		90.2	92.8	93.8	101.2	98.9	97.0	95.3	93.8	93.5	103.6	96.0
770	96.4	96.8	100.9	99.3	99.6	95.2	97.8	97.1	95.6			97.6
845		90.2	85.2									87.7
874	100.6	97.1	97.6	93.9								97.3
Avg.	98.4	92.8	93.5	95.4	96.6	95.9	95.8	95.2	94.5	96.5	101.5	

**Table 30: Average Elongations for Individual Mills, A 615 Grade 40
and 60 Bars, %**

	A 615 Grade 40				A 615 Grade 60										
	Bar Size				Bar Size										
Mill ID	3	4	5	6	3	4	5	6	7	8	9	10	11	14	18
41					9.1	7.8	8.0								
45	19.7	20.4	18.1		12.3	14.3	13.7								
80						12.4	12.9								
184					12.8	13.1	12.8	13.5	13.5	13.3	13.2	12.7	14.3		
227		16.8	16.2			11.6	11.3	11.2	11.0	11.3	10.8	11.0	10.1	8.1	9.6
241		19.3	17.8			14.9	15.2	14.6	14.7	13.7	12.6	13.3	12.9		
252						11.2									
323						15.7	15.5	15.3	15.7	15.4	15.7	16.4	16.3		
435					12.8	12.9	13.4	12.9	12.2	12.1	11.6	10.9	10.2	9.4	
437						12.9	12.9	11.8	12.1	12.2	12.3	12.1	11.9		
463	14.4				13.6	14.1	14.2	12.9	13.0	12.6					
539		18.9	19.8	20.9		12.0	14.1	14.7	16.0	15.9	15.2	15.6	16.9		
541	18.1	17.8	19.1		13.1	11.7	12.0	11.9	11.5	12.6	10.6	10.1	8.7		
561						11.5	13.8	11.8	13.0	13.0	13.0	11.7	12.3		
574					13.9	14.2	14.6								
575	16.7					12.4									
581					12.0	12.1	11.8	11.7	12.2	12.1	12.1	12.2	11.3		
638						12.5	13.4	11.1	12.6	11.0					
639	18.8	18.2	17.2	16.2	12.9	12.8	12.8	12.7	12.8	12.7	12.1	12.6	12.2	14.2	14.2
676		23.1	22.7			13.3	13.5	12.7	12.5	12.7	11.8	12.4	10.8	12.5	10.3
681	19.5				13.2	13.5	13.1	13.1	12.2	13.5	13.1	14.2	13.7		
687	21.0	19.0	19.0		14.0	13.0	13.0	14.0	13.0	12.0	12.0	11.0	12.0	14.0	13.0
690					13.1	13.8									
739		20.1	19.6	21.0		13.6	13.4	13.6	12.5	12.4	11.1	11.5	10.2		
770		17.8	16.4	18.6	13.0	12.7	12.0	11.4	12.1	12.3	11.8	10.3	10.1	9.1	
808	17.4	19.9			12.8	13.7									
829														11.6	11.3
845	19.0	19.0	19.0		13.0	13.0	14.0	15.0	15.0	15.0	15.0	14.0	15.0		
896						14.4	14.2	15.6	15.1	16.1	13.3	13.2	13.3		
947						12.5	13.3	13.4	13.1	13.1	13.0	13.3	12.4		
973		16.2	15.0			13.0	13.5	13.2	14.7	11.9	12.9	13.1	11.8		
Avg.	18.3	19.0	18.3	19.2	12.8	12.9	13.2	13.1	13.2	13.0	12.7	12.6	12.3	11.3	11.7

**Table 31: Average Elongations for Individual Mills, A 615 Grade 75
and A 616 Grade 60 Bars, %**

	A 615 Grade 75									A 616 Grade 60	
	Bar Size									Bar Size	
Mill ID	4	5	6	7	8	9	10	11	14	8	10
639		12.2	12.3	12.8	12.2	12.0	11.0	10.5			
681										12.5	14.5
687					12.0	12.0	13.0				
770	11.9							10.0	6.0		
829									14.2		
Avg.	11.9	12.2	12.3	12.8	12.1	12.0	12.0	10.2	10.1	12.5	14.5

**Table 32: Average Elongations for Individual Mills, A 706 Grade
60 Bars, %**

	A 706 Grade 60										
	Bar Size										
Mill ID	3	4	5	6	7	8	9	10	11	14	18
227		15.5	15.4	14.6	14.9	15.7	15.9	15.3	14.9	12.8	12.2
435		16.2	16.5	17.1	17.7	16.3	18.0	16.6	17.0	14.0	
541		15.2	15.5	15.6							
638		16.8									
639	14.8	15.7	16.5	16.9	16.7	15.5	16.1	15.7	16.2		
681				17.7	14.8	19.9	15.0				
687		17.0	17.0	17.0	18.0	15.0	17.0	14.0	16.0	13.0	12.0
770	15.0	15.5	14.4	14.6	14.0	14.3	14.5	14.0	12.0		
845		17.0	18.0								
Avg.	14.9	16.1	16.2	16.2	16.0	16.1	16.1	15.1	15.2	13.3	12.1

Table 33: Percent of Nominal Weights for Individual Mills, A 615 Grade 40 and 60 Bars

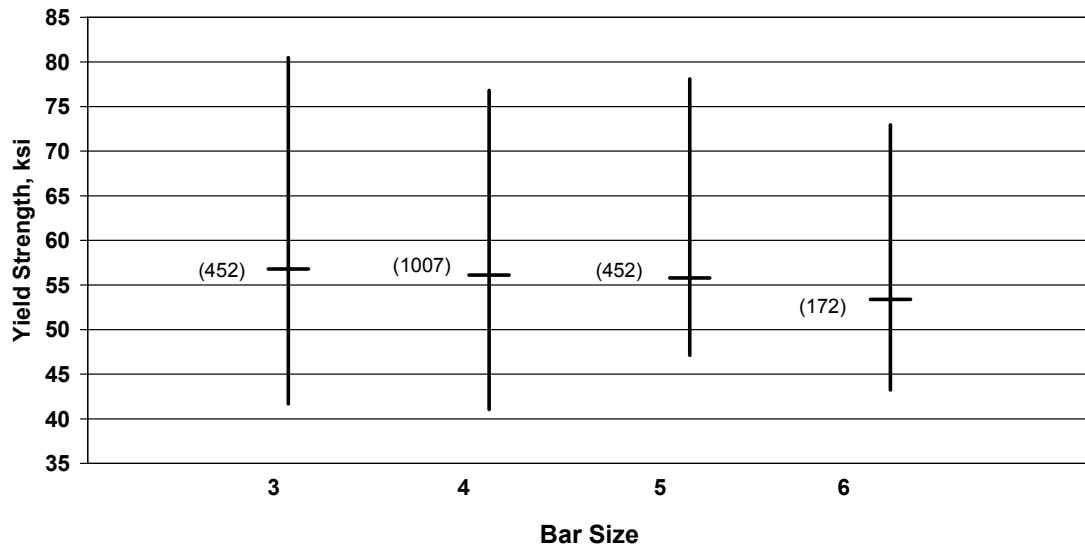
Mill ID	A 615 Grade 40					A 615 Grade 60											
	Bar Size					Bar Size											
	3	4	5	6	Avg.	3	4	5	6	7	8	9	10	11	14	18	Avg.
227		97.0	96.6		96.8		96.6	96.9	96.6	96.6	97.3	96.6	96.7	96.8	97.5	96.9	96.9
463	95.8				95.8	96.0	95.9	95.7	95.4	95.6	95.3						95.7
539		96.2	95.9	96.3	96.1		96.4	96.0	95.8	95.6	95.9	96.1	95.6	95.5			95.9
541	97.8	96.5	96.6		97.0	97.5	96.5	96.5	96.8	96.6	96.7	96.7	96.7	96.7			96.7
638							95.9	95.6	95.6	96.0	95.5						95.7
676		99.1	96.8		97.9		97.9	96.7	97.0	97.2	97.6	97.1	96.8	96.8	100.6	97.5	97.5
681	99.0				99.0	98.9	98.0	97.5	97.6	97.7	96.5	97.5	96.9	97.4			97.6
687	96.8	96.0	96.5		96.4	97.7	96.5	96.1	96.4	97.1	96.8	96.4	96.5	96.6	97.4	97.6	96.8
690						98.1	97.9										98.0
808	98.4	98.7			98.6	98.9	98.2										98.5
829															97.5	97.3	97.4
Avg.	97.6	97.2	96.5	96.3		97.8	97.0	96.4	96.4	96.6	96.4	96.7	96.5	96.6	98.2	97.3	

Table 34: Percent of Nominal Weights for Individual Mills, A 615 Grade 75 and A 616 Grade 60 Bars

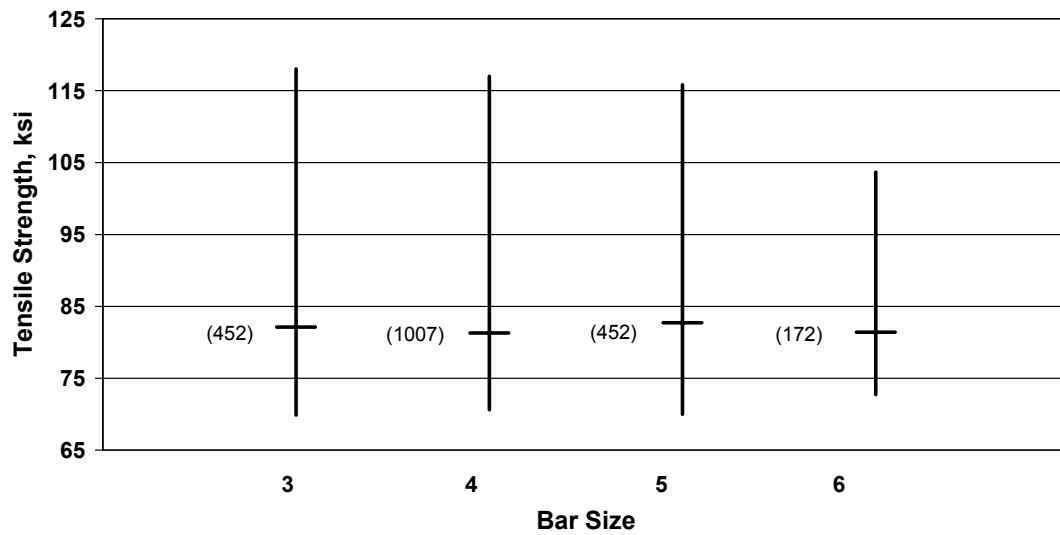
Mill ID	A 615 Grade 75										A 616 Grade 60		
	Bar Size										Bar Size		
	4	5	6	7	8	9	10	11	14	Avg.	8	10	Avg.
681											96.6	96.2	96.4
687					96.4	96.1				96.3			
829									98.5	98.5			
Avg.					96.4	96.1			98.5		96.6	96.2	

Table 35: Percent of Nominal Weights for Individual Mills, A 706 Grade 60 Bars

Mill ID	A 706 Grade 60											
	Bar Size											
	3	4	5	6	7	8	9	10	11	14	18	Avg.
227		96.9	96.9	96.8	96.8	96.8	96.4	96.5	96.8	97.2	97.3	96.8
541		96.4	96.7	96.5								96.5
638		95.3										95.3
681				97.6	97.6	96.8	96.6					97.1
687		98.0	97.4	96.3	97.1	96.8	96.8	96.8	97.3	97.0	97.0	97.0
Avg.		96.6	97.0	96.8	97.2	96.8	96.6	96.6	97.1	97.1	97.2	

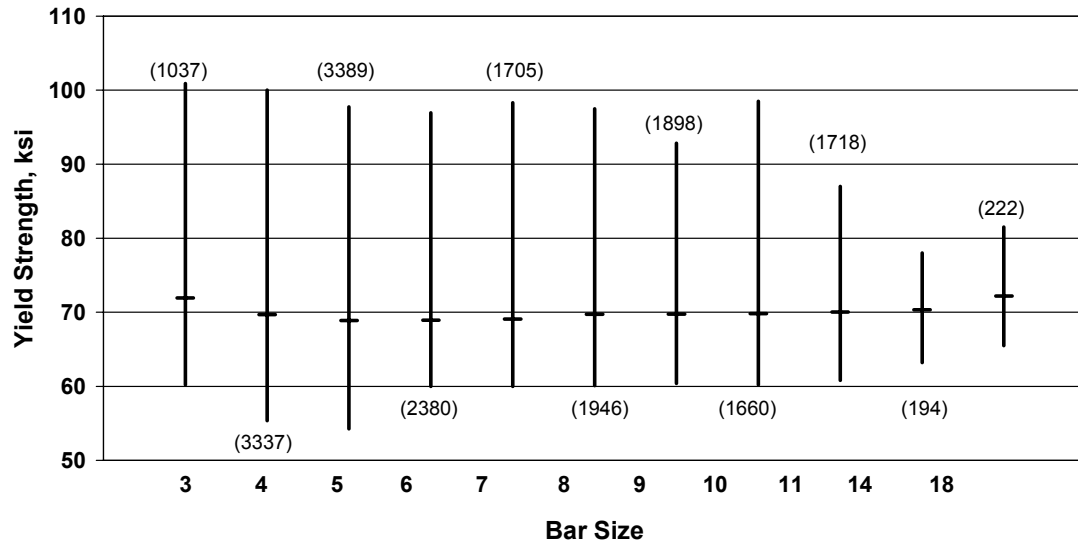


(a)

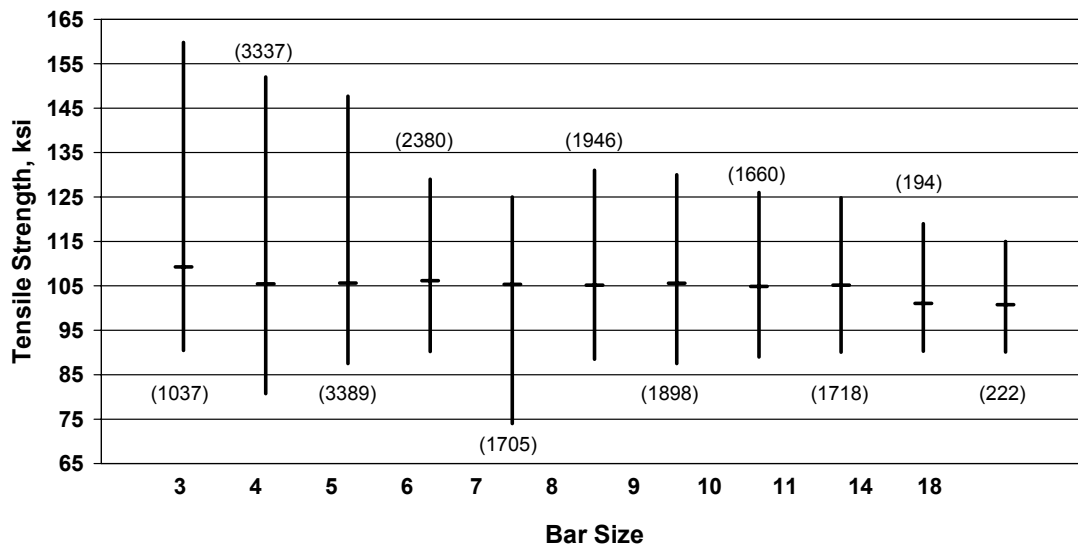


(b)

Figure 1: Mean and range of (a) Yield Strength and (b) Tensile Strength for A 615 Grade 40 reinforcement (number of heats)

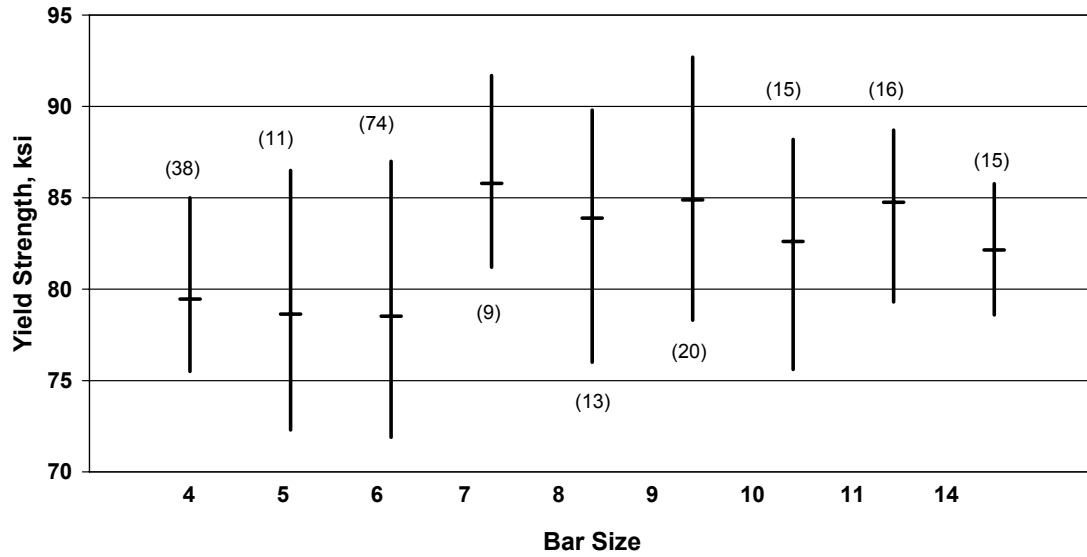


(a)

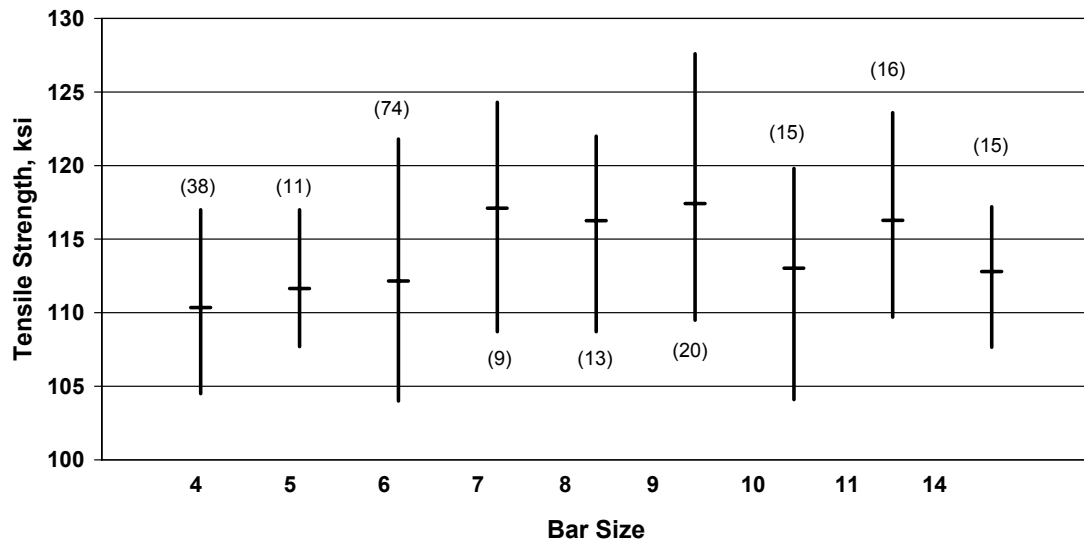


(b)

Figure 2: Mean and range of (a) Yield Strength and (b) Tensile Strength for A 615 Grade 60 reinforcement (number of heats)

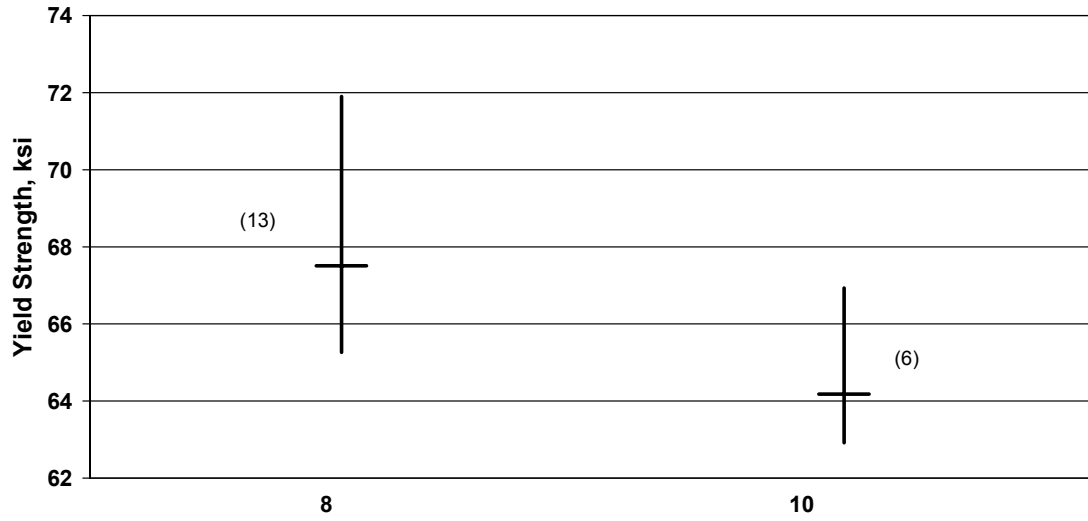


(a)

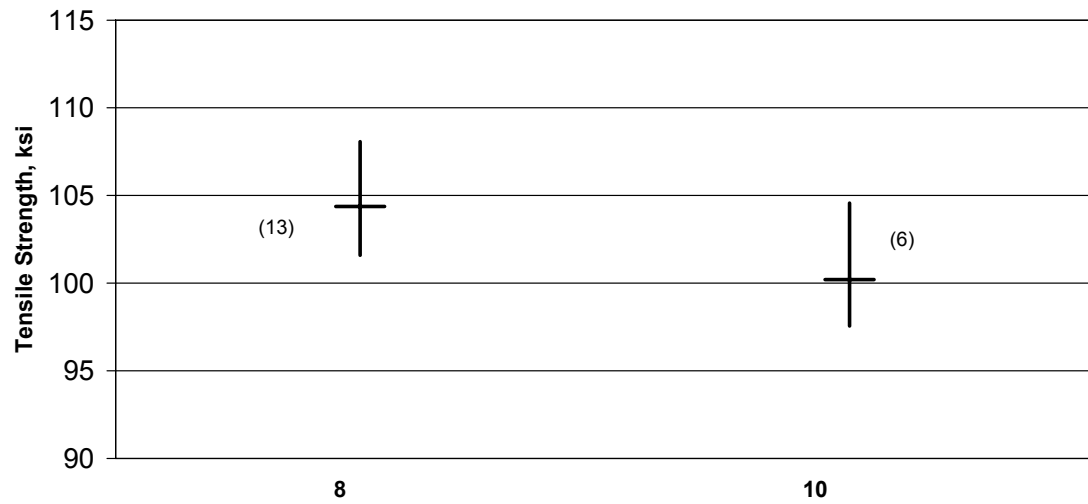


(b)

Figure 3: Mean and range of (a) Yield Strength and (b) Tensile Strength for A 615 Grade 75 reinforcement (number of heats)

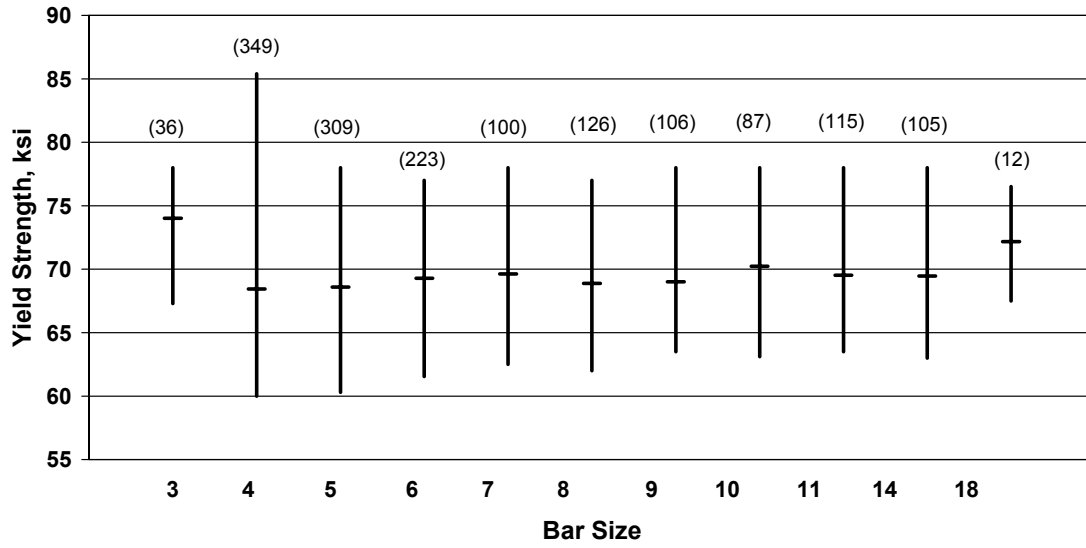


(a)

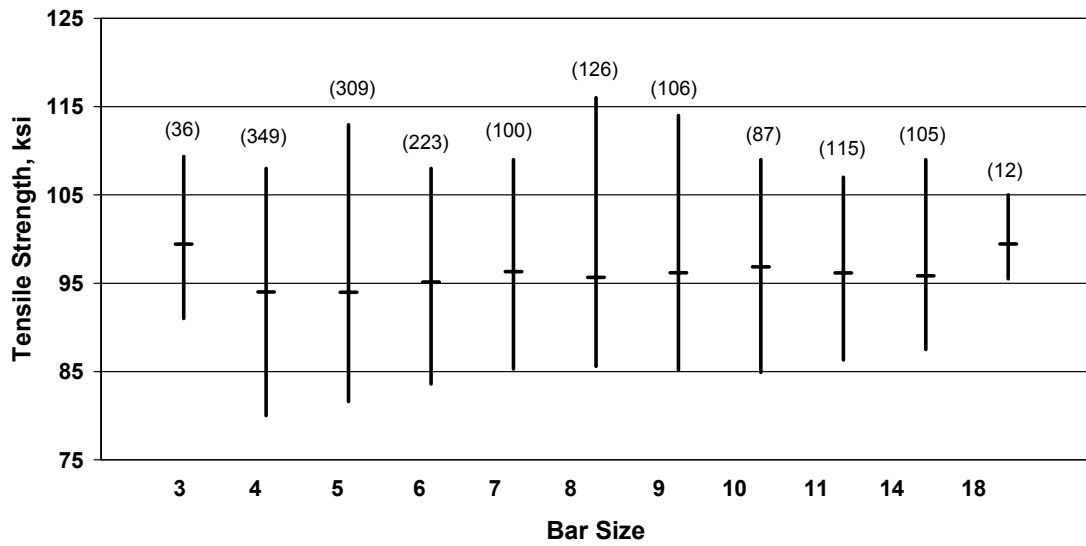


(b)

Figure 4: Mean and range of (a) Yield Strength and (b) Tensile Strength for A 616 Grade 60 reinforcement (number of heats)

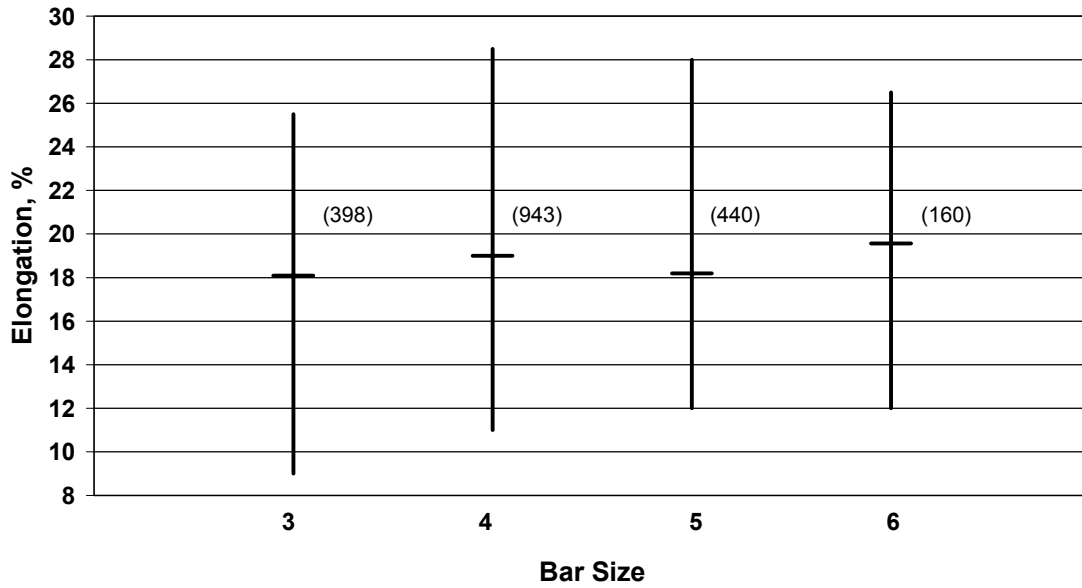


(a)

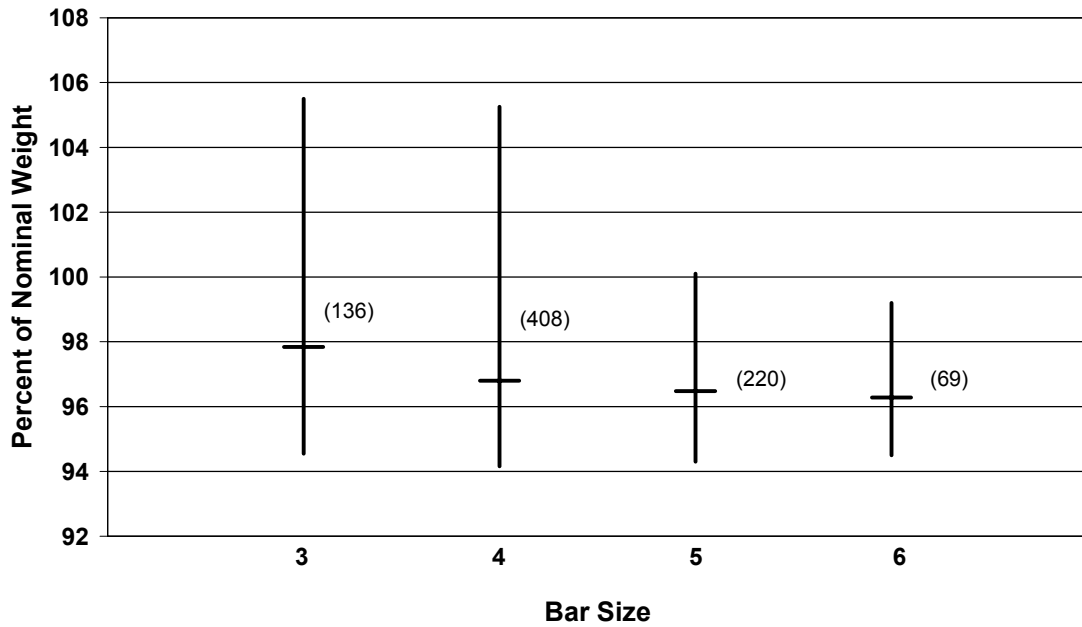


(b)

Figure 5: Mean and range of (a) Yield Strength and (b) Tensile Strength for A 706 Grade 60 reinforcement (number of heats)

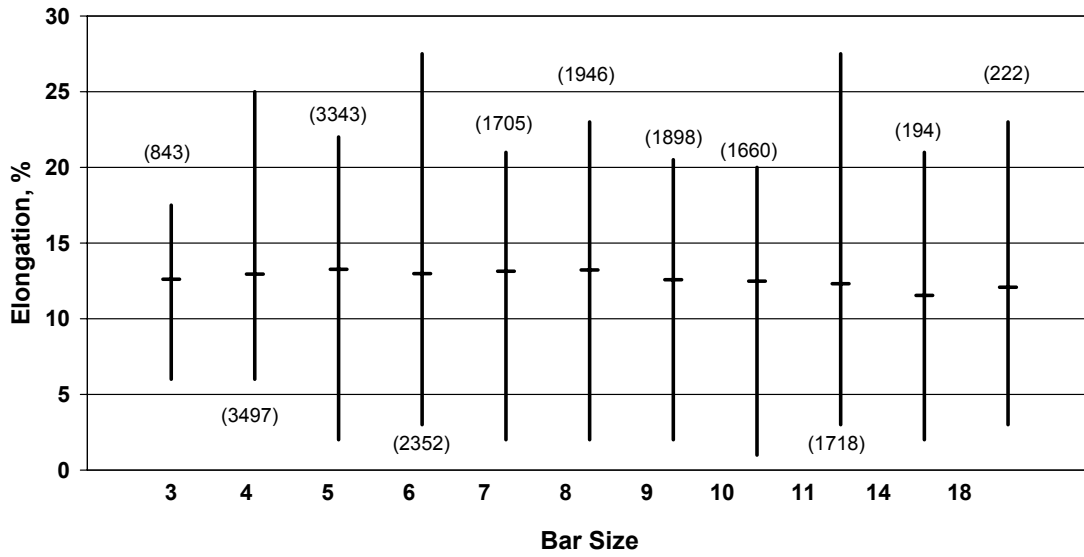


(a)

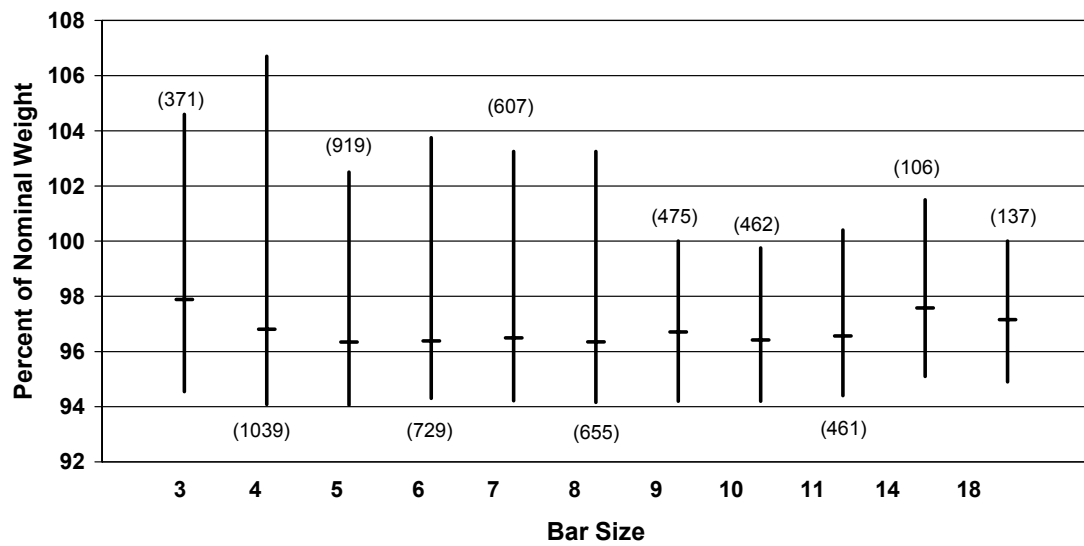


(b)

Figure 6: Mean and range of (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 40 reinforcement (number of heats)

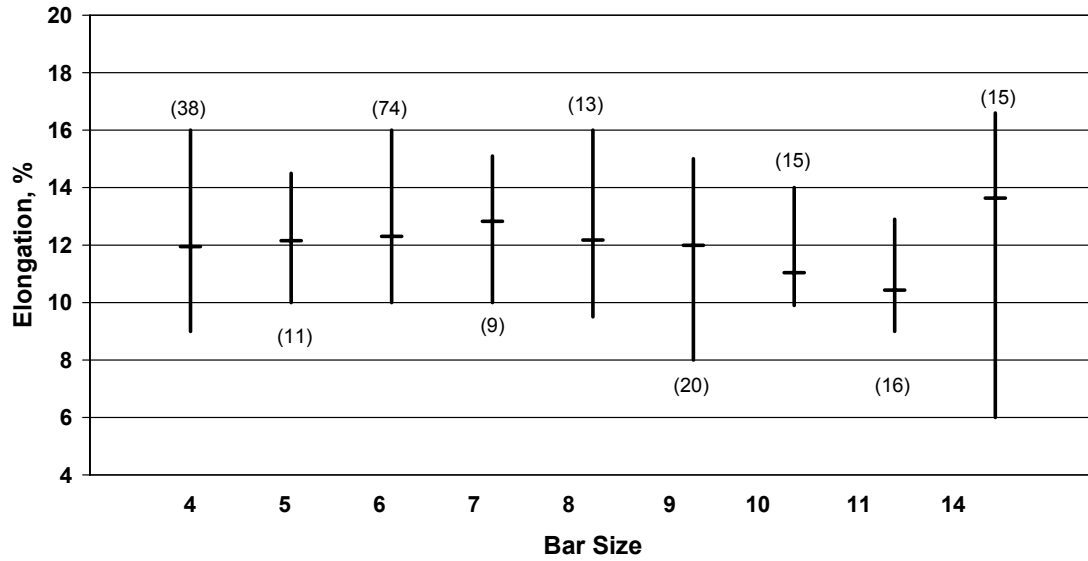


(a)

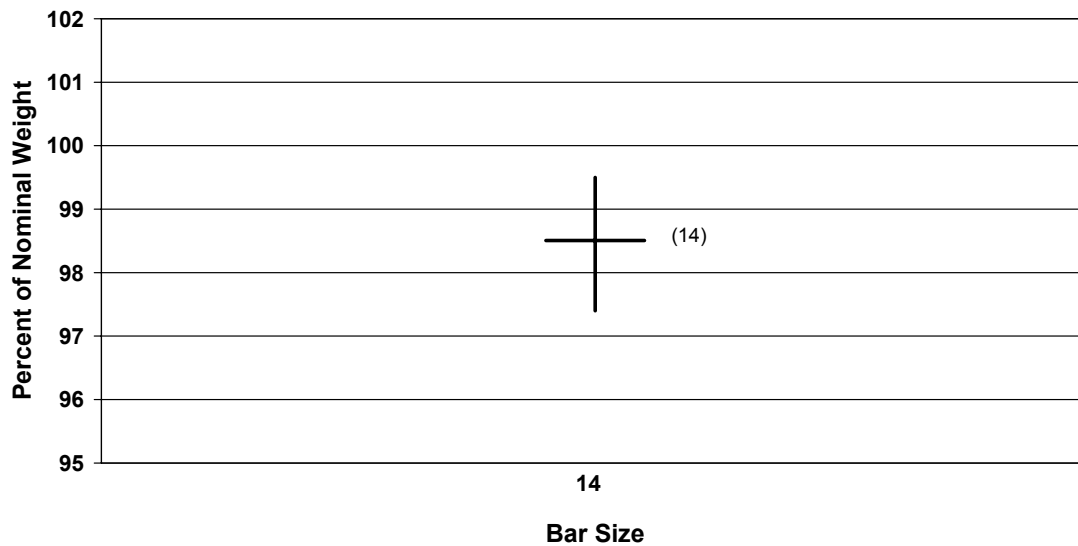


(b)

Figure 7: Mean and range of (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 reinforcement (number of heats)

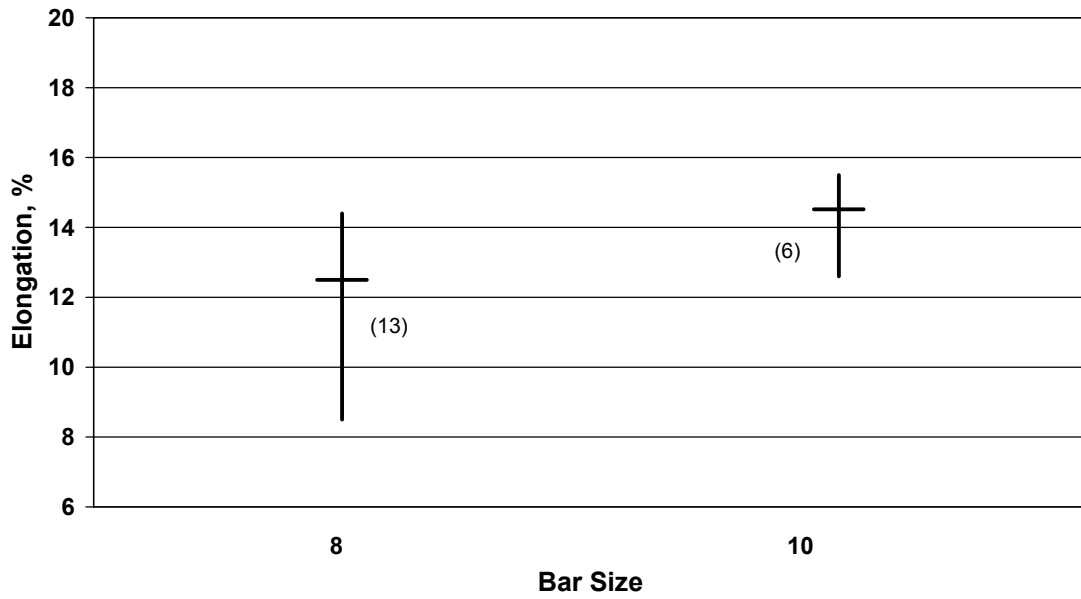


(a)

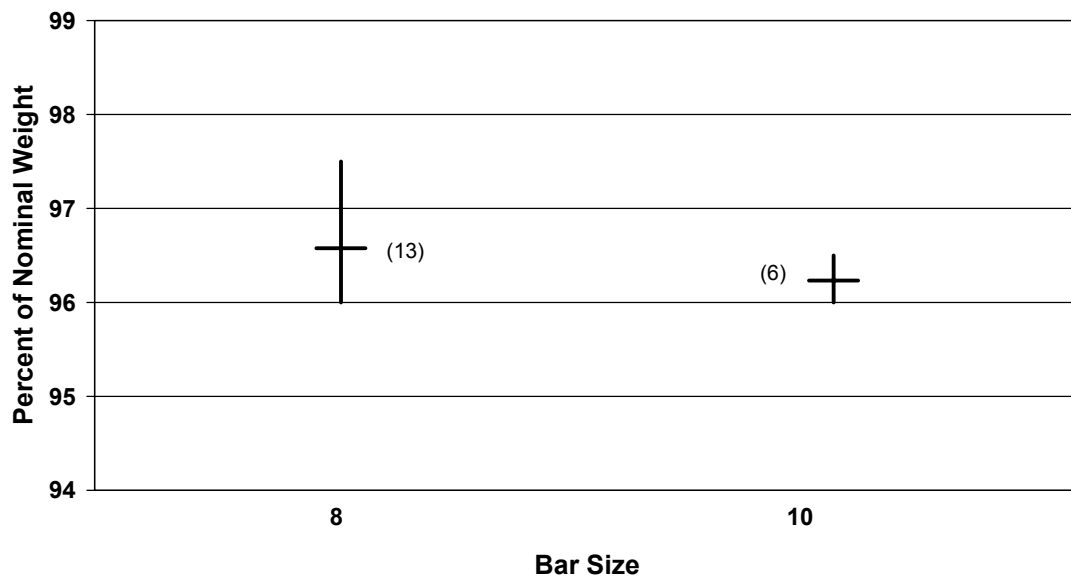


(b)

Figure 8: Mean and range of (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 75 reinforcement (number of heats)



(a)



(b)

Figure 9: Mean and range of (a) Elongation and (b) Percent of Nominal Weight for A 616 Grade 60 reinforcement (number of heats)

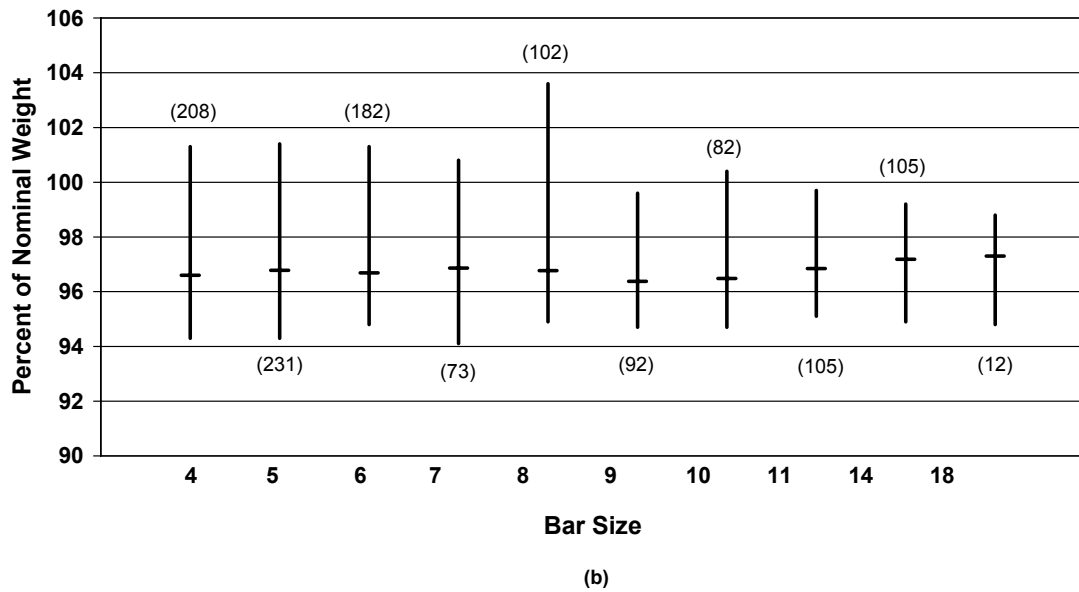
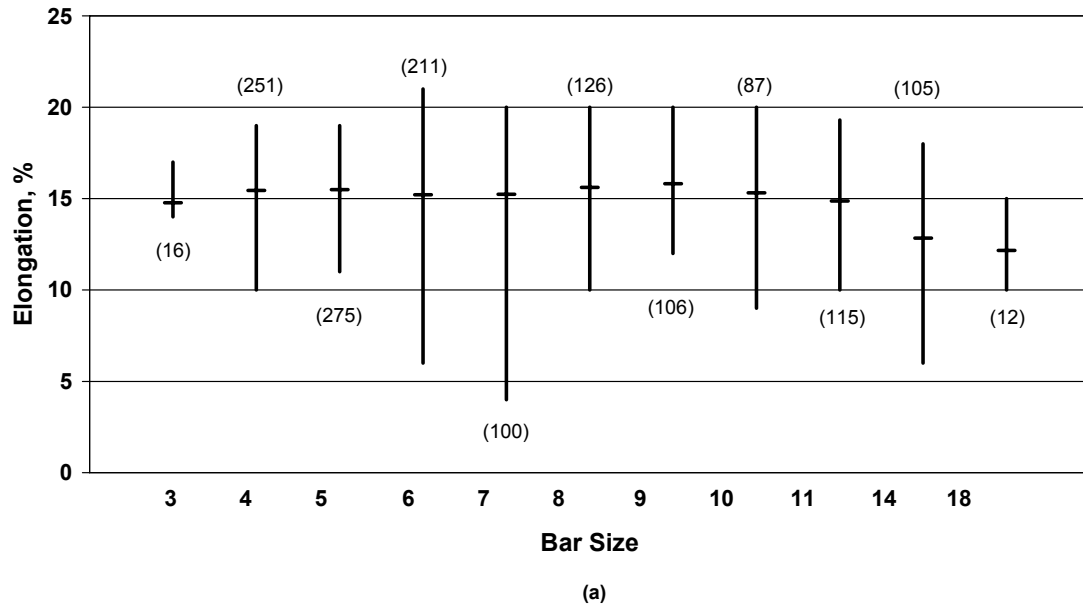


Figure 10: Mean and range of (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 reinforcement (number of heats)

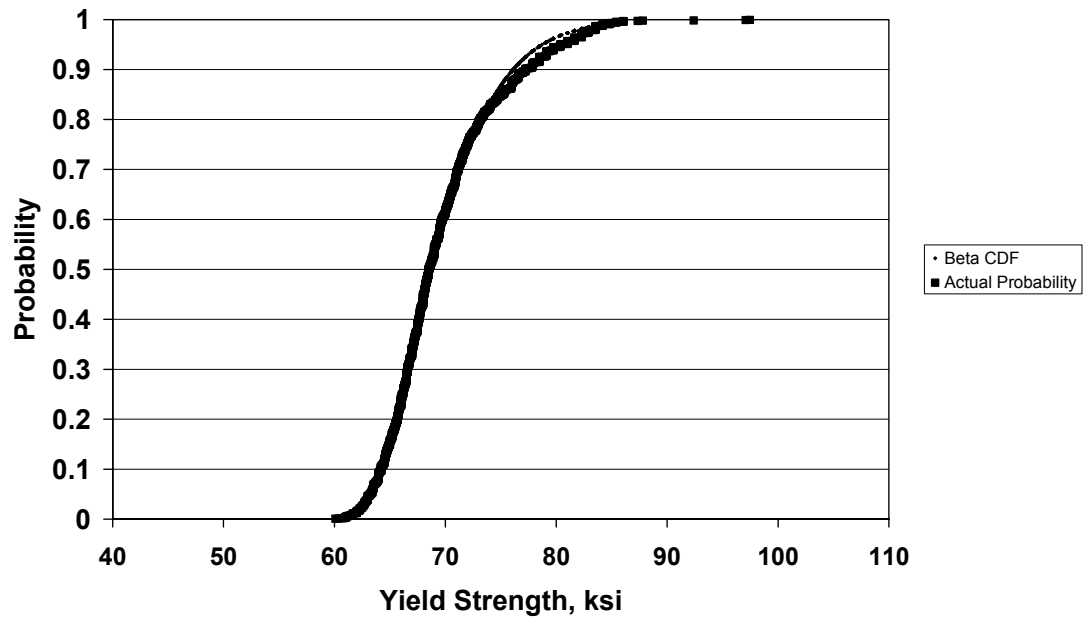
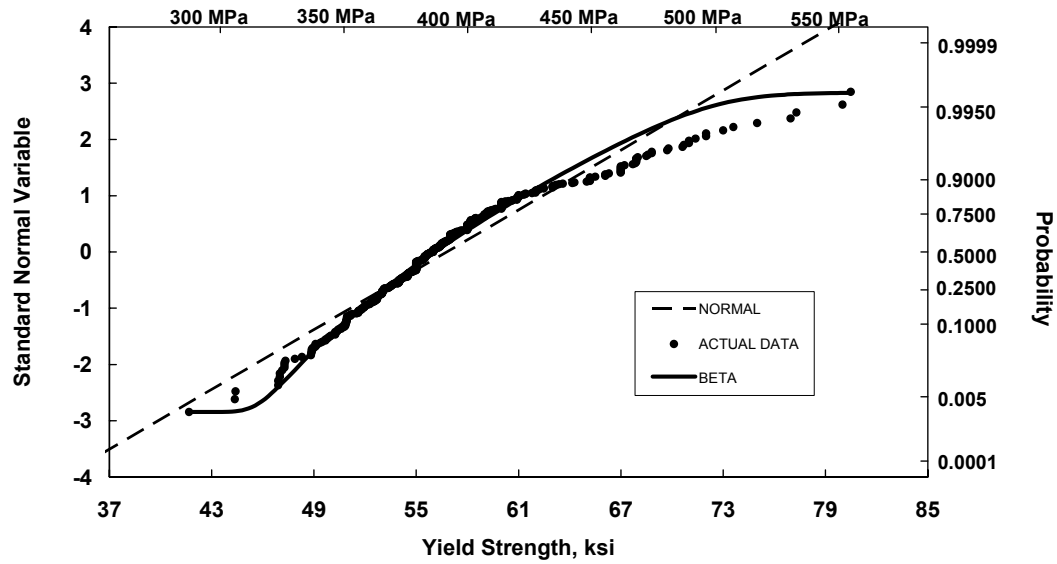
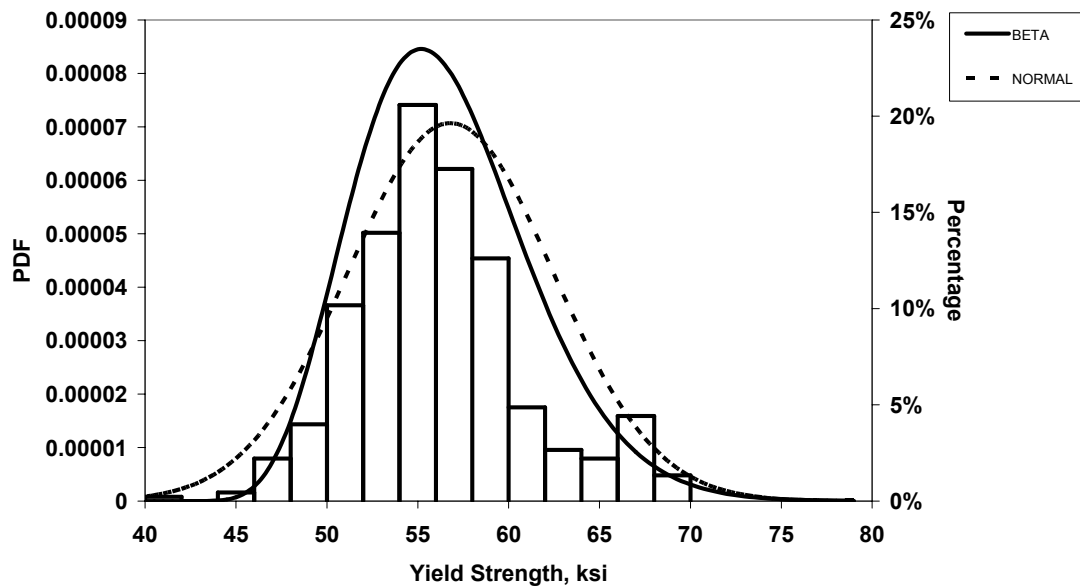


Figure 11: Example of Beta Cumulative Density Function and Actual Probability Distribution Prior to Transfer to Normal Probability Paper

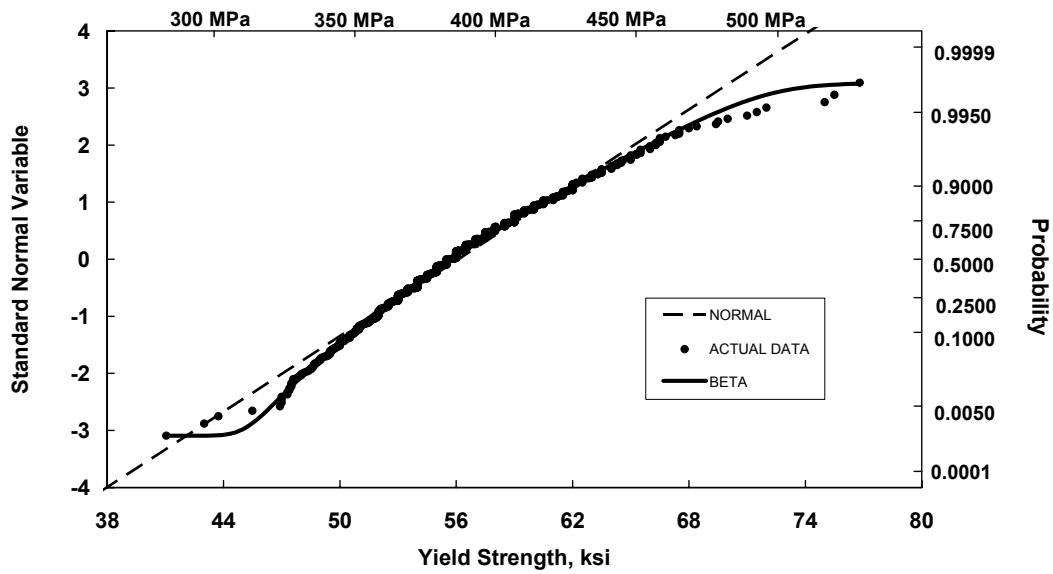


(a)

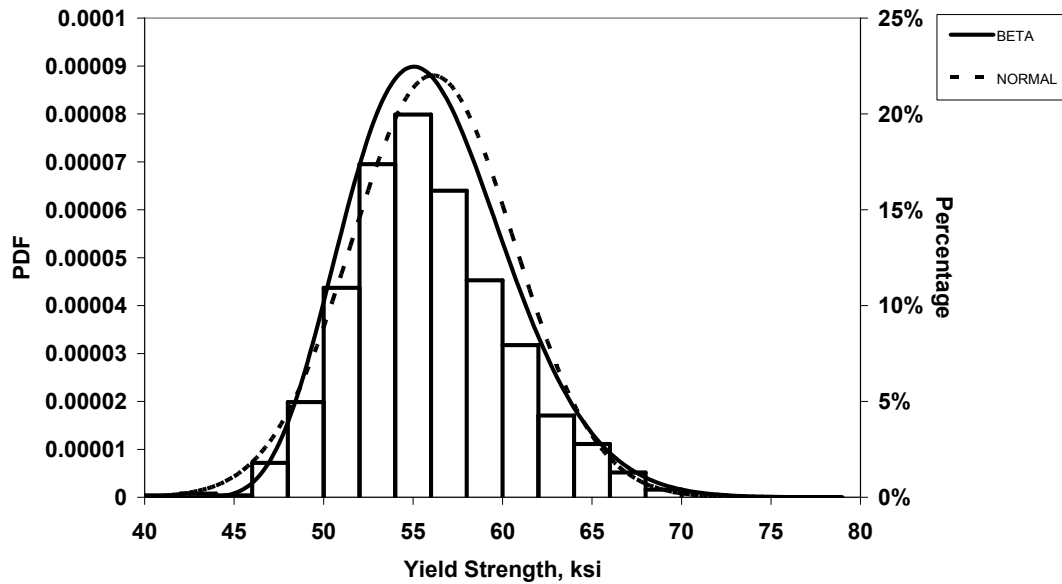


(b)

Figure 12: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 40 No. 3 bars

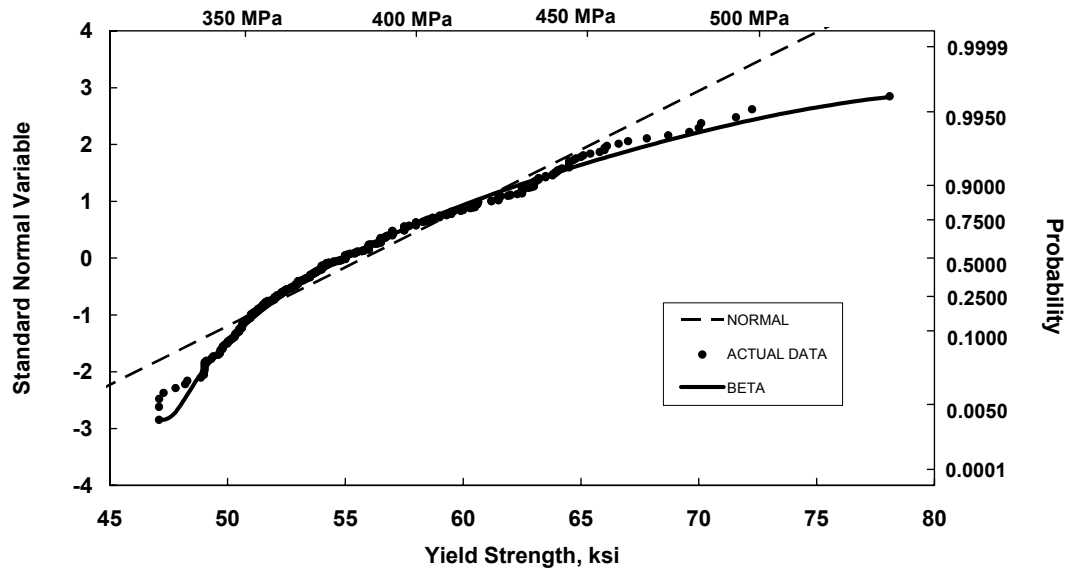


(a)

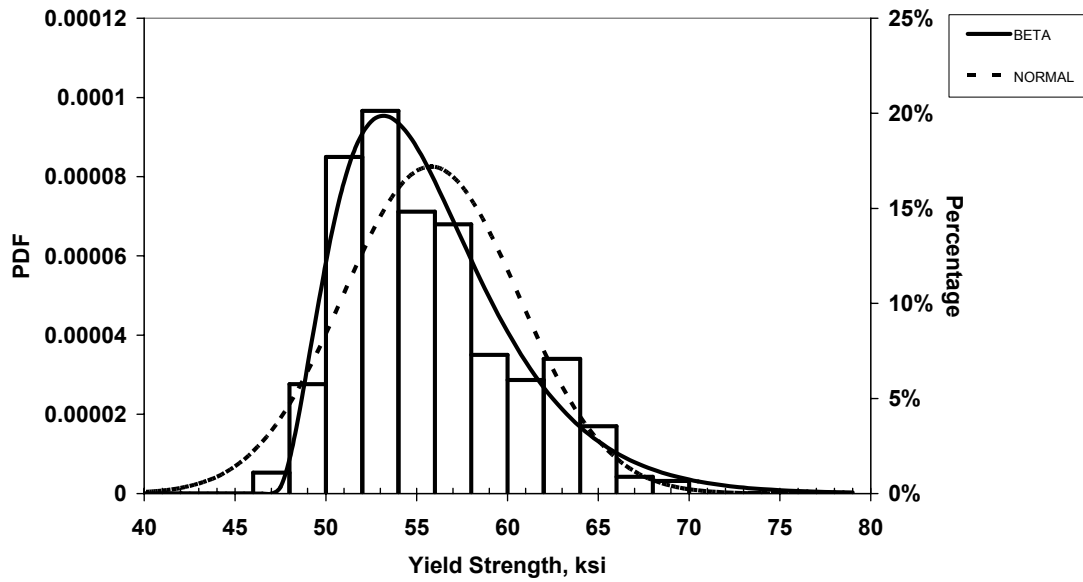


(b)

Figure 13: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 40 No. 4 bars

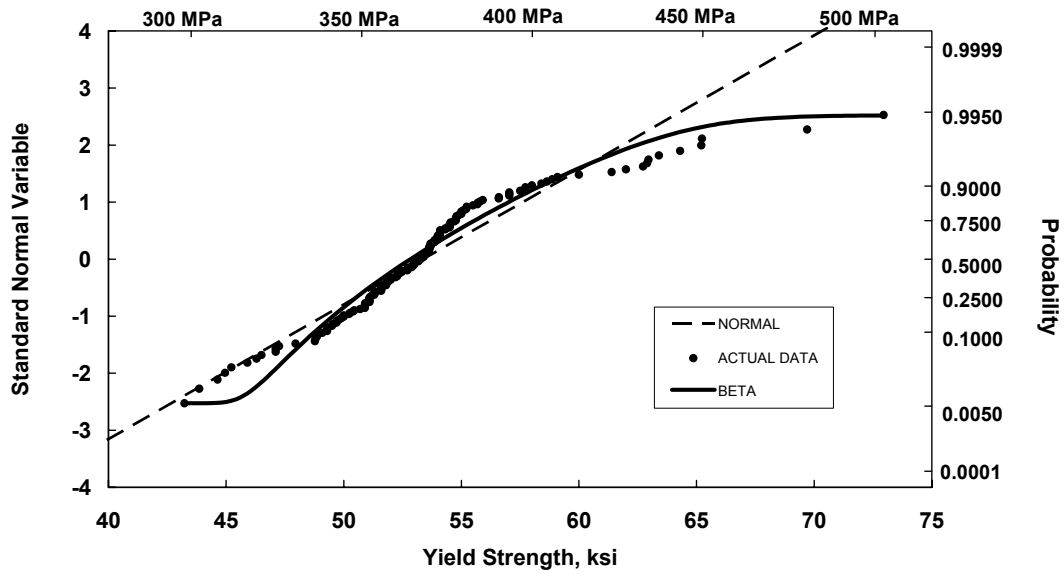


(a)

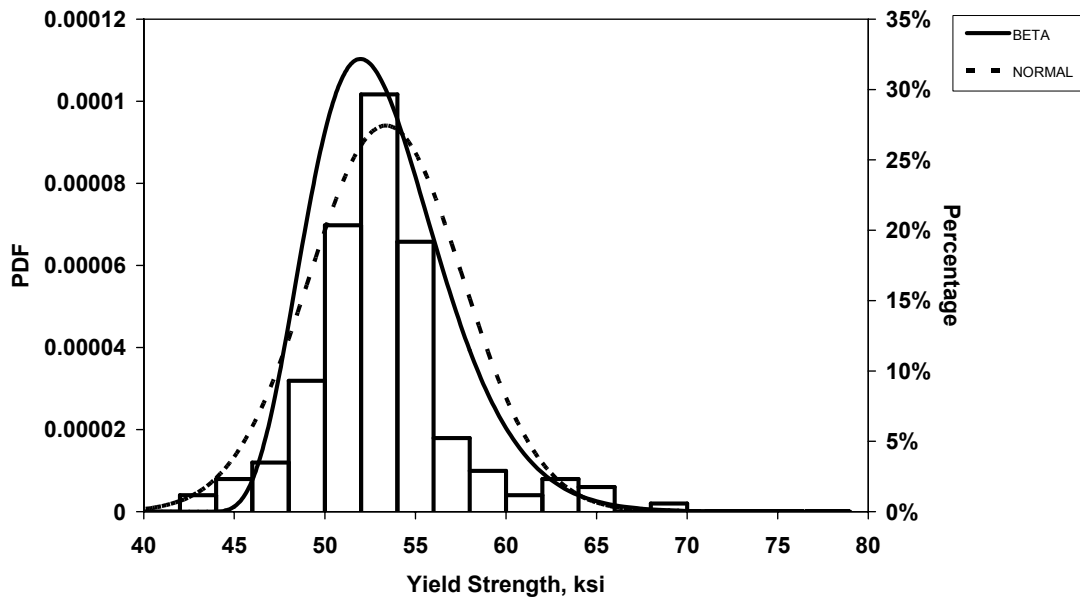


(b)

Figure 14: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 40 No. 5 bars

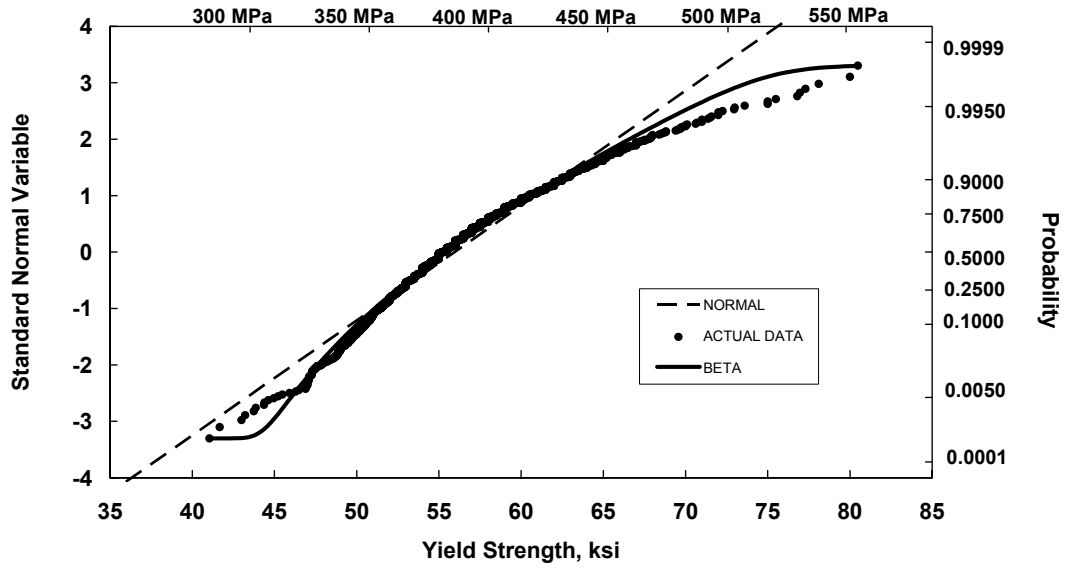


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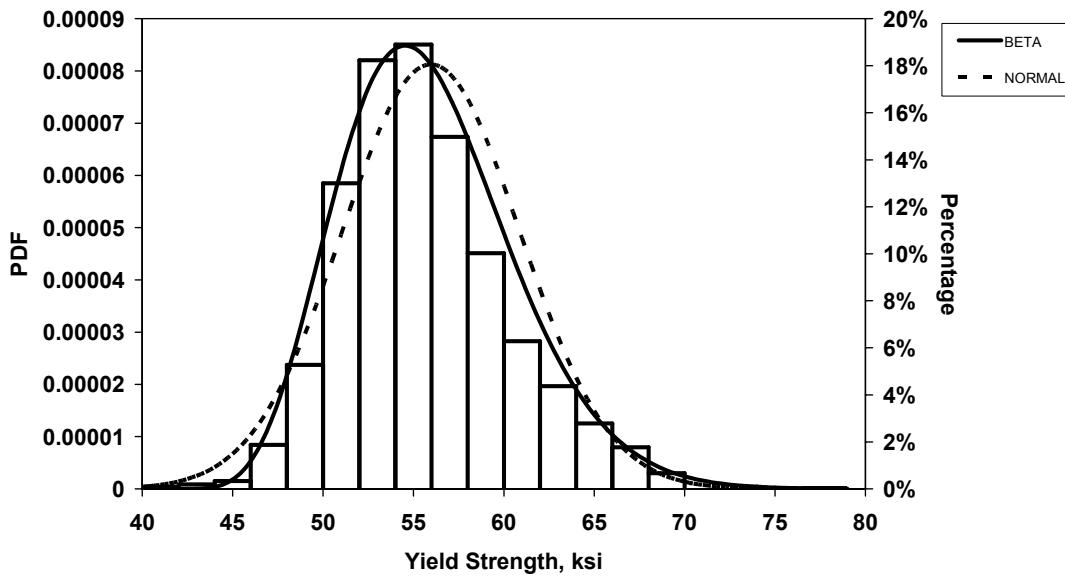


(b)

Figure 15: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 40 No. 6 bars

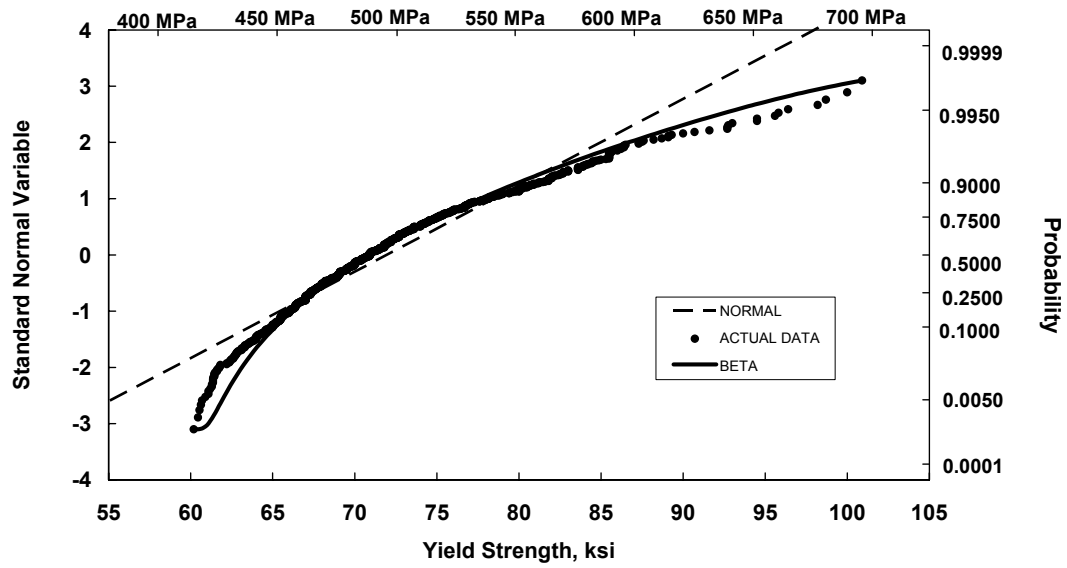


(a)

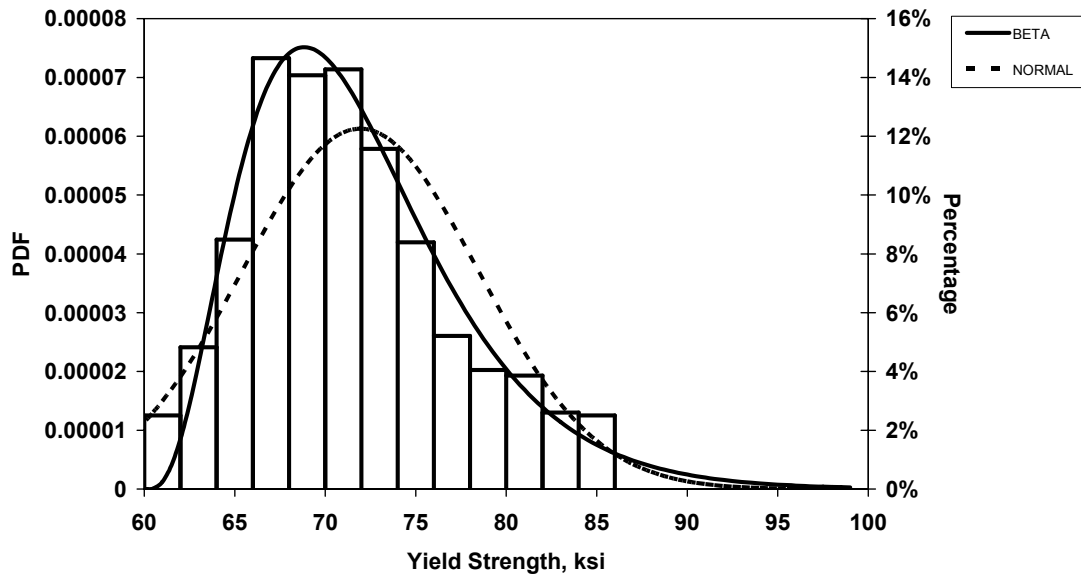


(b)

Figure 16: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of All A 615 Grade 40 bars

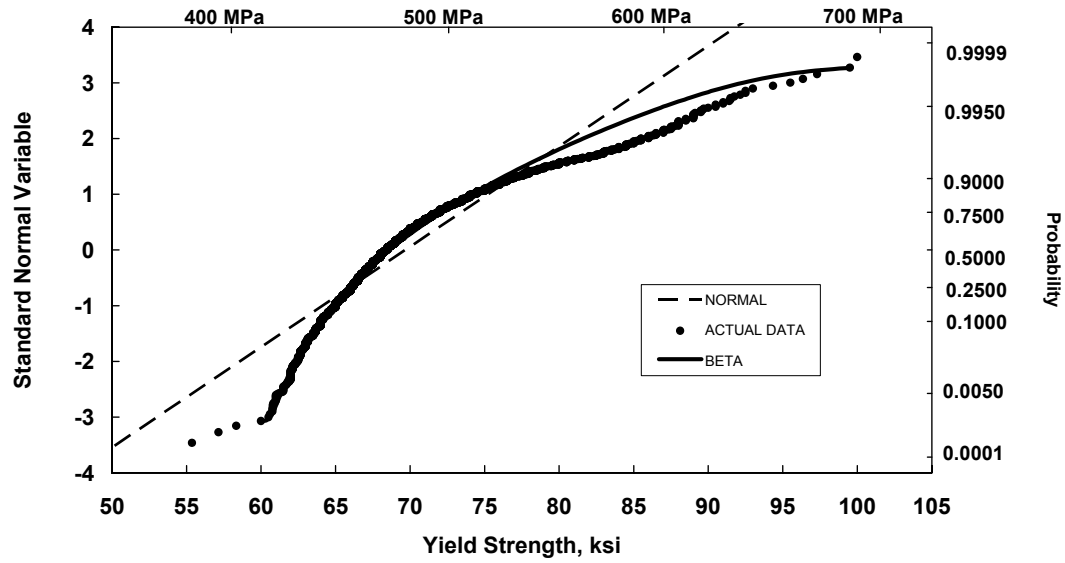


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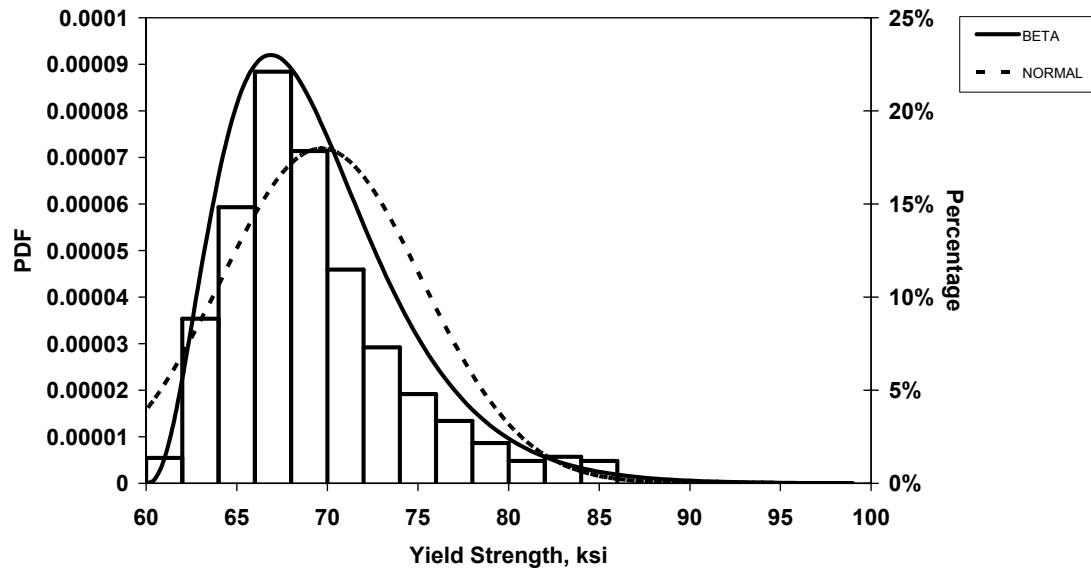


(b)

Figure 17: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 3 bars

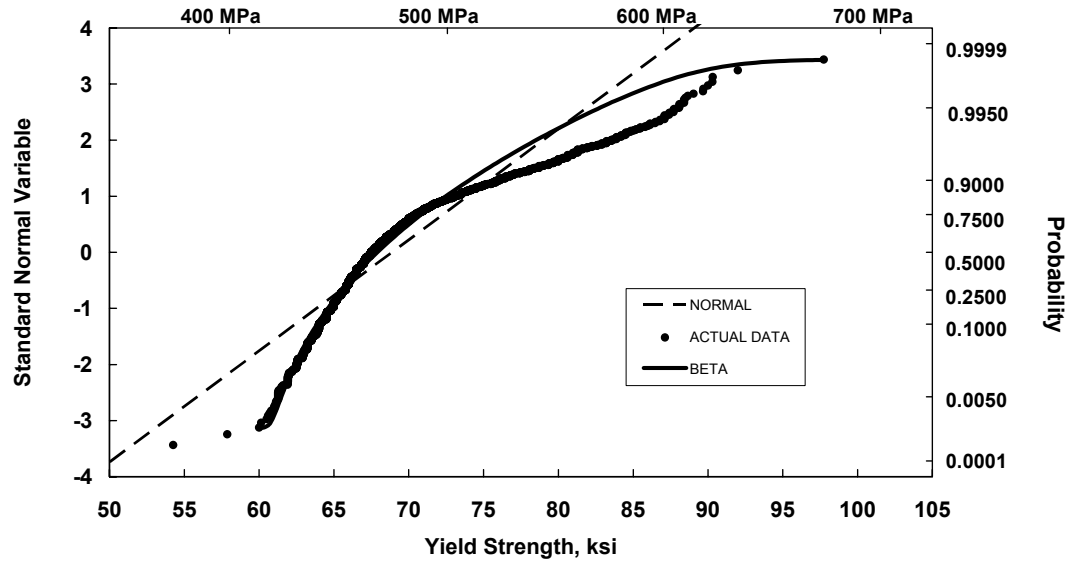


(a)

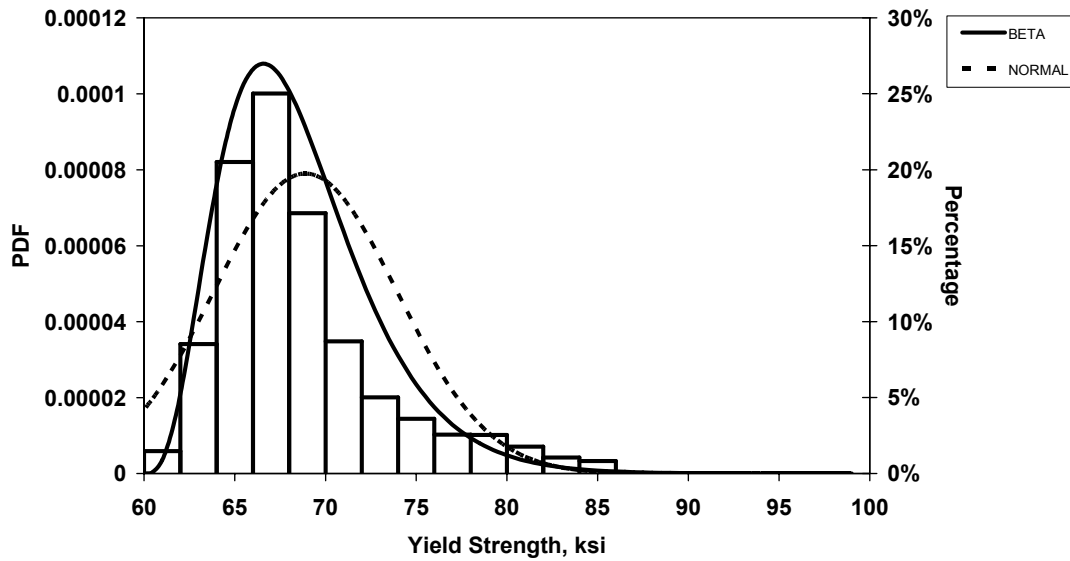


(b)

Figure 18: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 4 bars

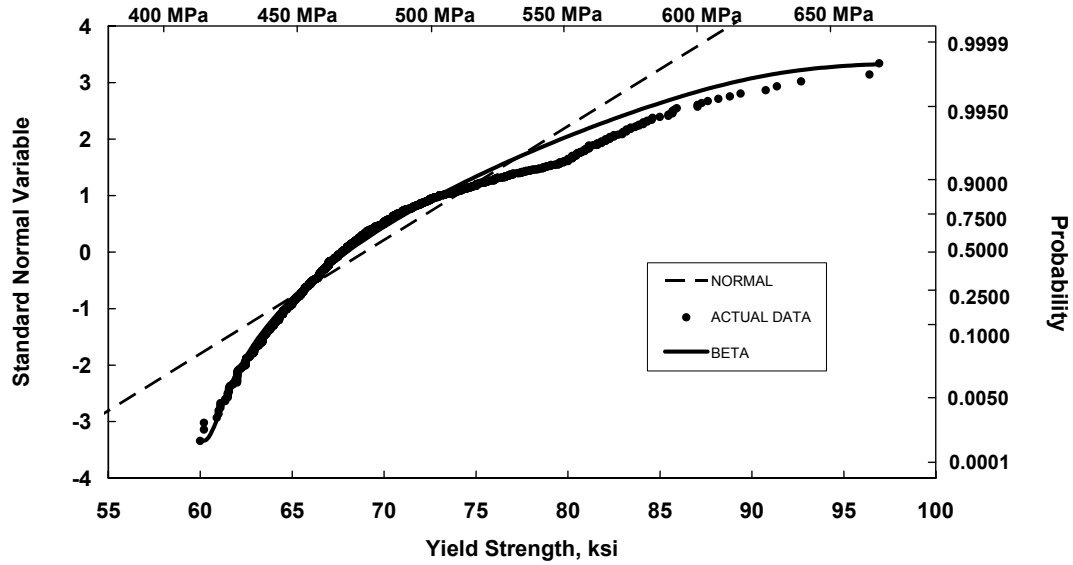


(a)

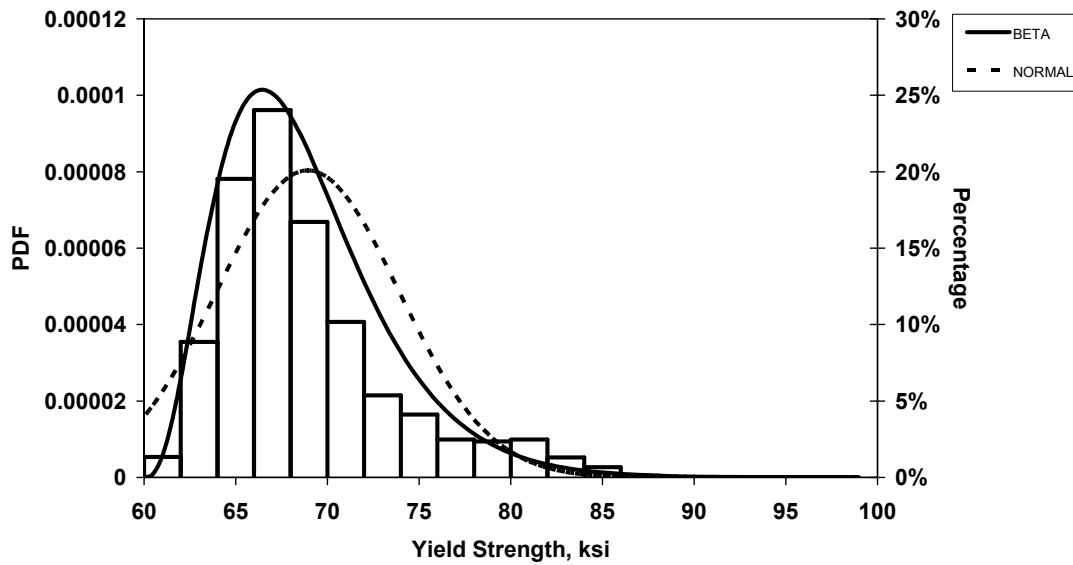


(b)

Figure 19: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 5 bars

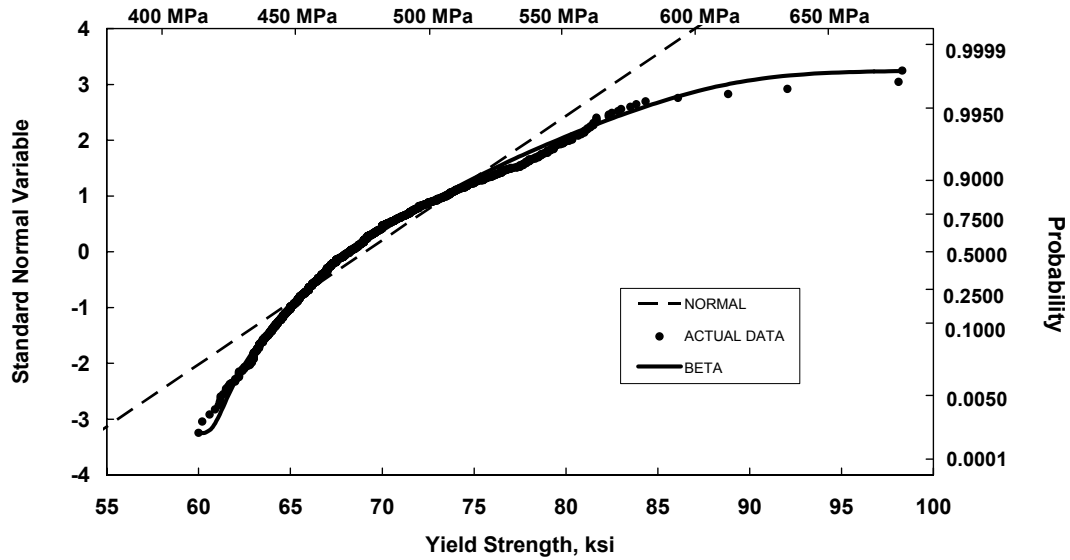


(a)

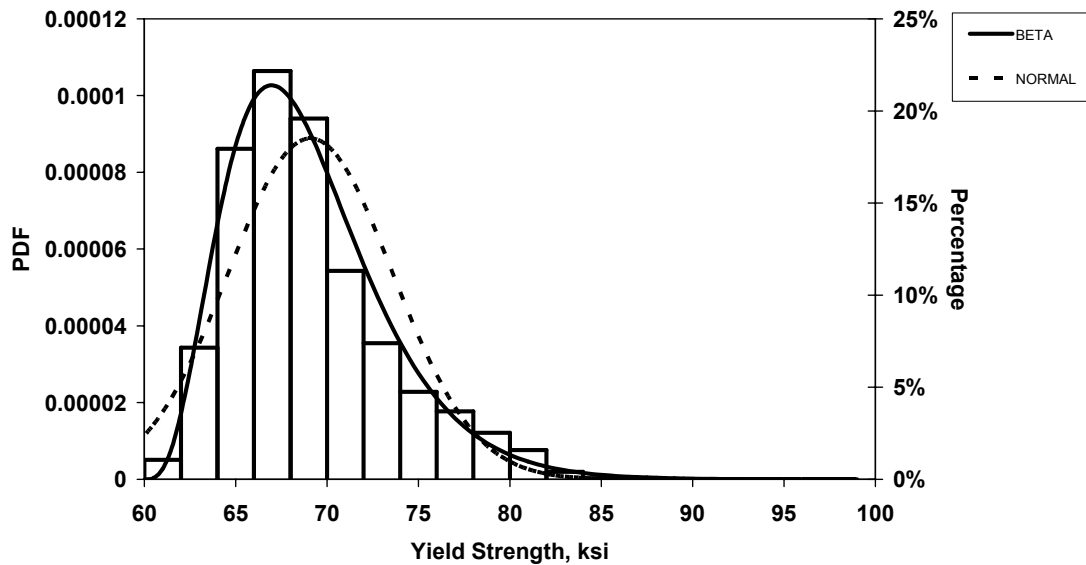


(b)

Figure 20: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 6 bars

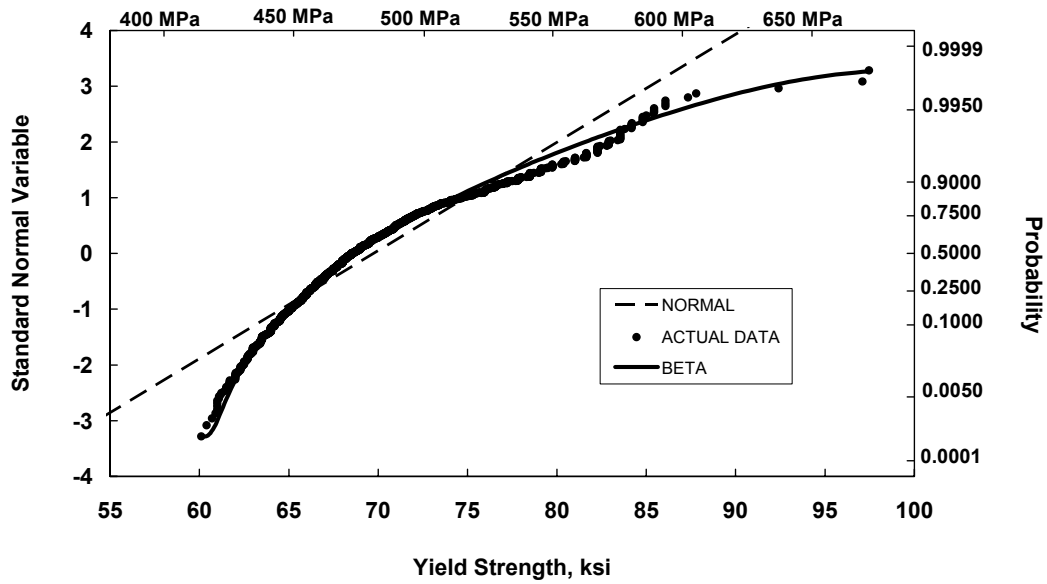


(a)

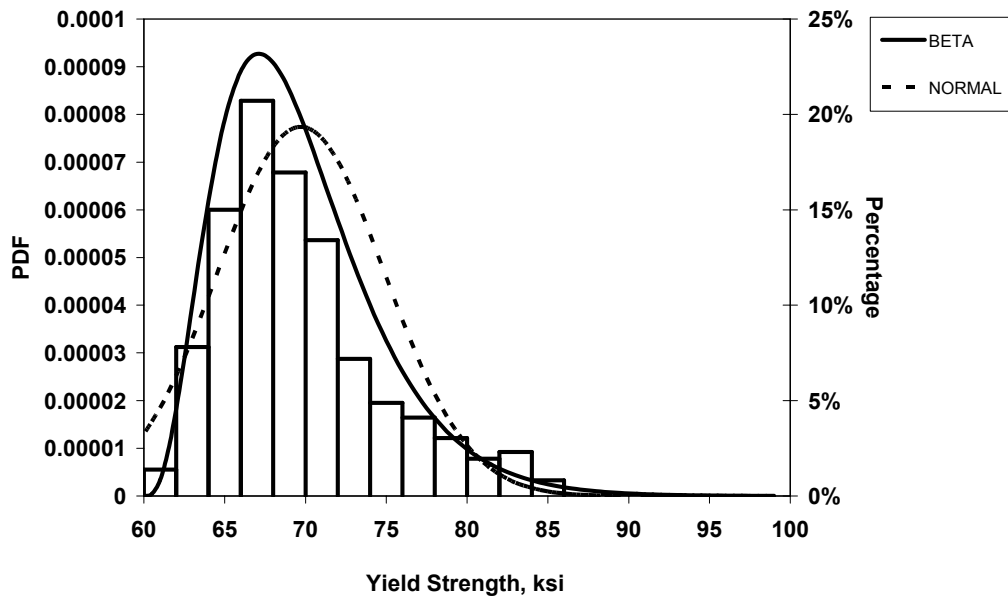


(b)

Figure 21: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 7 bars

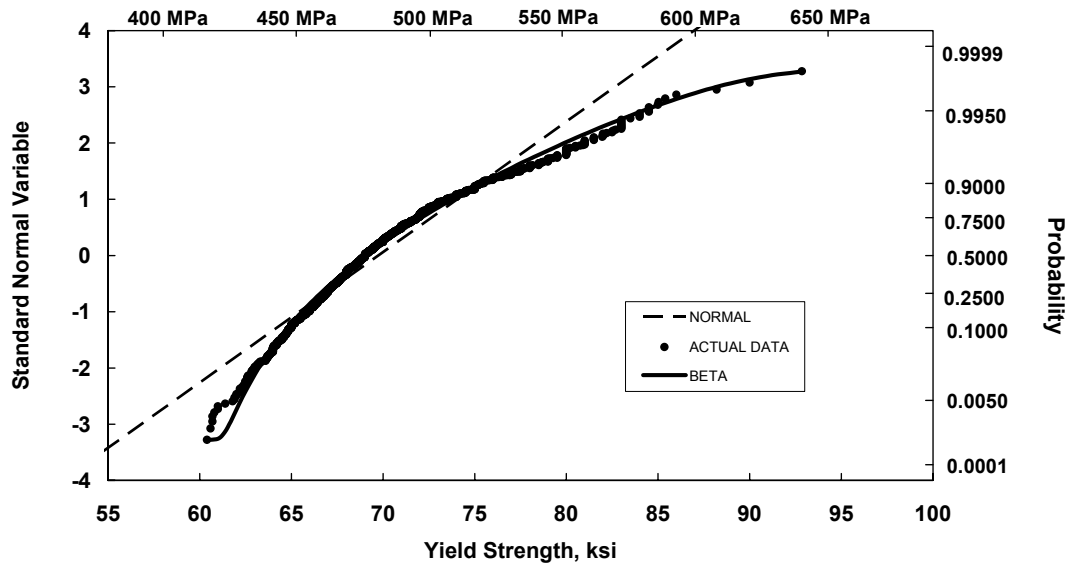


(a)

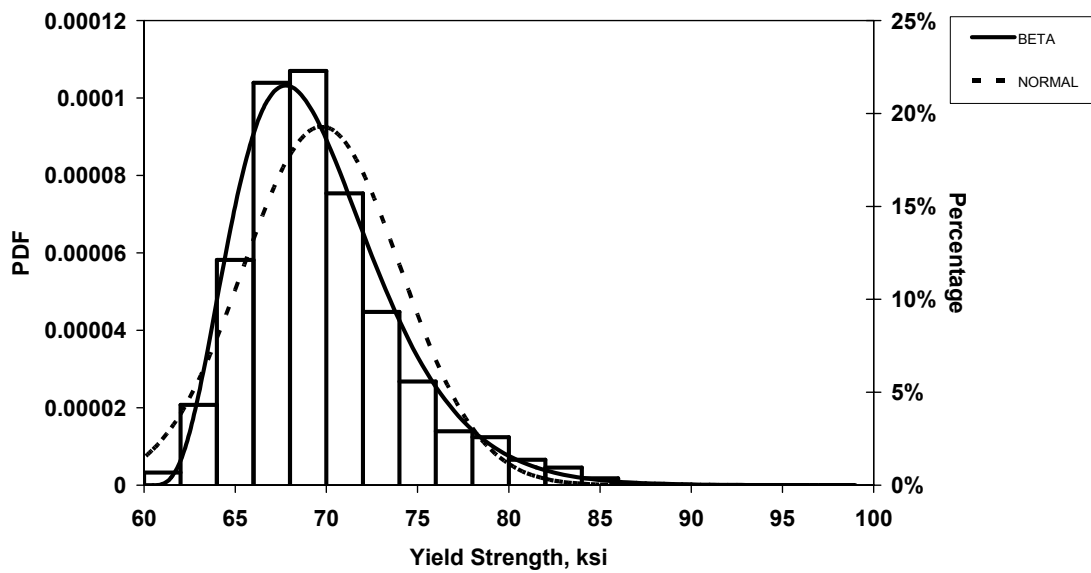


(b)

Figure 22: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 8 bars

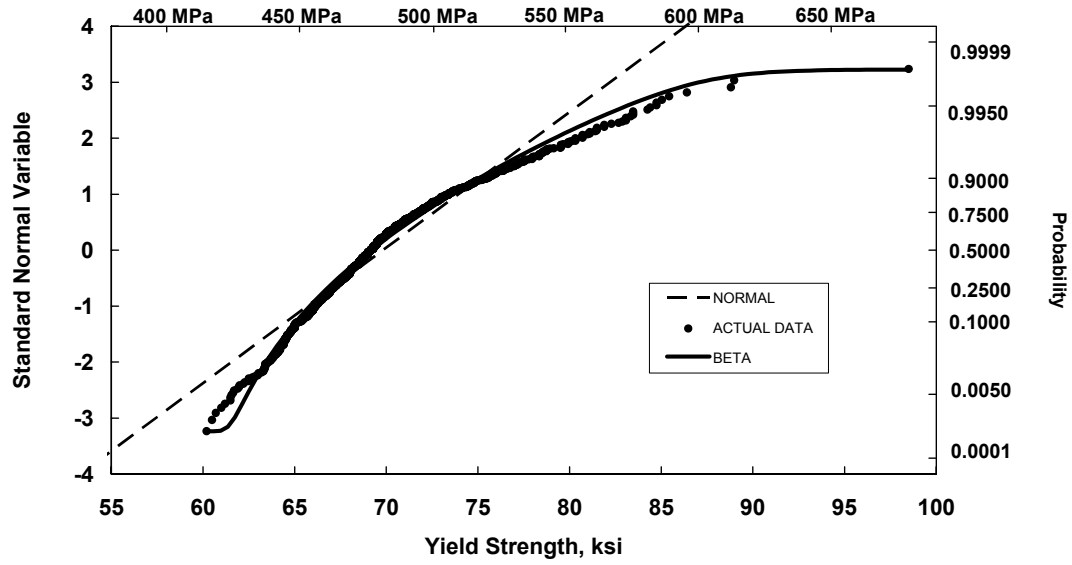


(a)

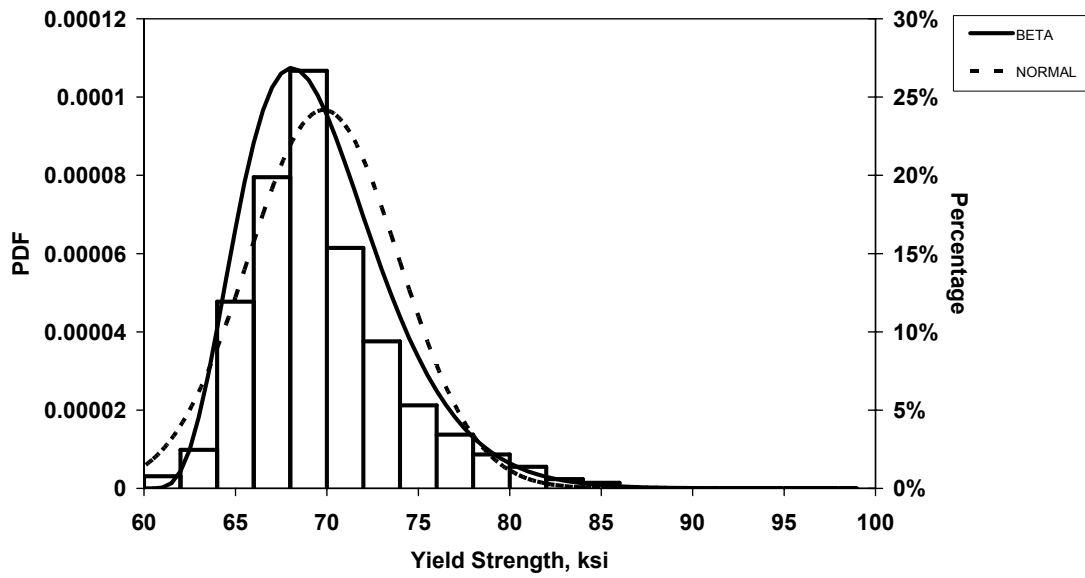


(b)

Figure 23: (a) Cumulative Density Function and (b) Histogram with Probability Density Function of Yield Strength for A 615 Grade 60 No. 9 bars

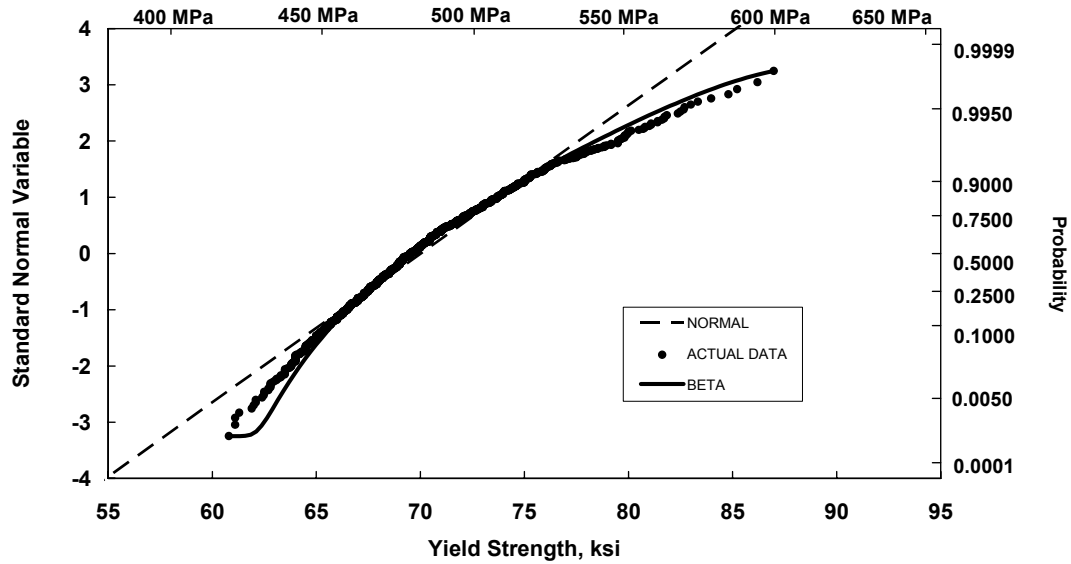


(a)

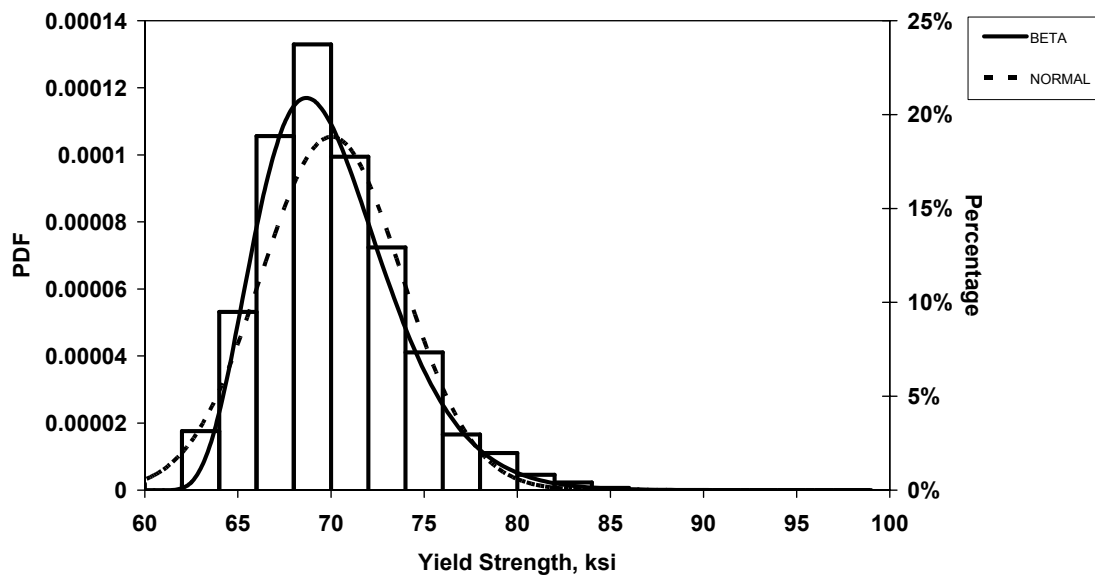


(b)

Figure 24: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 10 bars

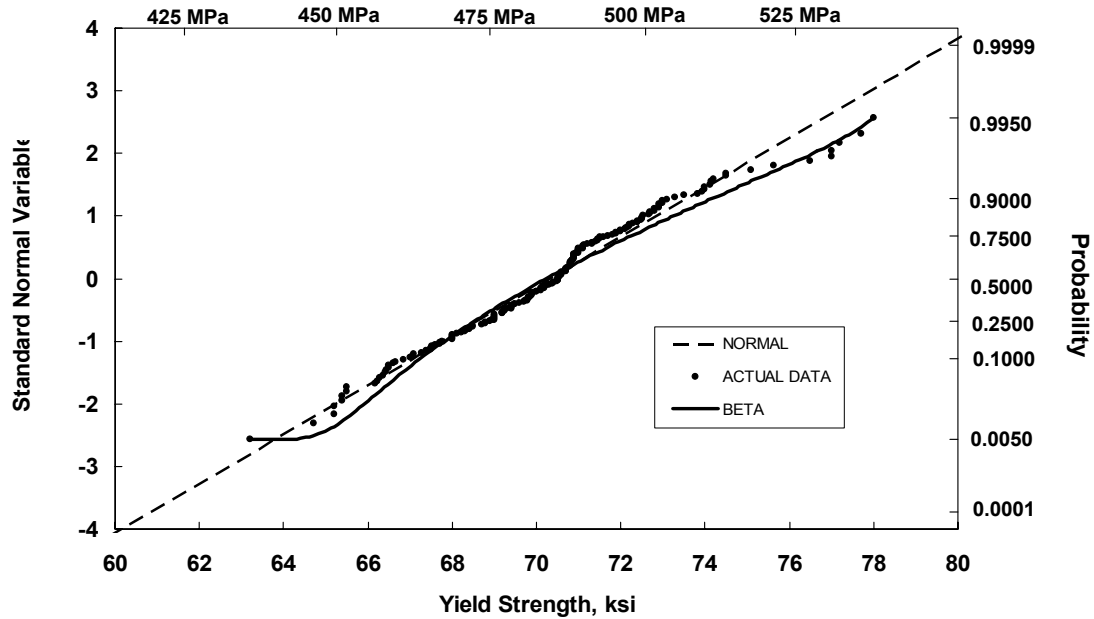


(a)

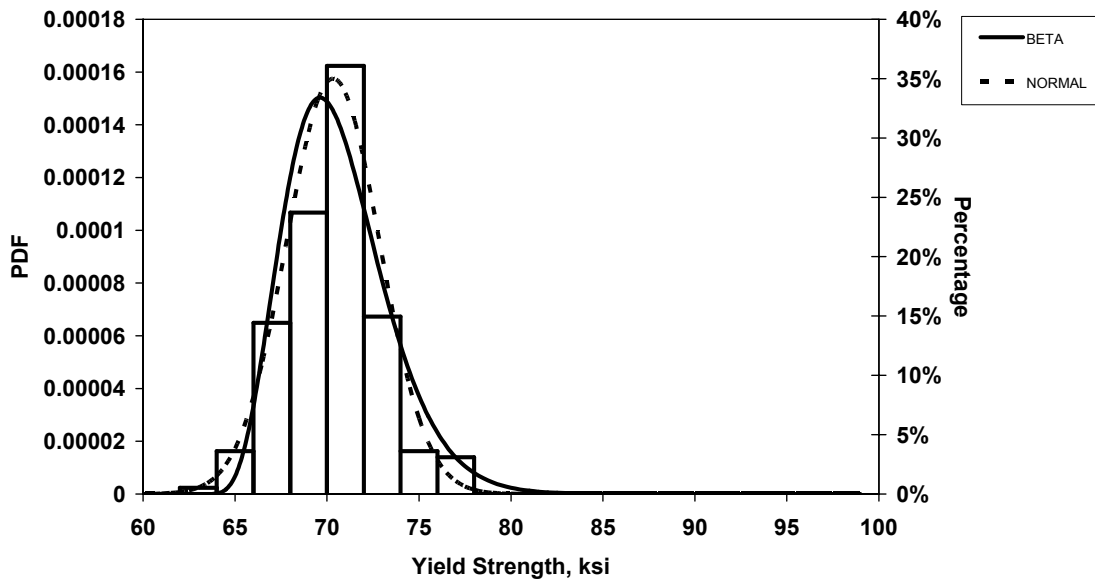


(b)

Figure 25: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 11 bars

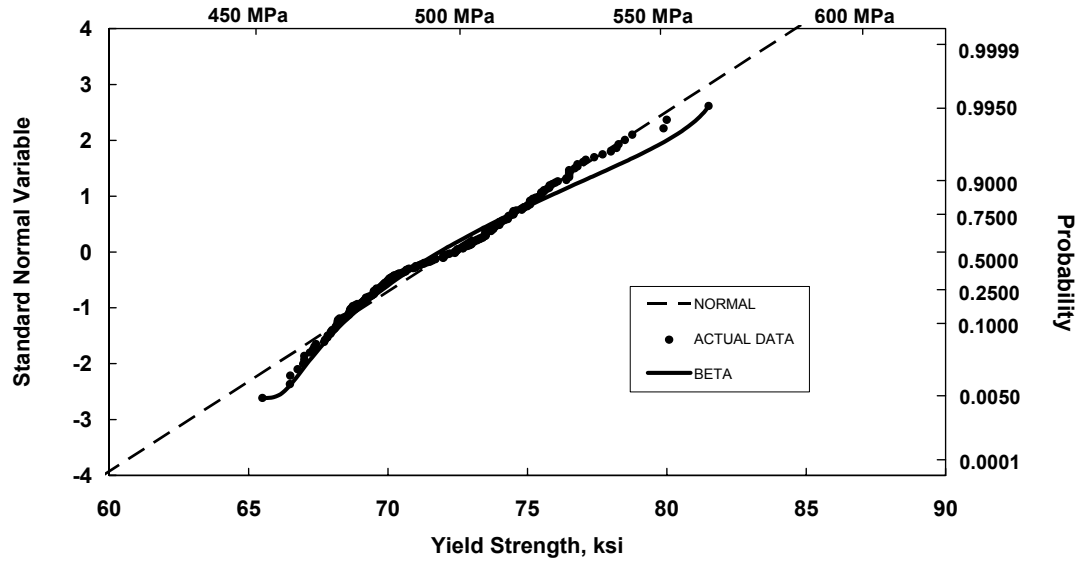


(a)

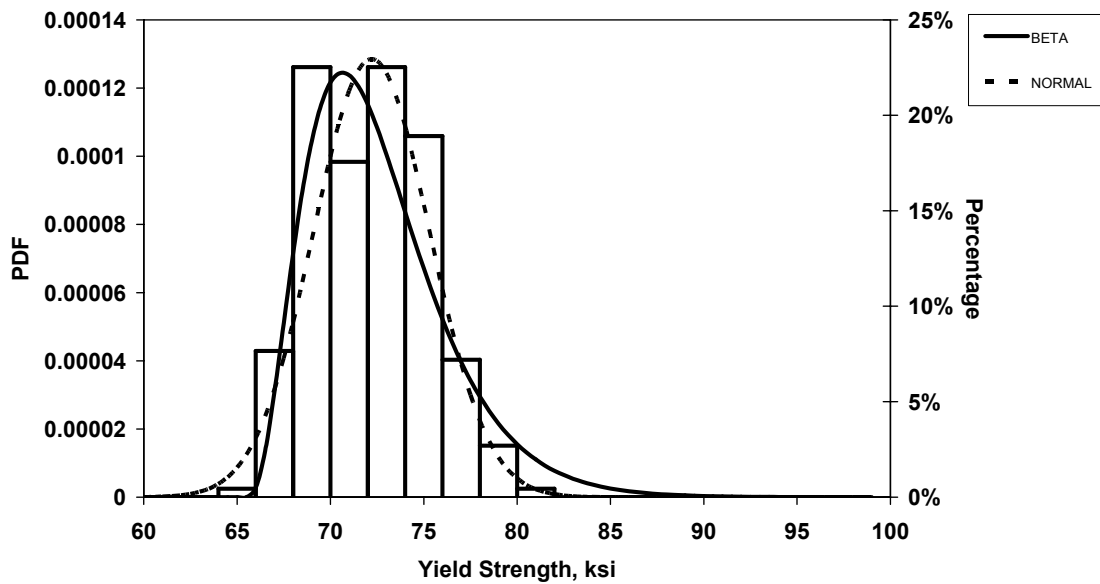


(b)

Figure 26: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 14 bars

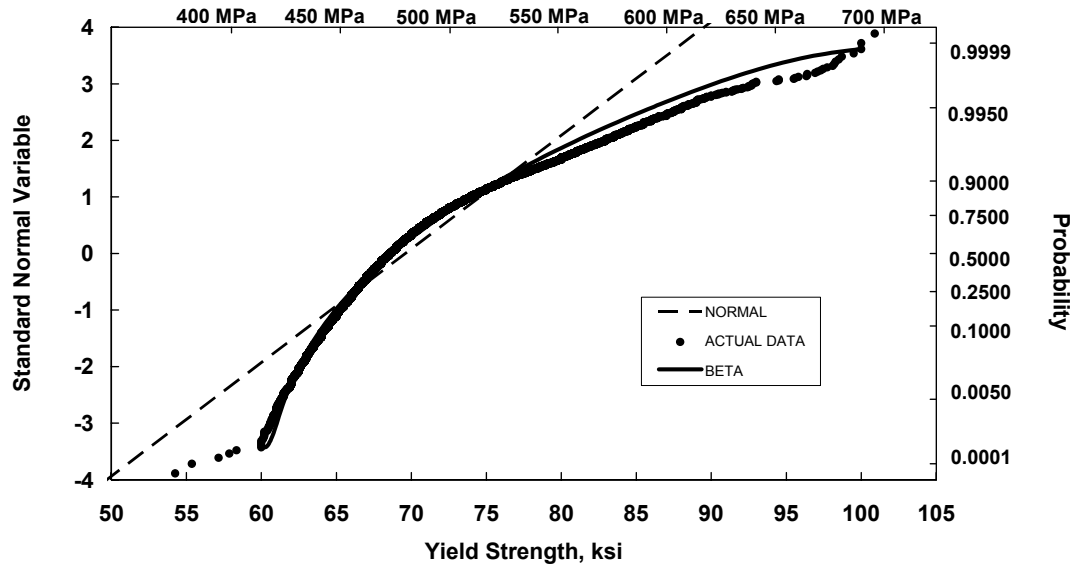


(a)

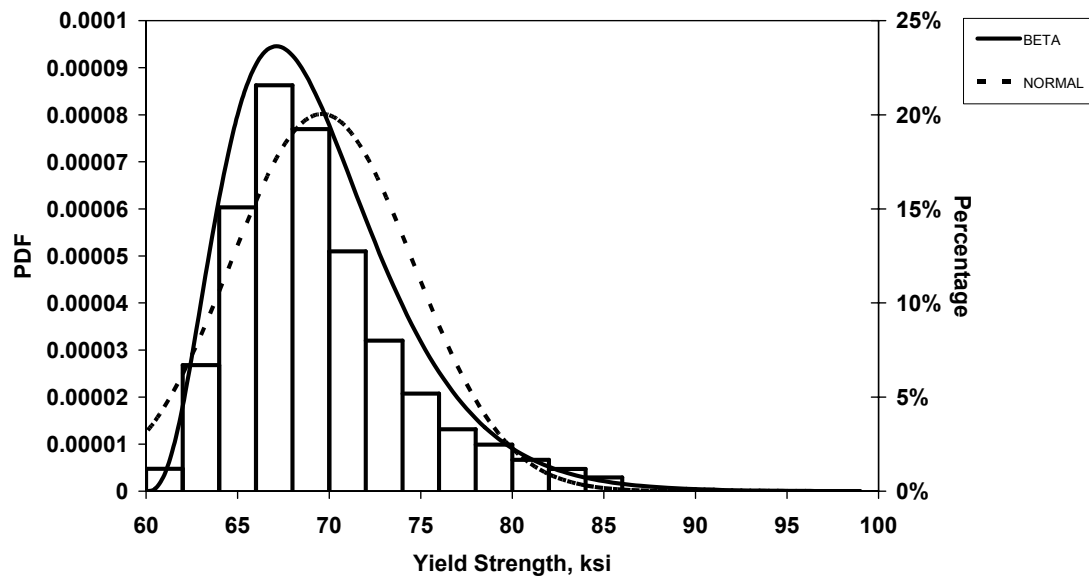


(b)

Figure 27: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 60 No. 18 bars

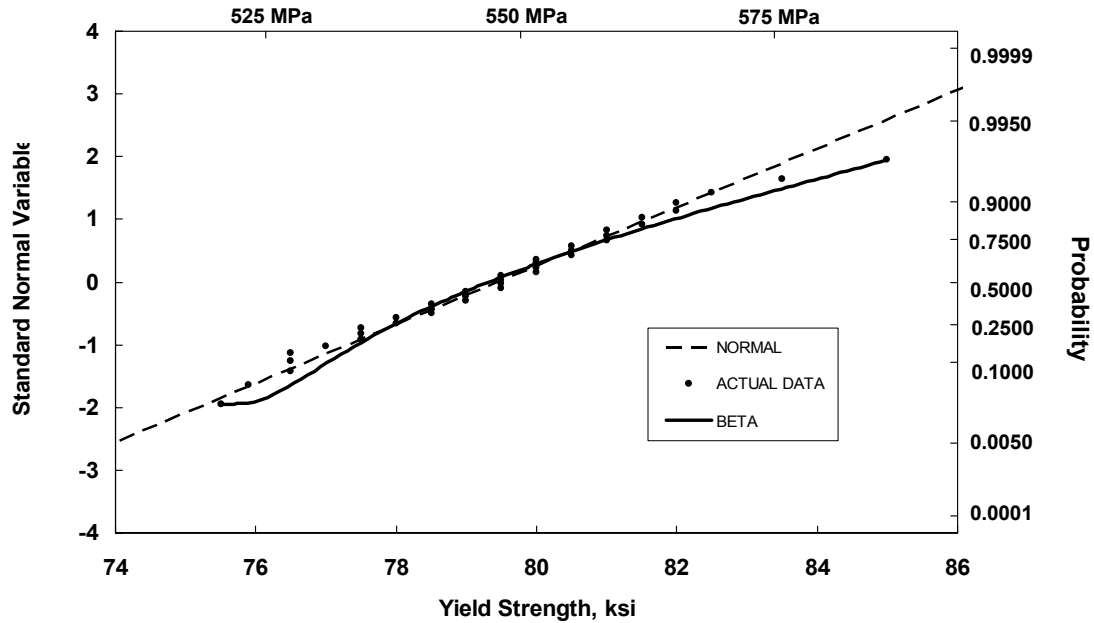


(a)

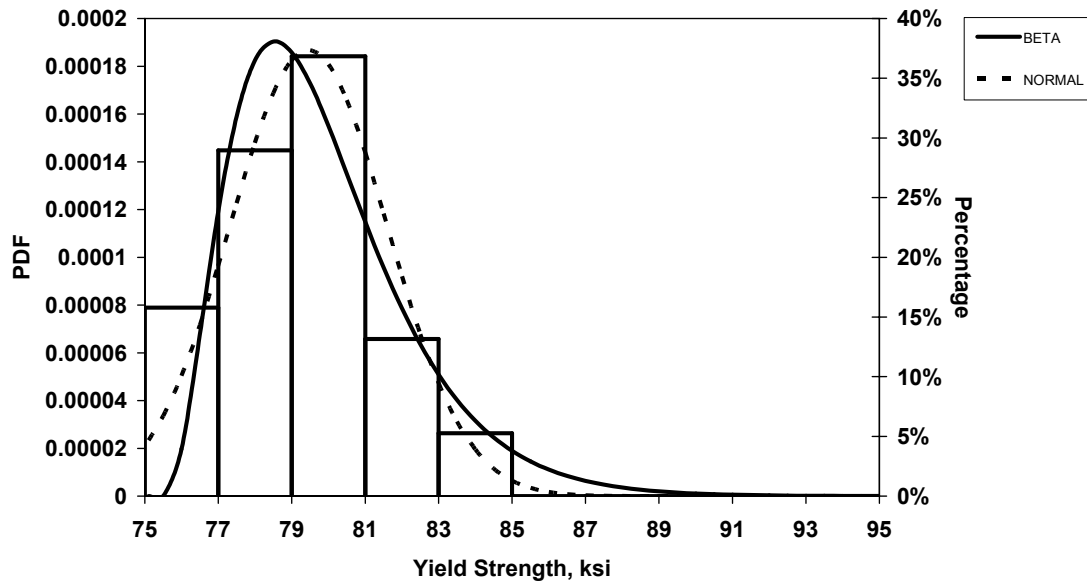


(b)

Figure 28: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of All A 615 Grade 60 bars



(a)



(b)

Figure 29: (a) Cumulative Density Function and (b) Histogram with Probability Density Function of Yield Strength of A 615 Grade 75 No. 4 bars

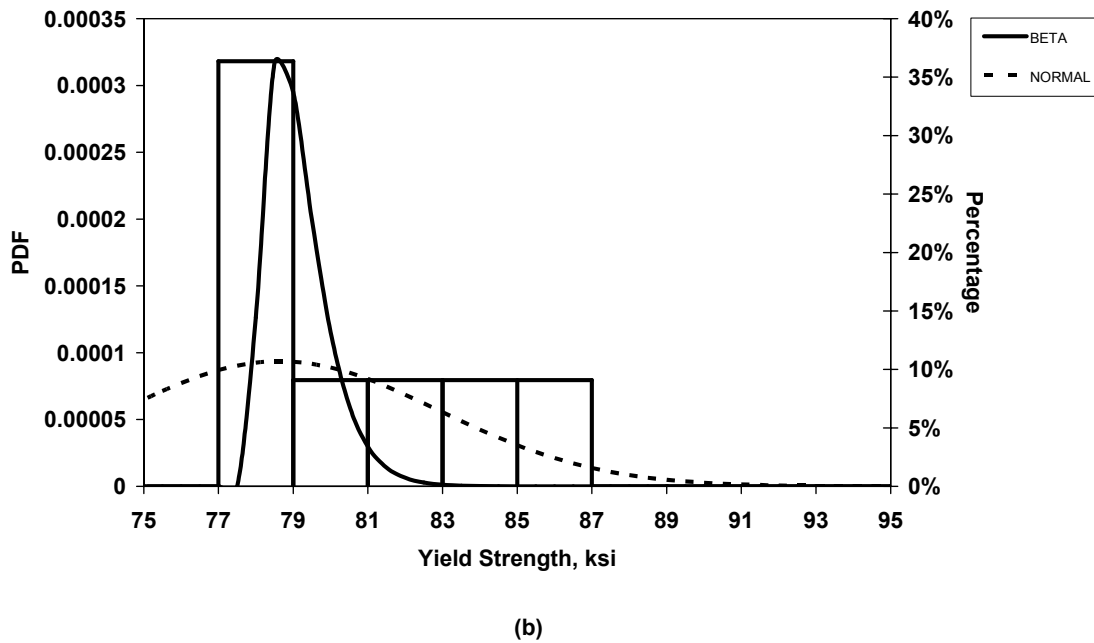
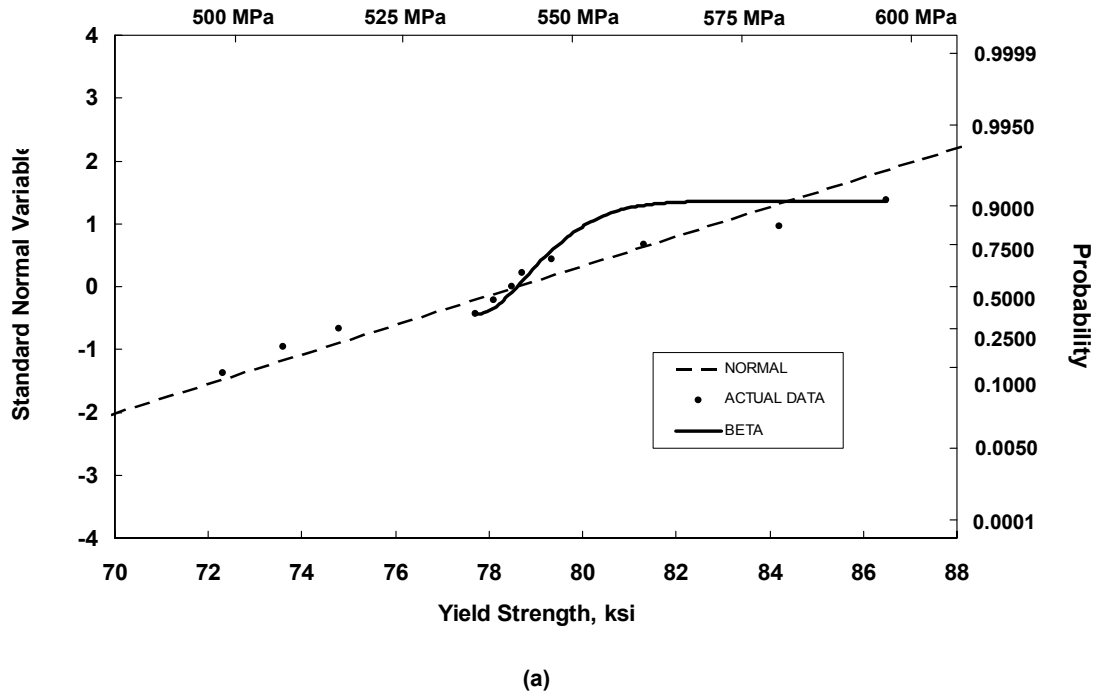
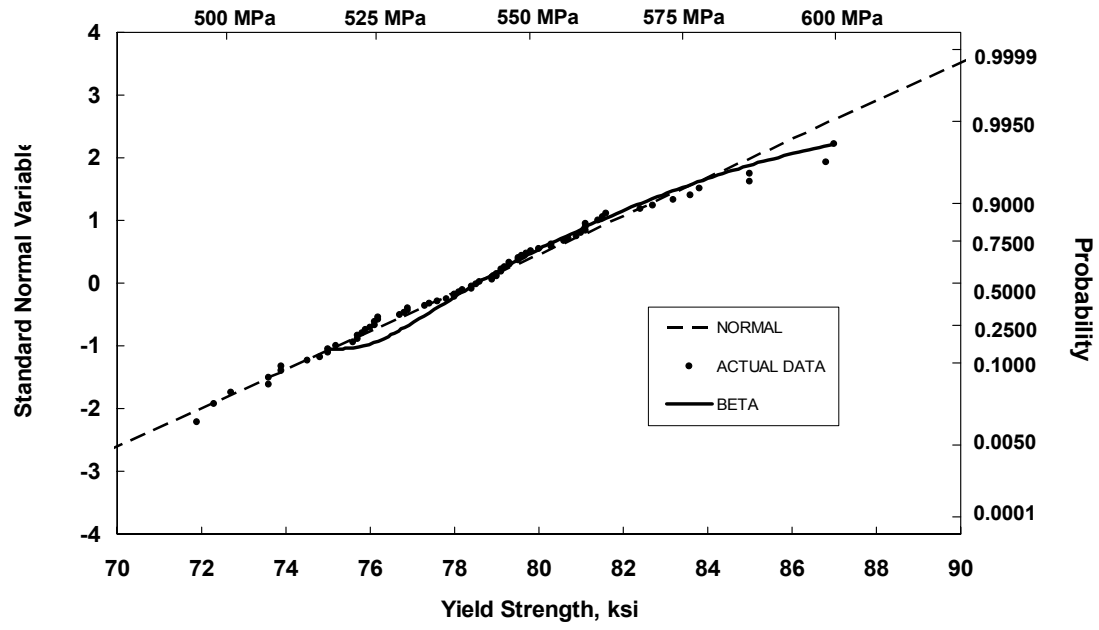
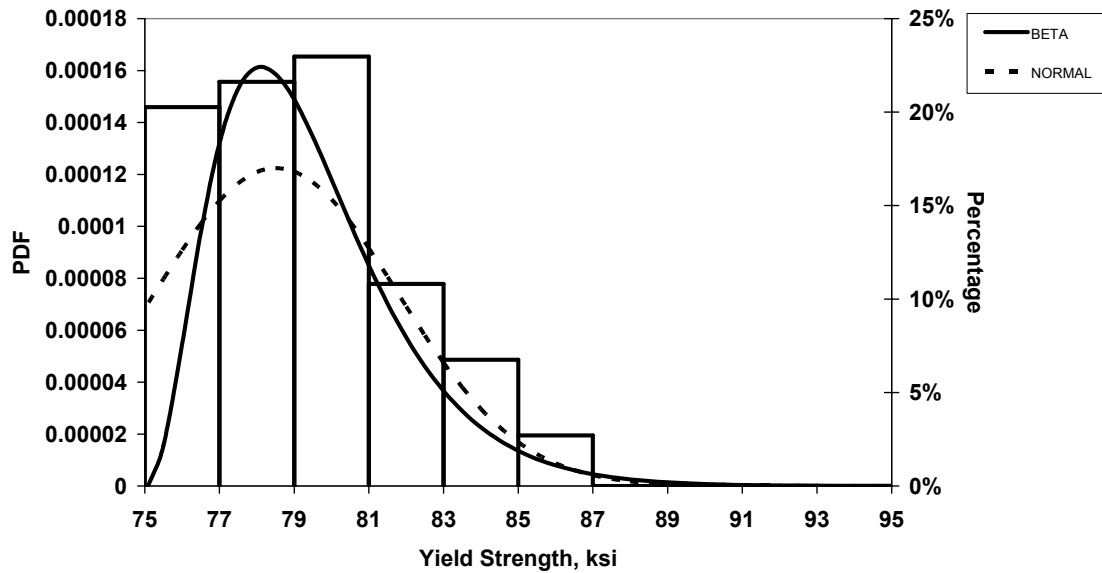


Figure 30: (a) Cumulative Density Function and (b) Histogram with Probability Density Function of Yield Strength for A 615 Grade 75 No. 5 bars

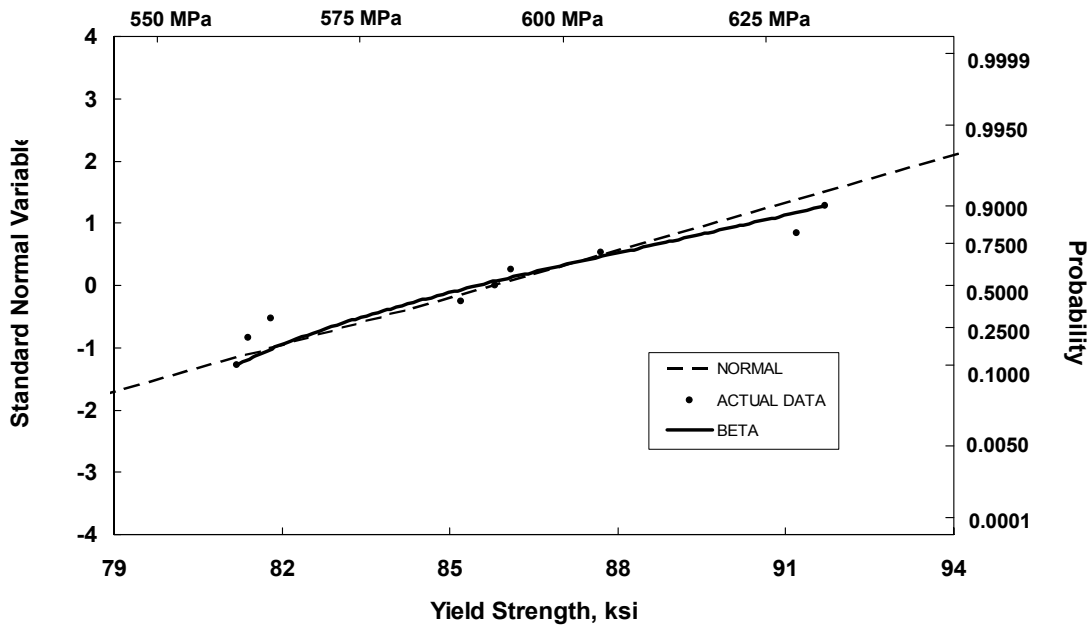


(a)



(b)

Figure 31: (a) Cumulative Density Function and (b) Histogram with Probability Density Function of Yield Strength for A 615 Grade 75 No. 6 bars



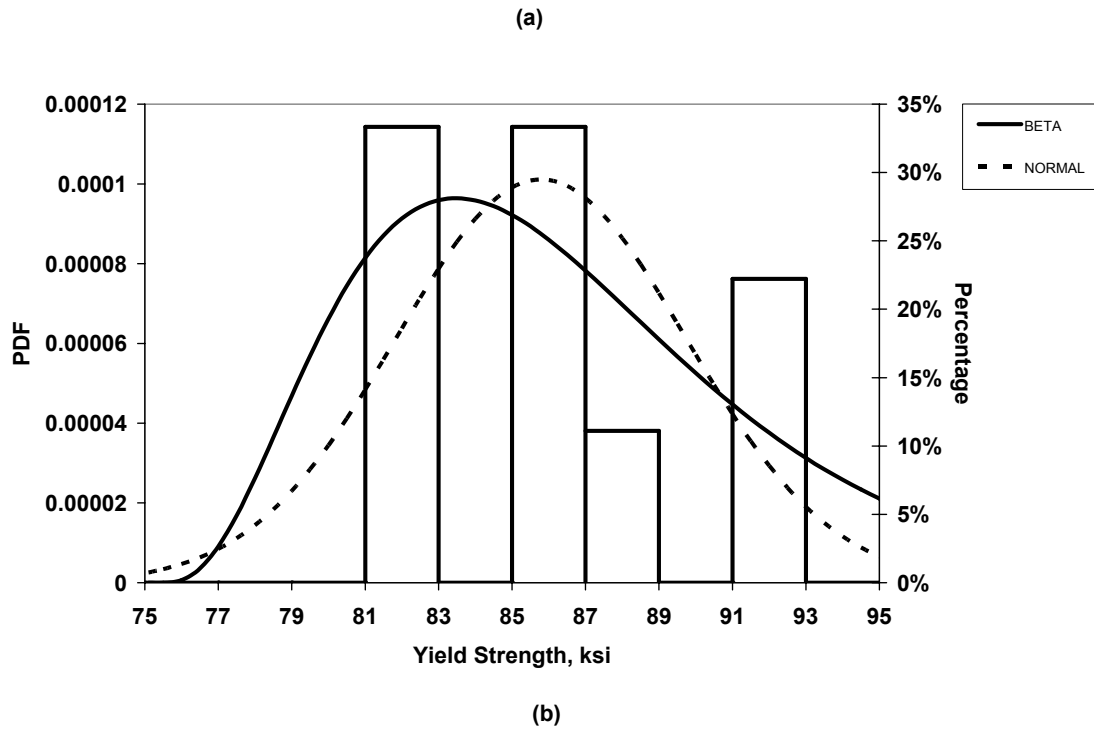
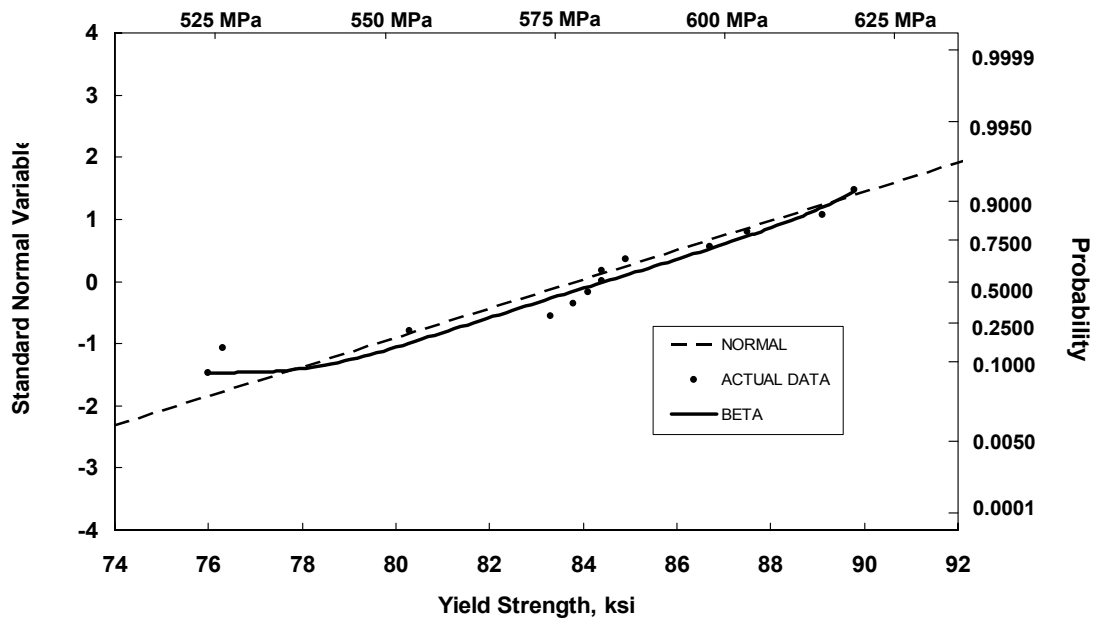


Figure 32: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 75 No. 7 bars



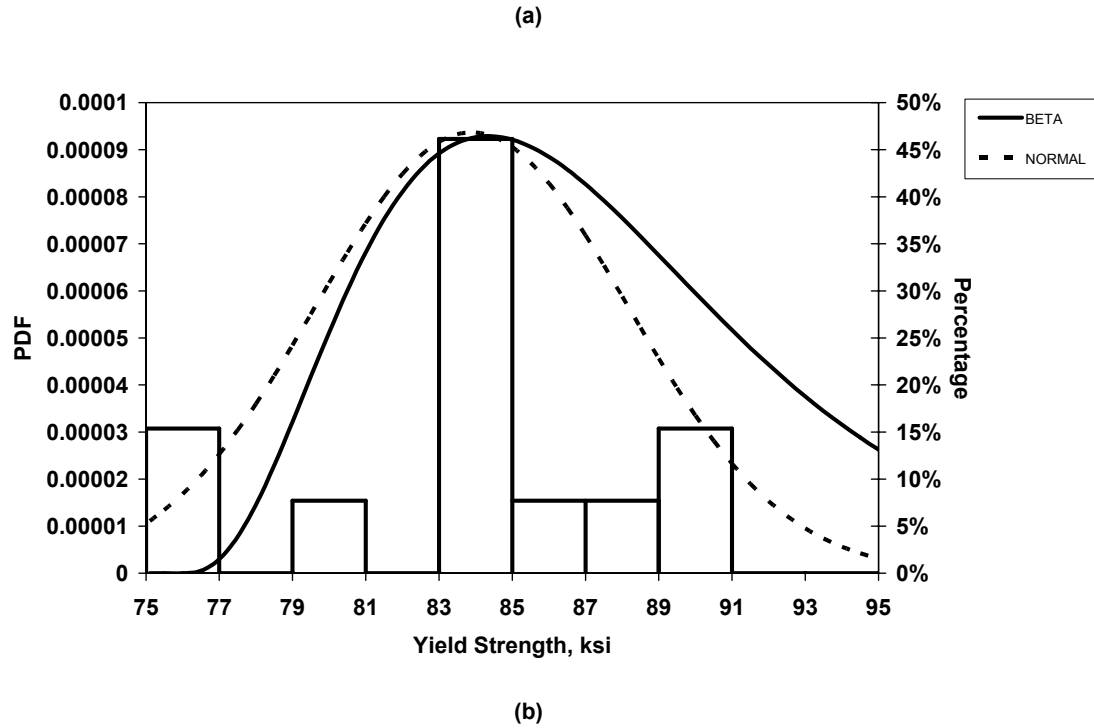
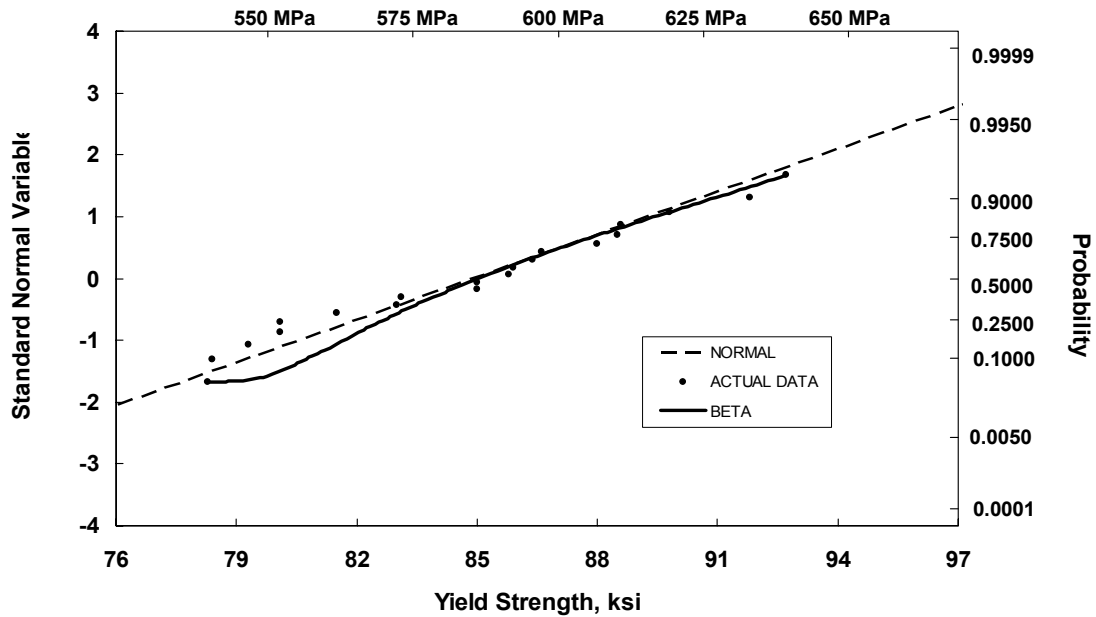


Figure 33: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 75 No. 8 bars



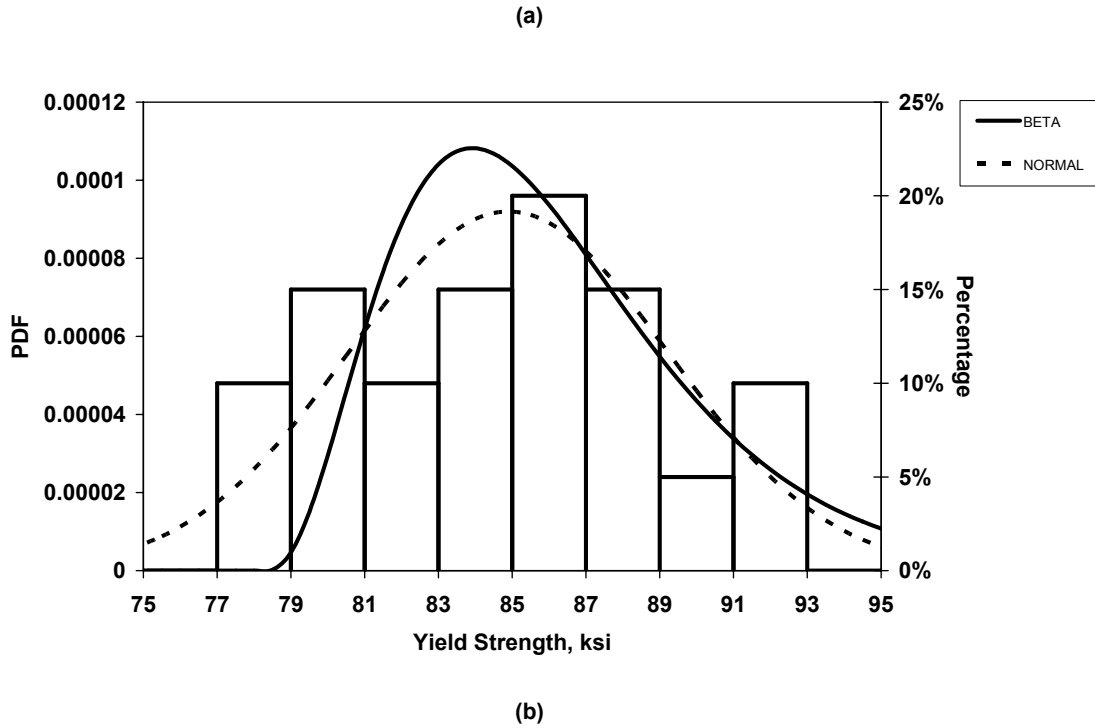
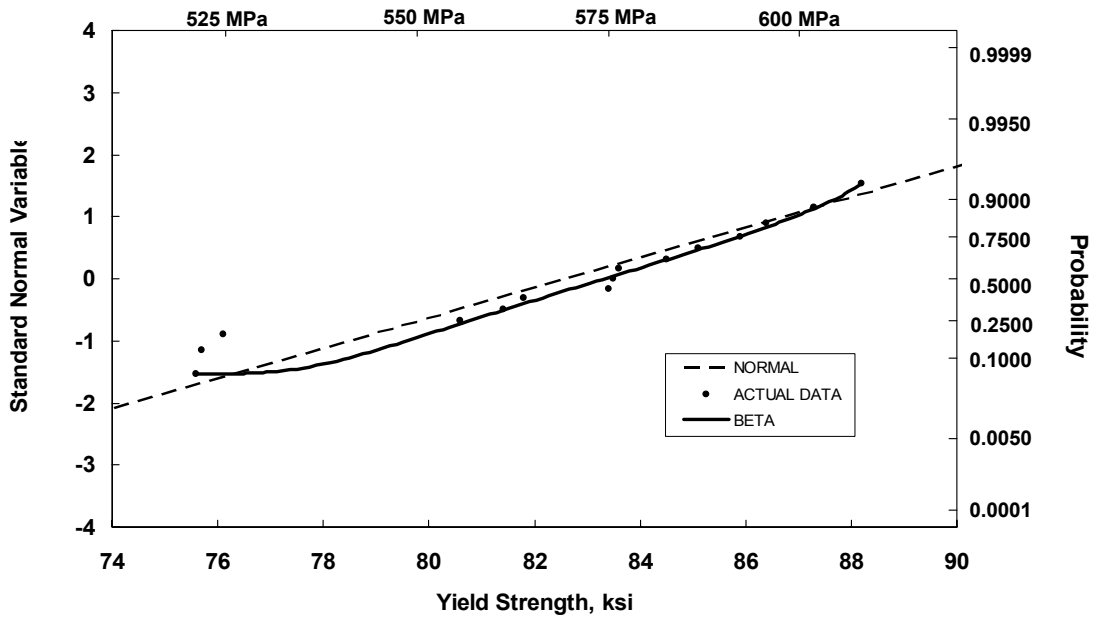


Figure 34: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 75 No. 9 bars



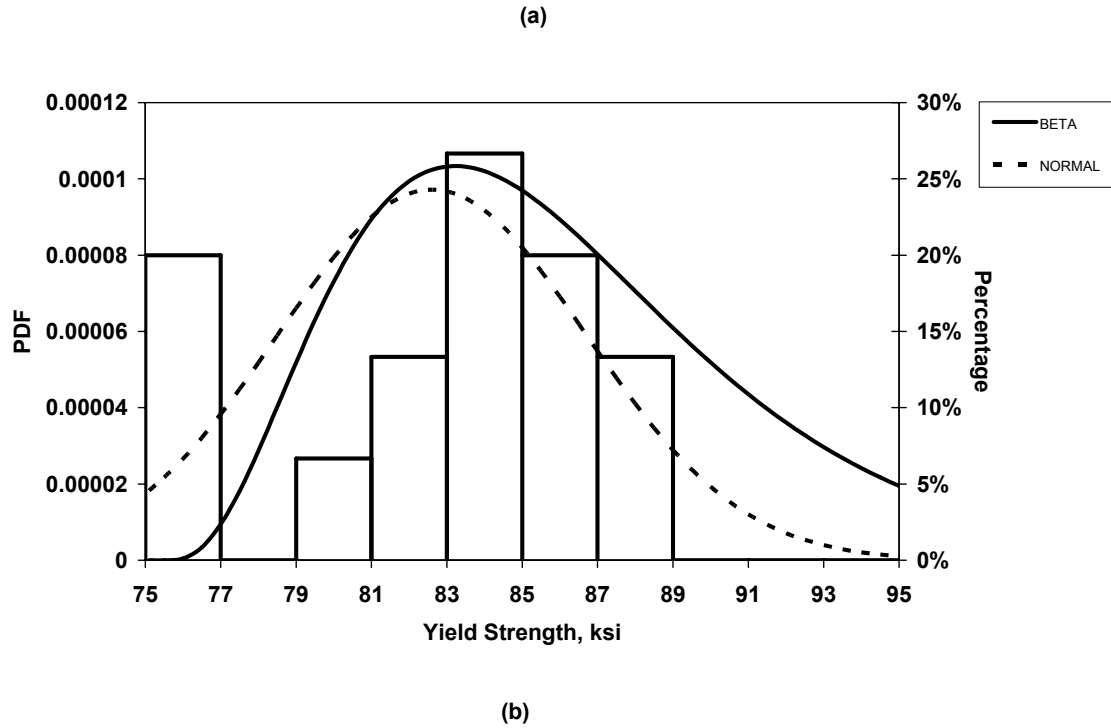
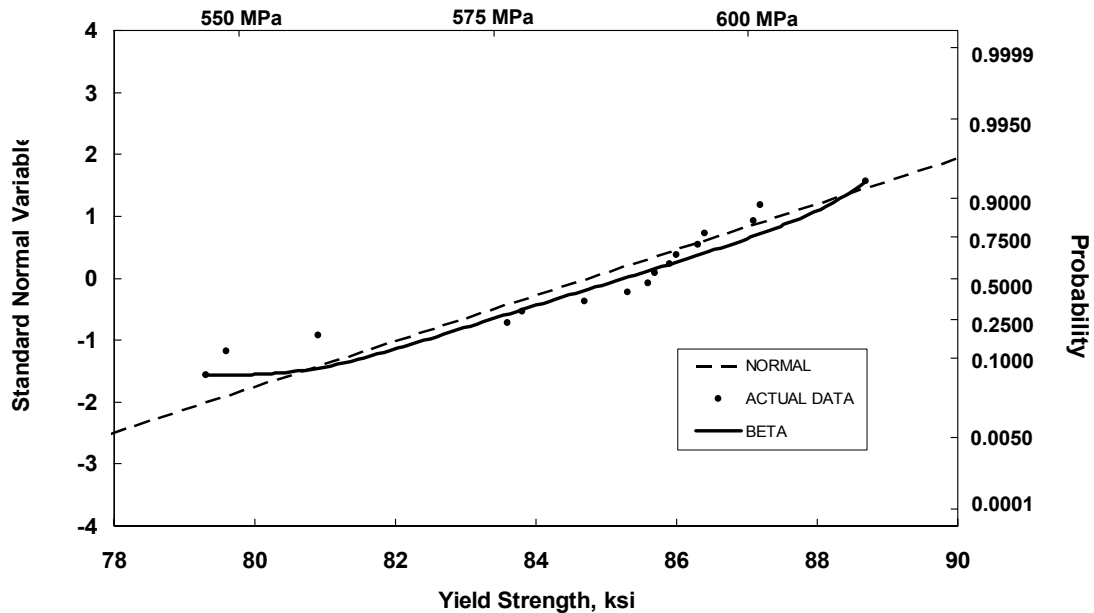


Figure 35: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 75 No. 10 bars



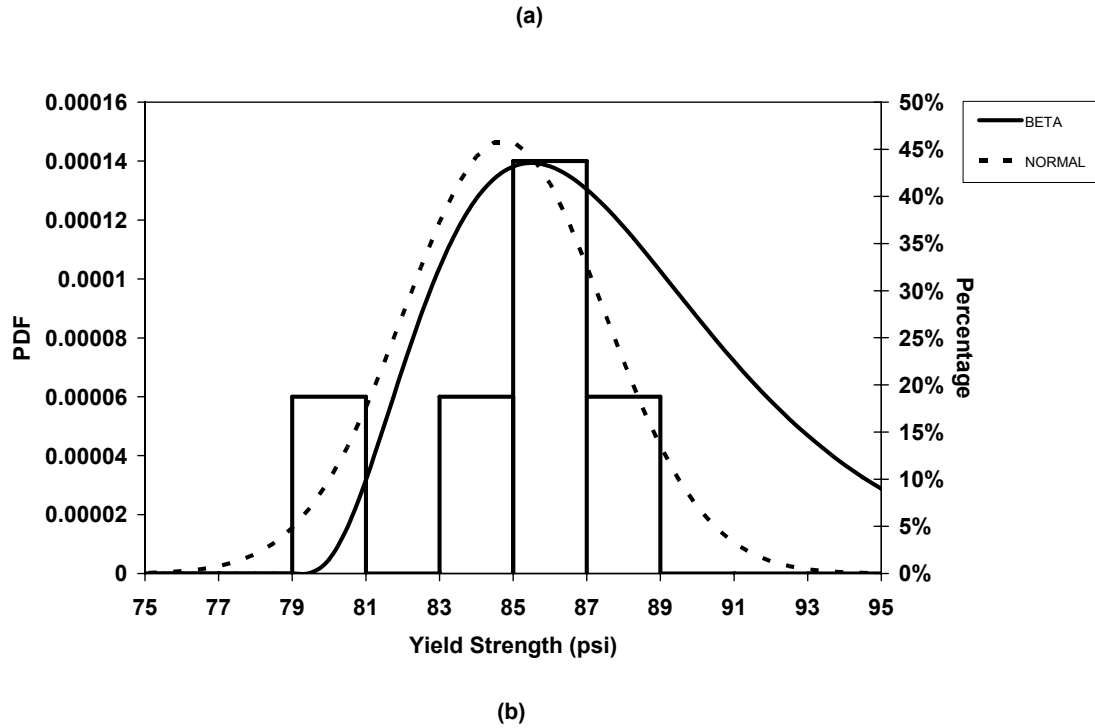
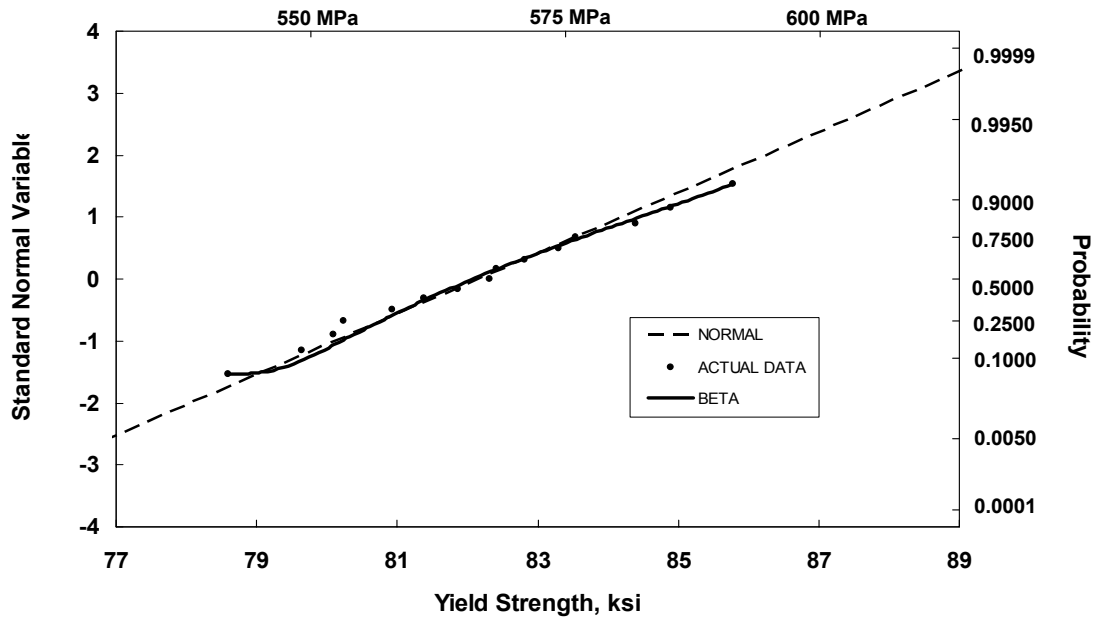


Figure 36: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 75 No. 11 bars



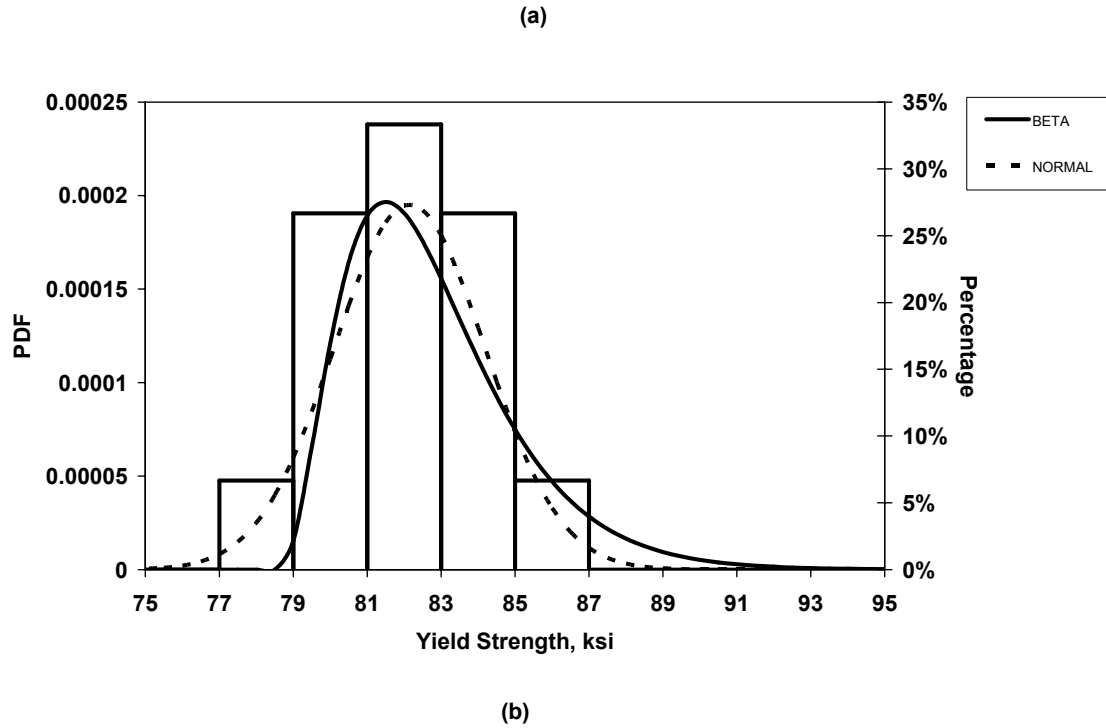
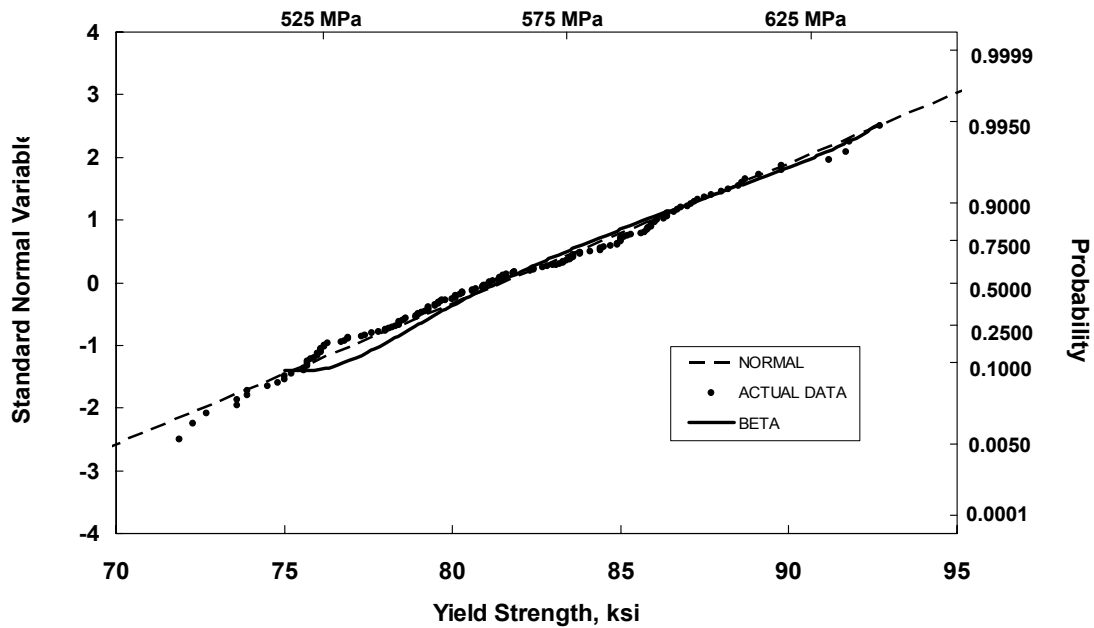


Figure 37: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 615 Grade 75 No. 14 bars



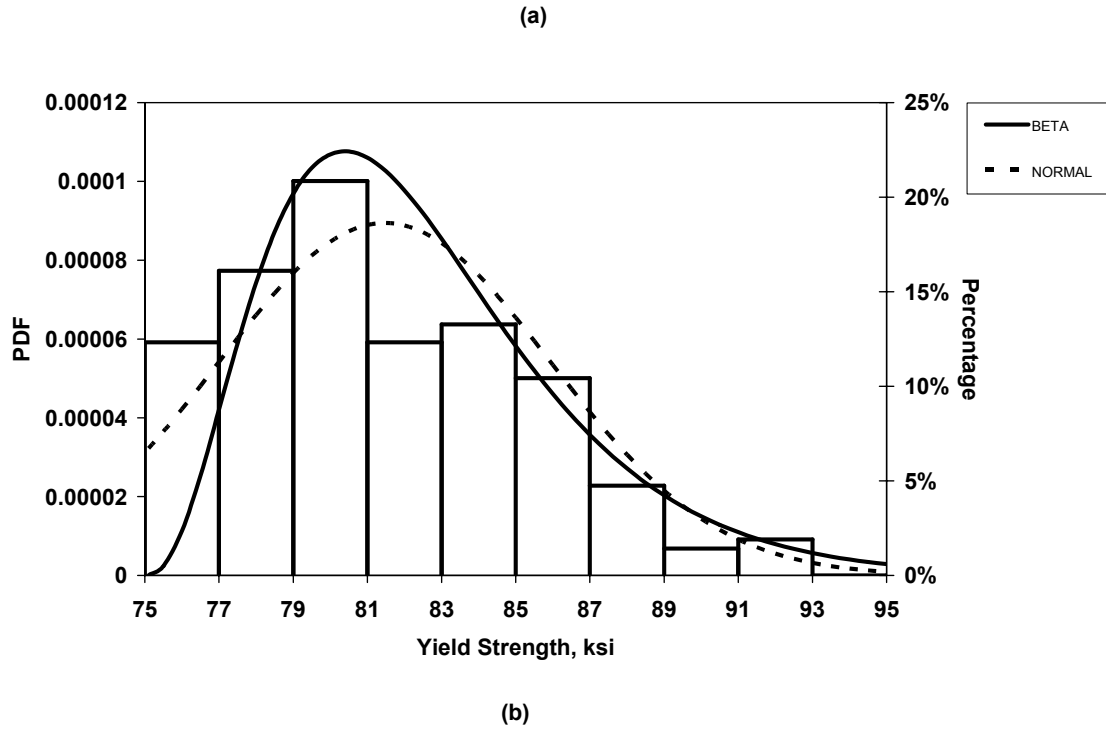
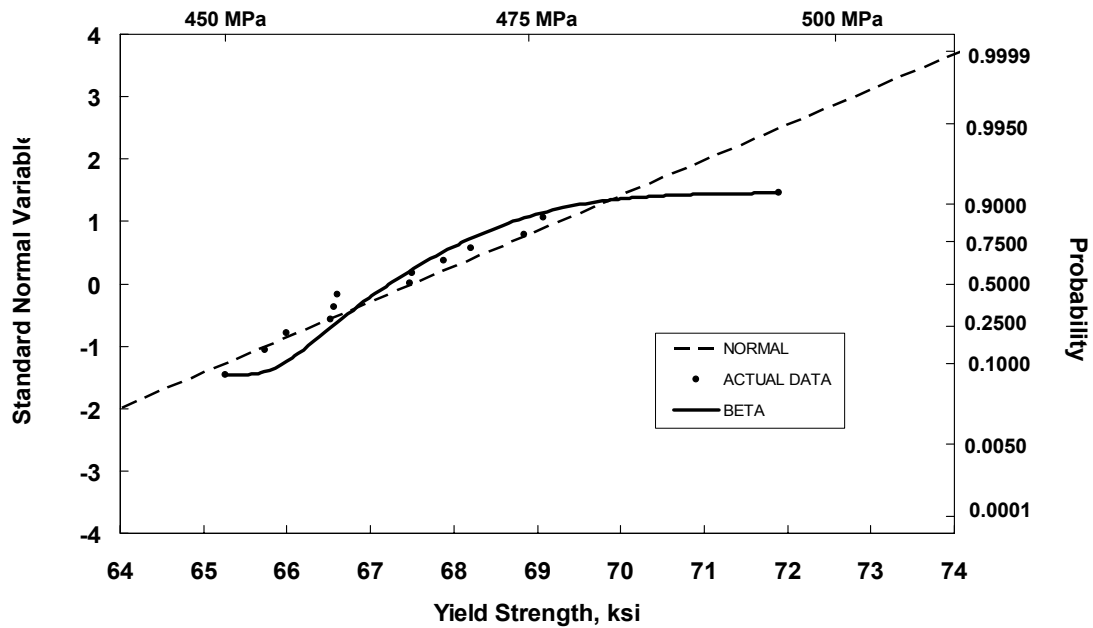
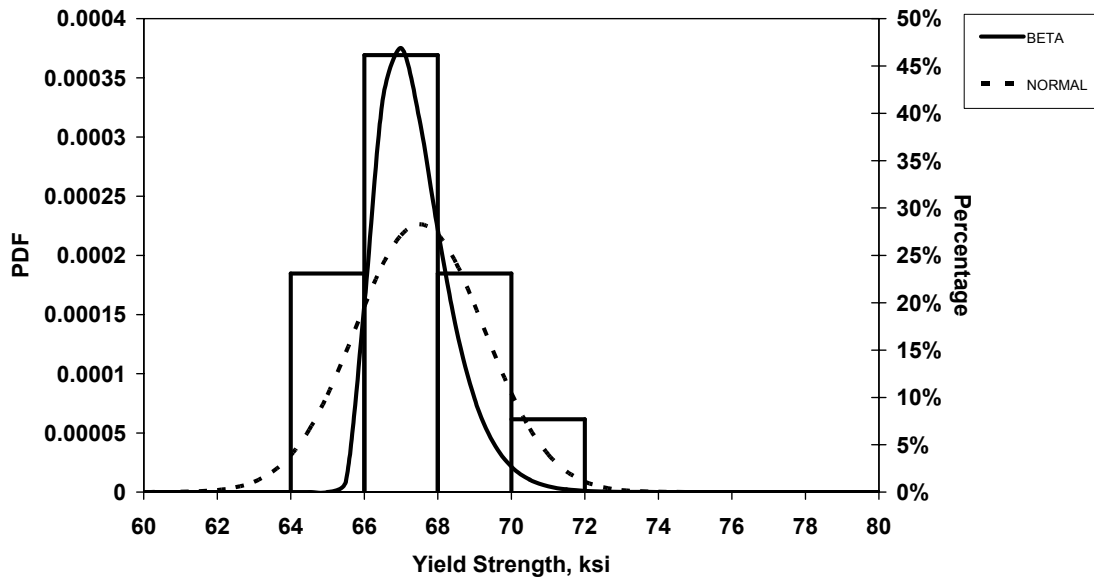


Figure 38: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of All A 615 Grade 75 bars

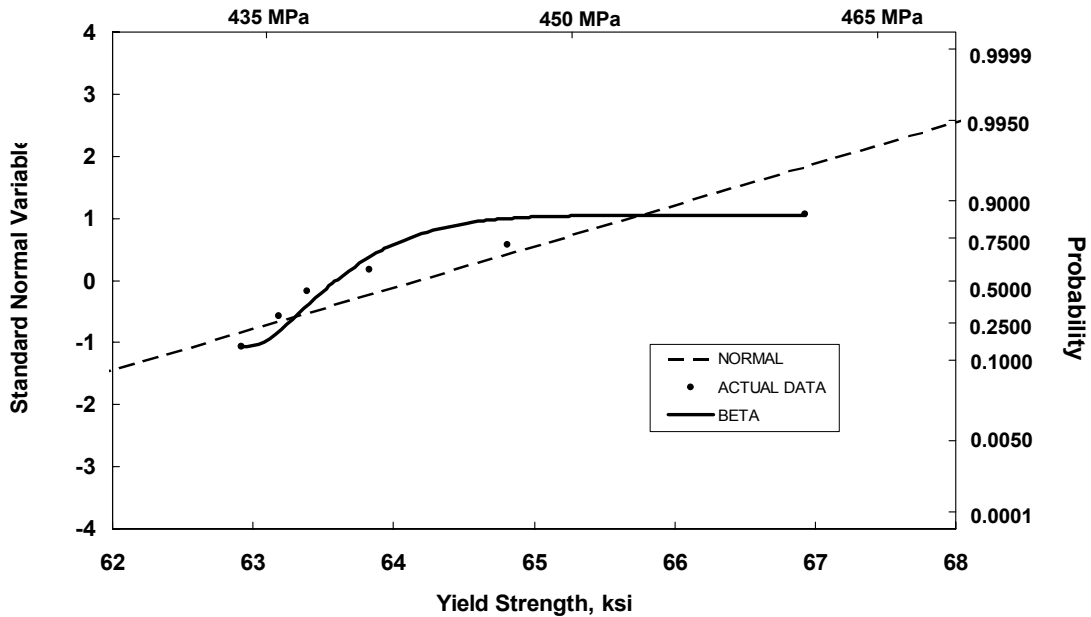


(a)



(b)

Figure 39: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 616 Grade 60 No. 8 bars



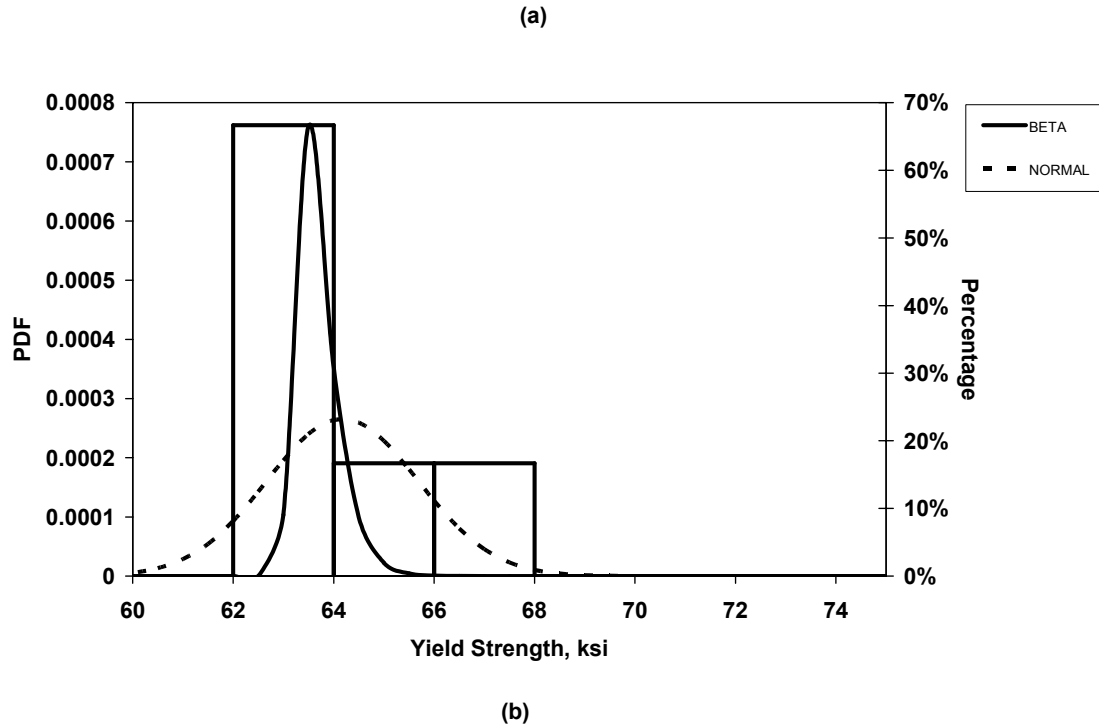
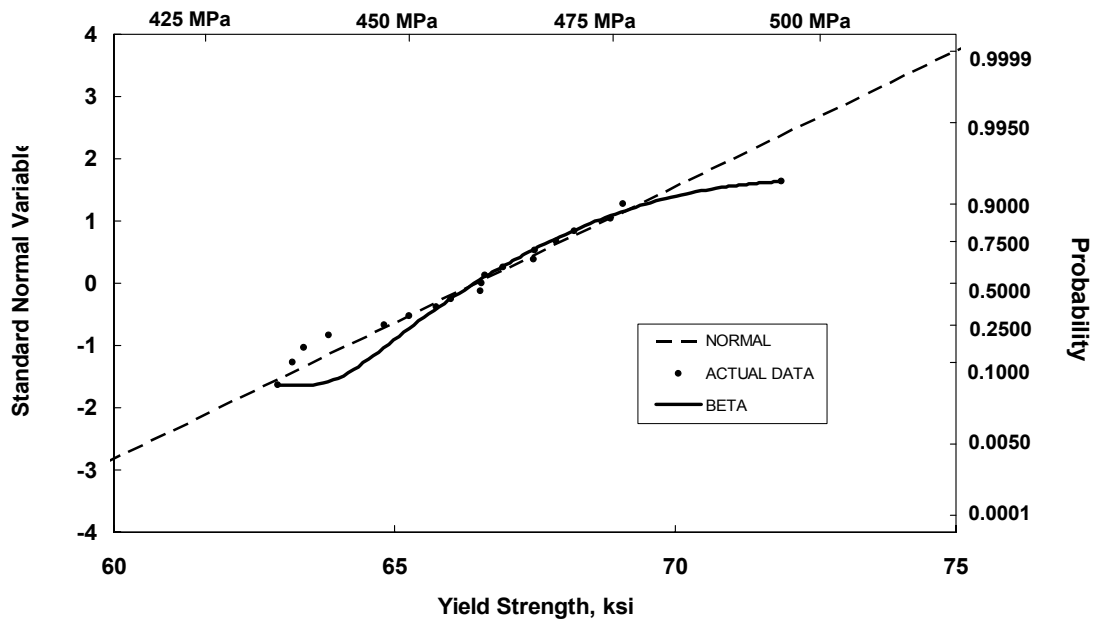


Figure 40: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 616 Grade 60 No. 10 bars



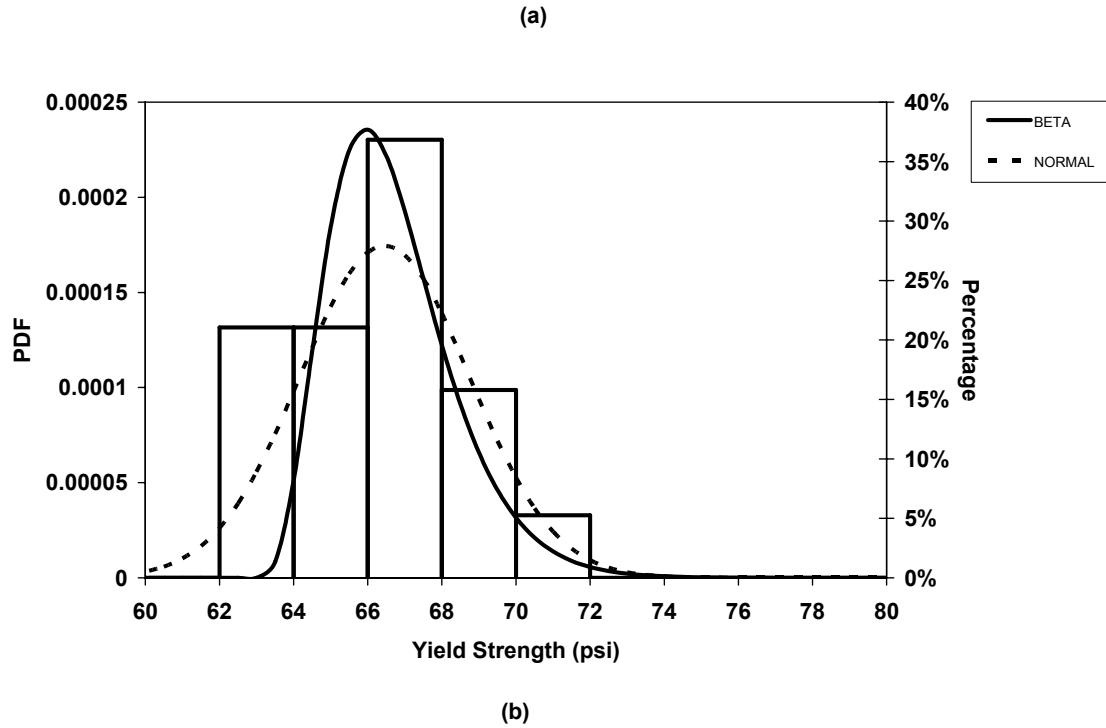
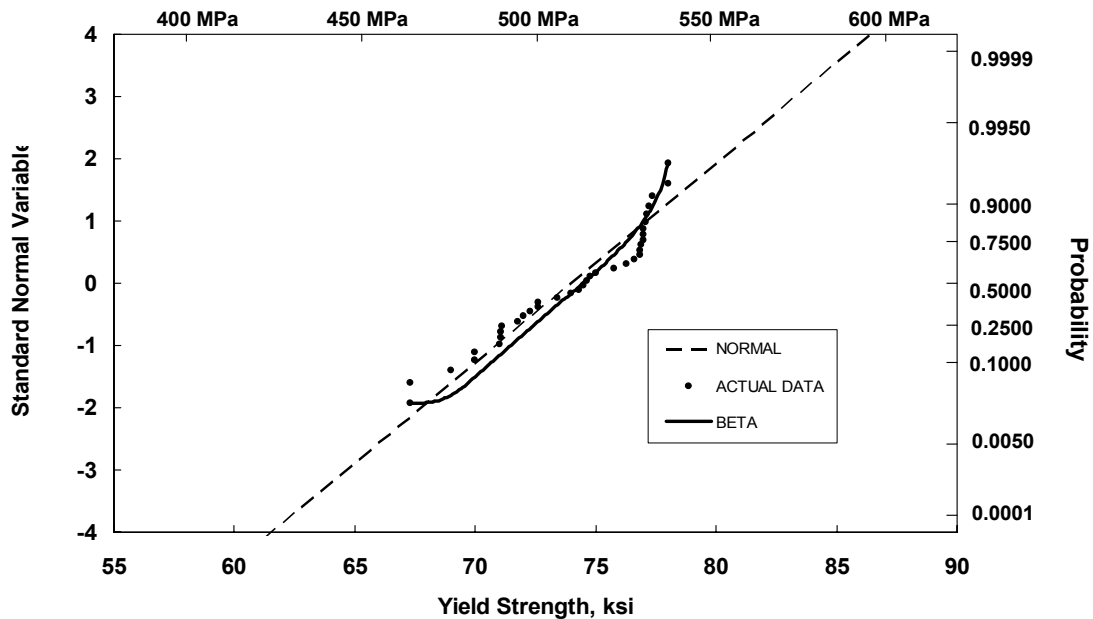


Figure 41: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of All A 616 Grade 60 bars



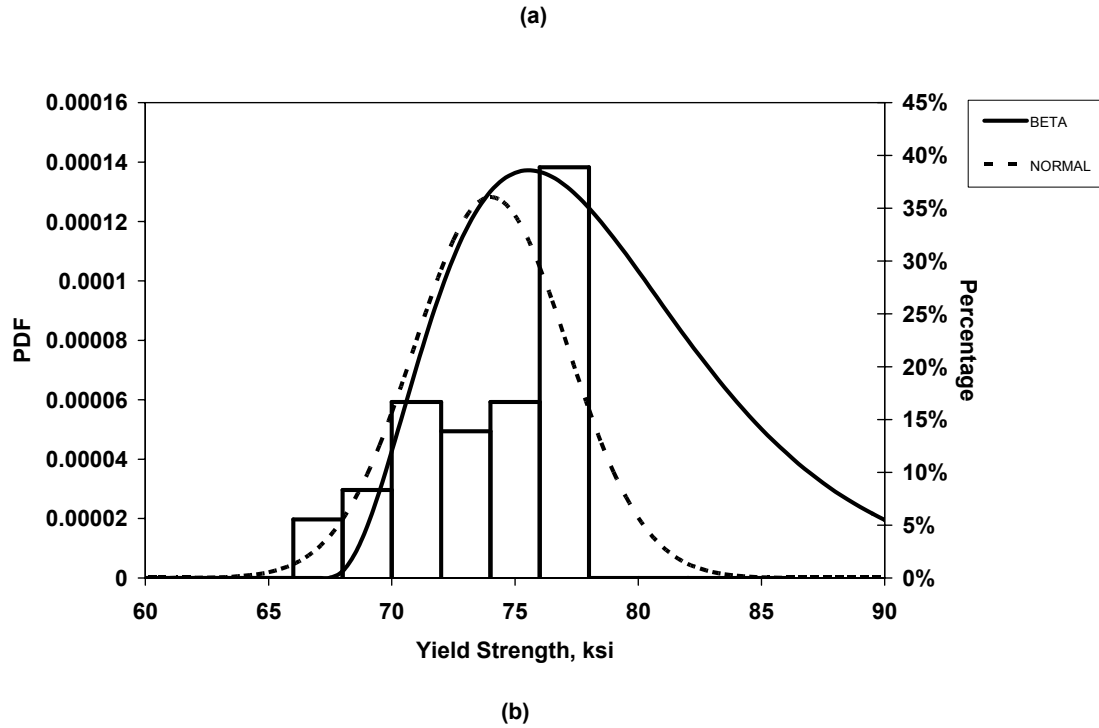
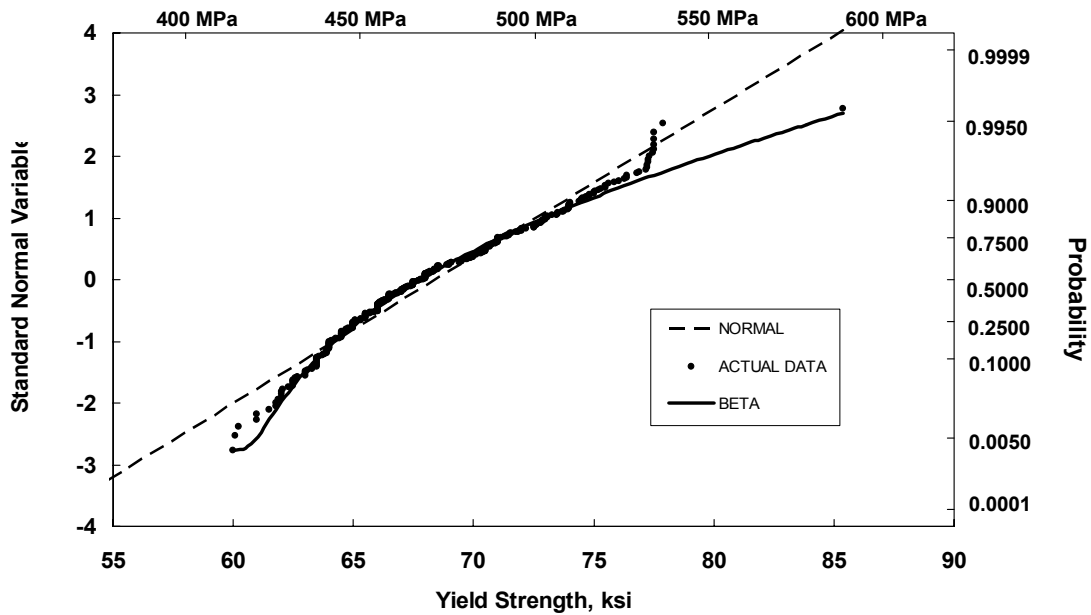
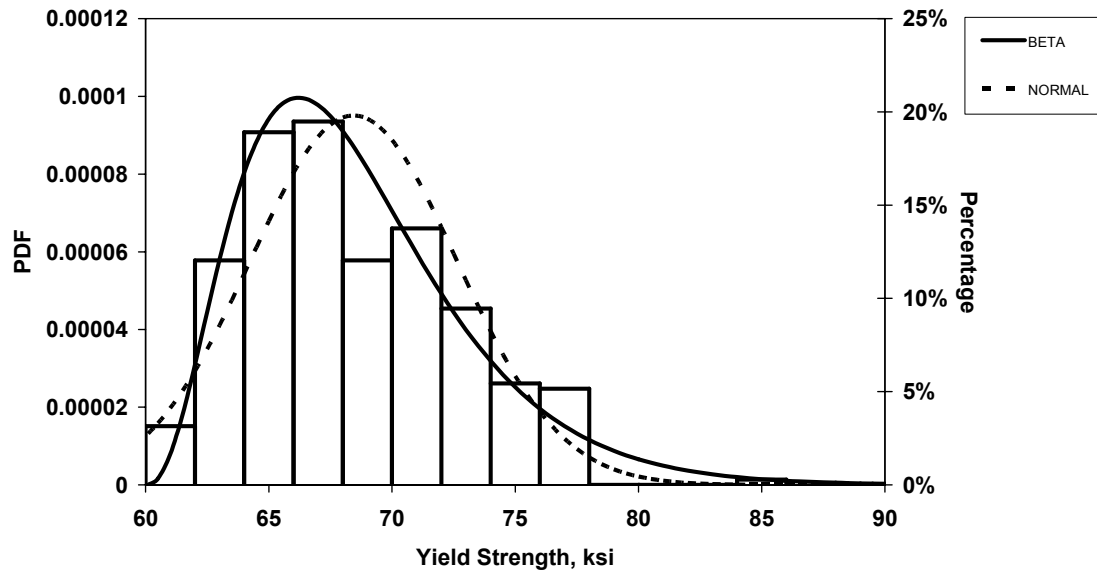


Figure 42: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 3 bars

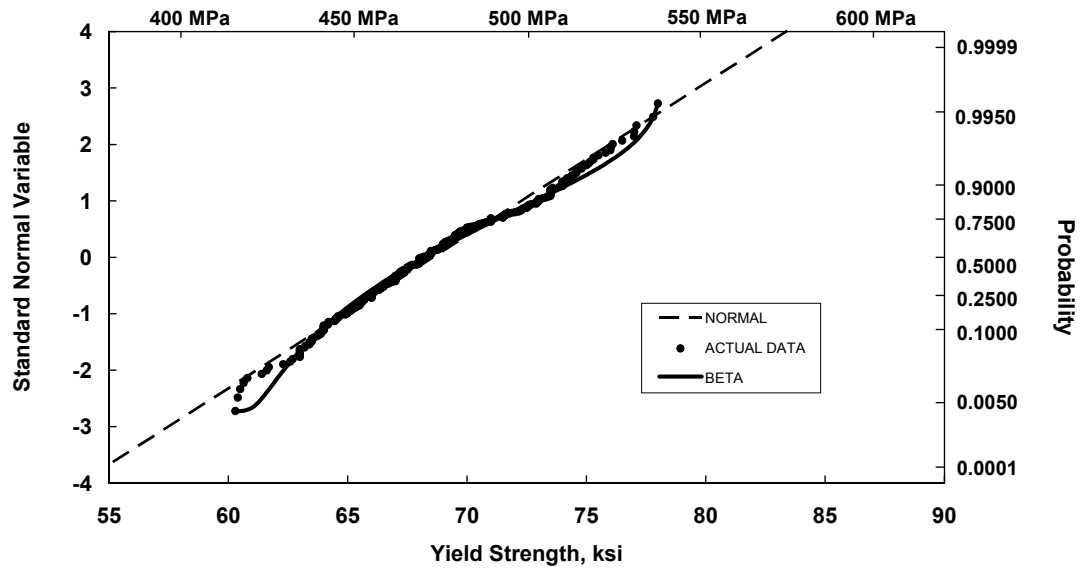


(a)

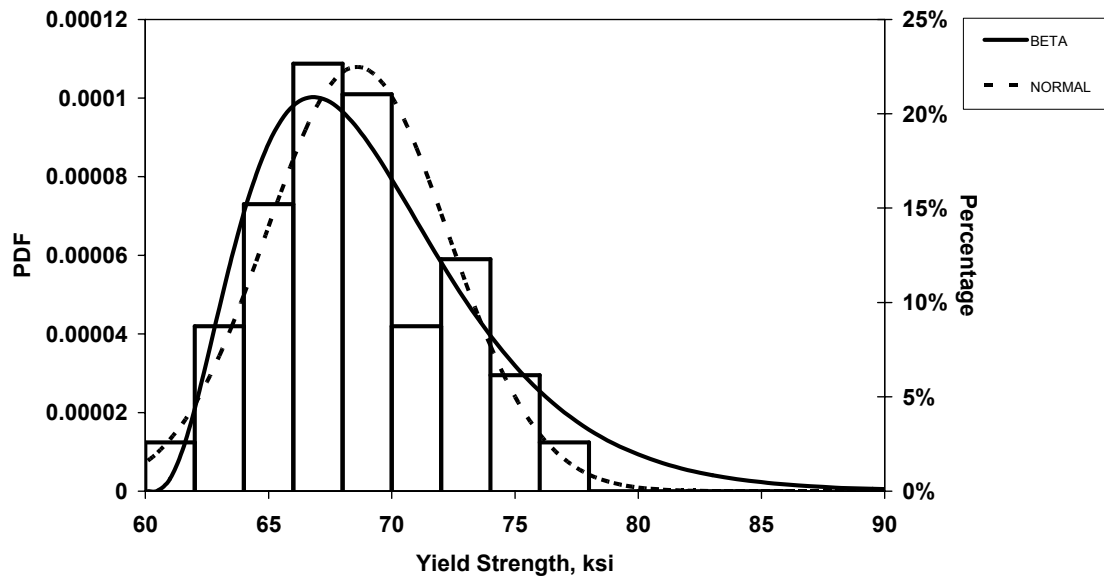


(b)

Figure 43: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 4 bars

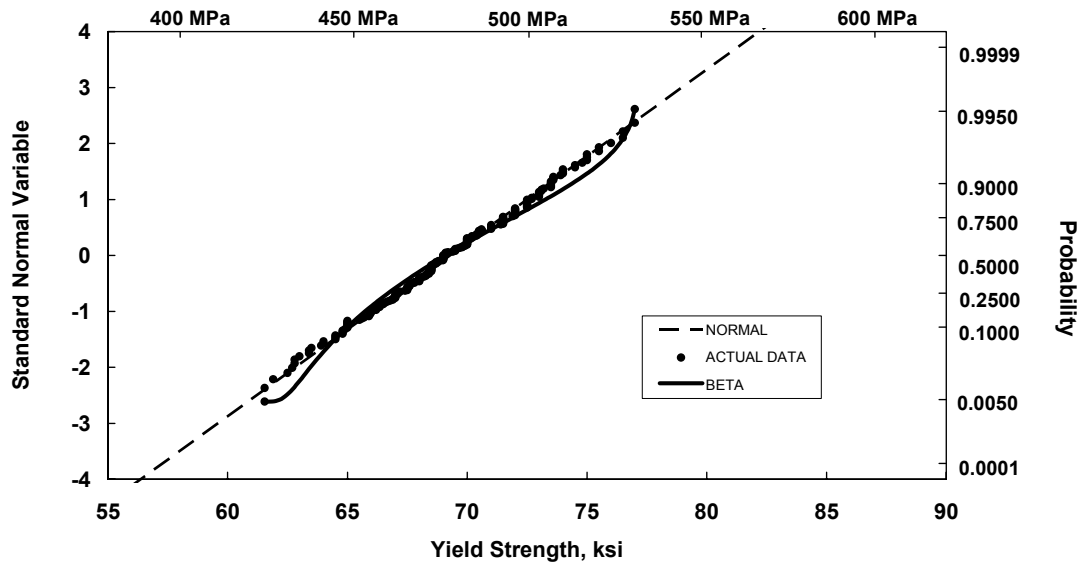


(a)

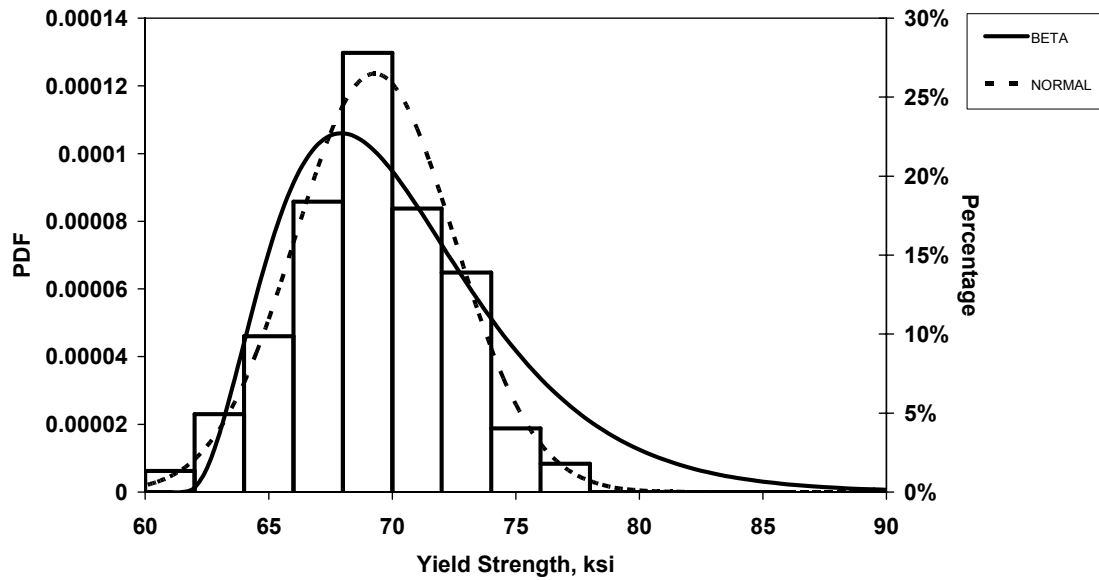


(b)

Figure 44: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 5 bars

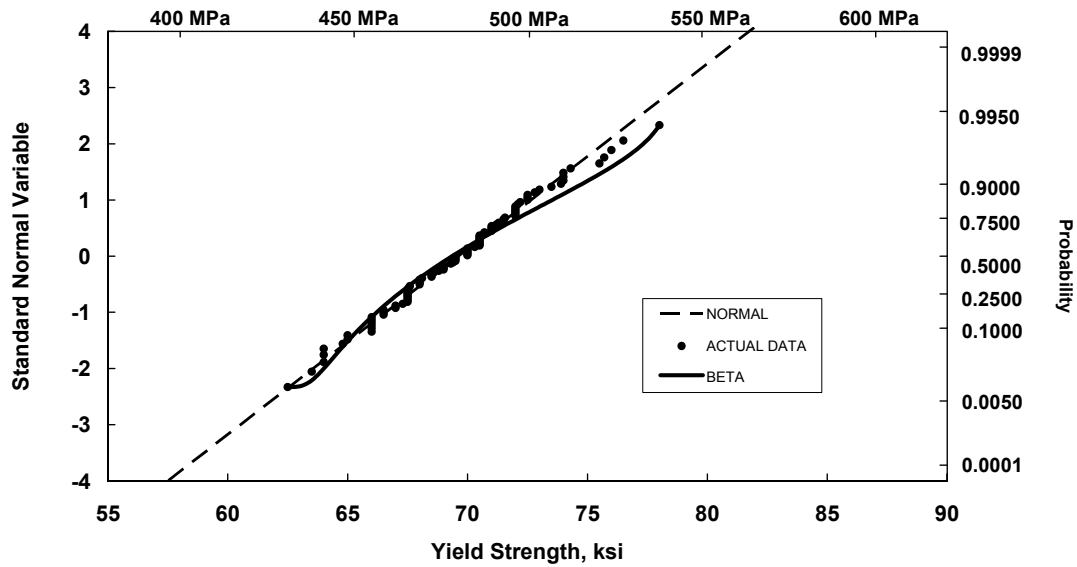


(a)

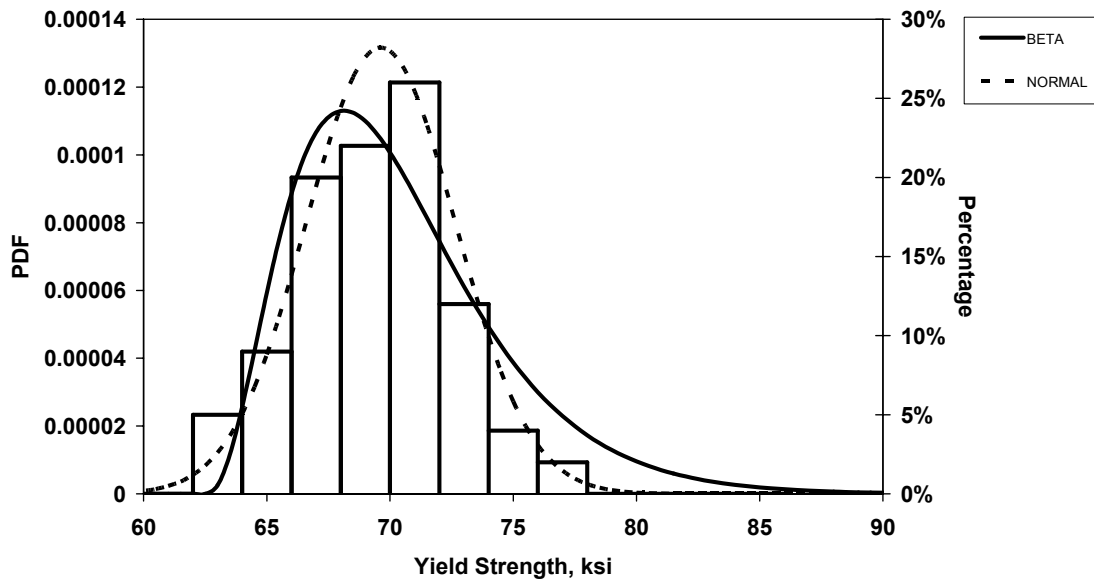


(b)

Figure 45: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 6 bars

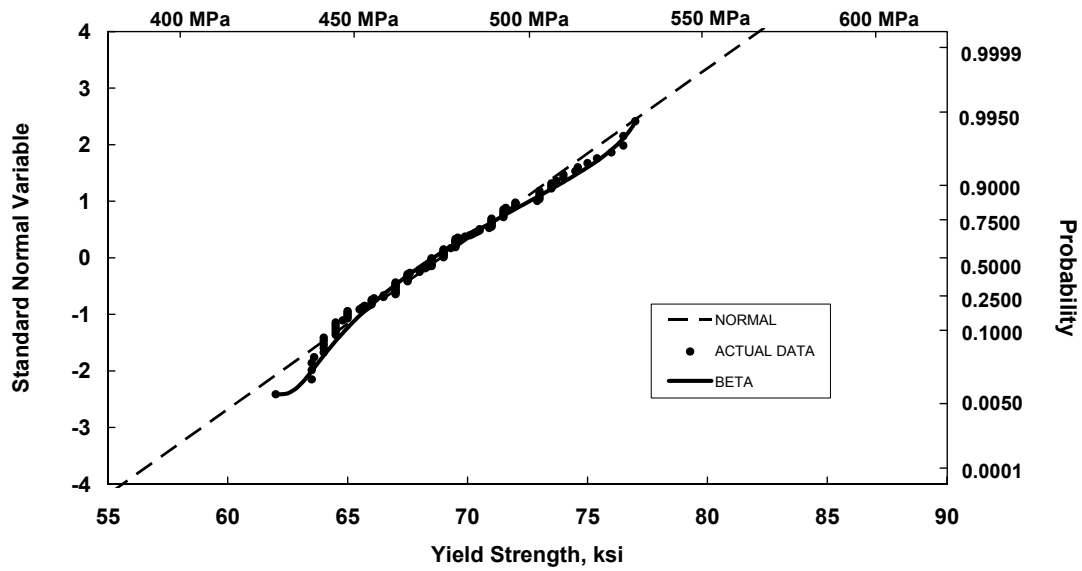


(a)

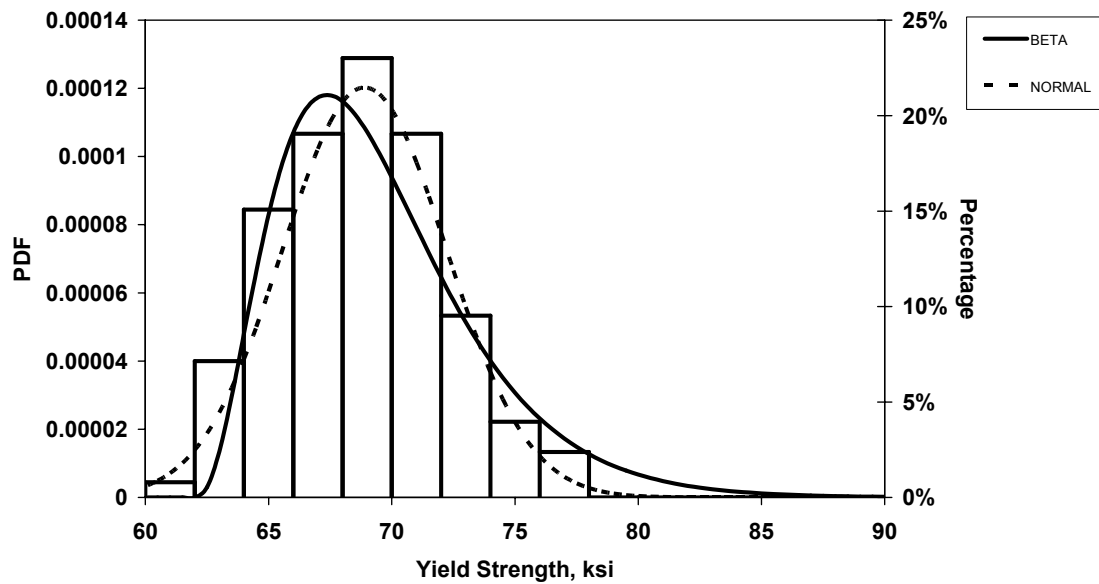


(b)

Figure 46: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 7 bars

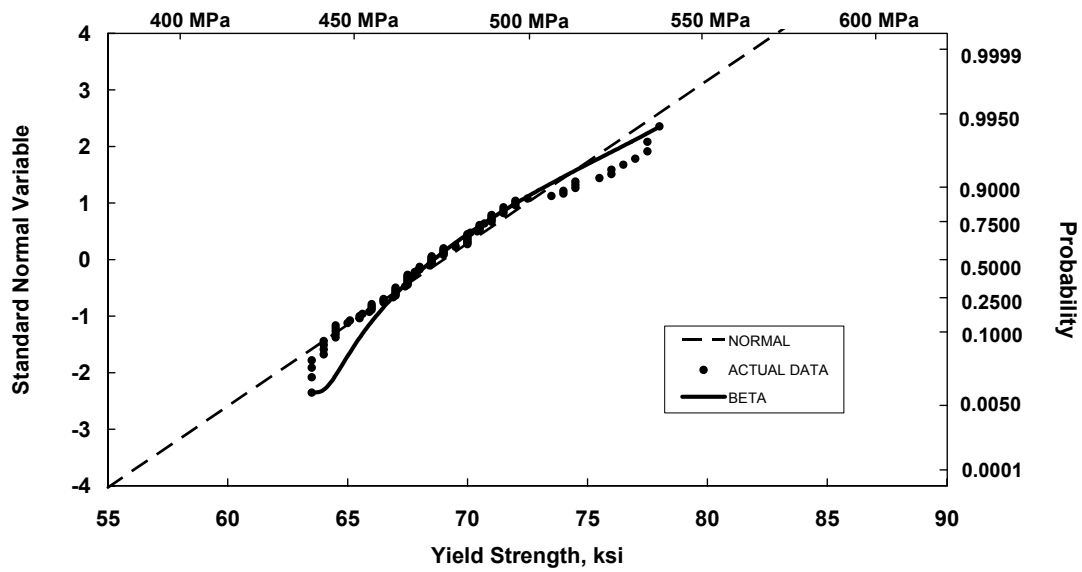


(a)

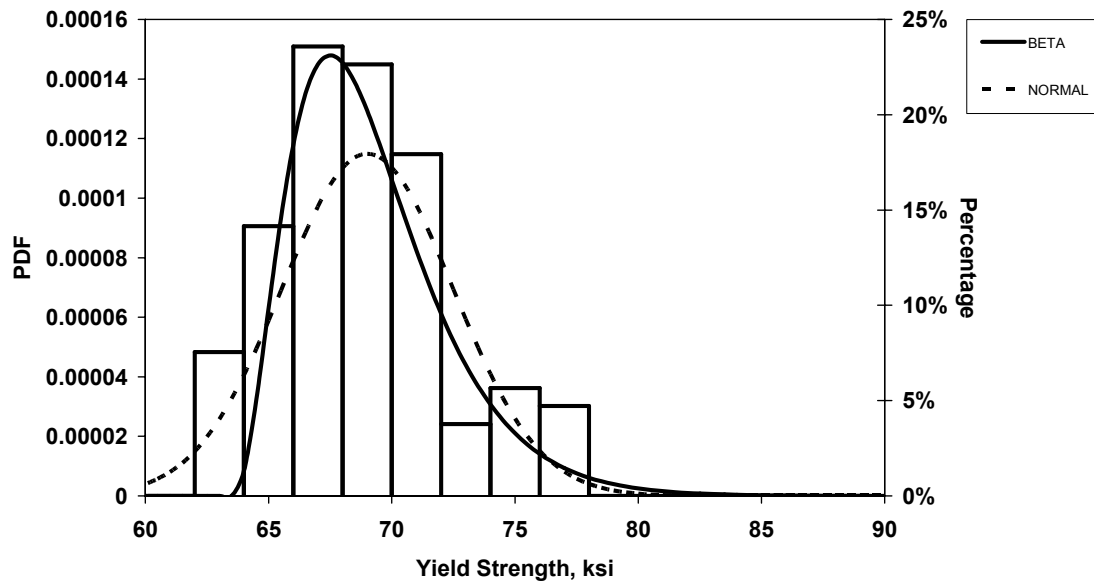


(b)

Figure 47: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 8 bars

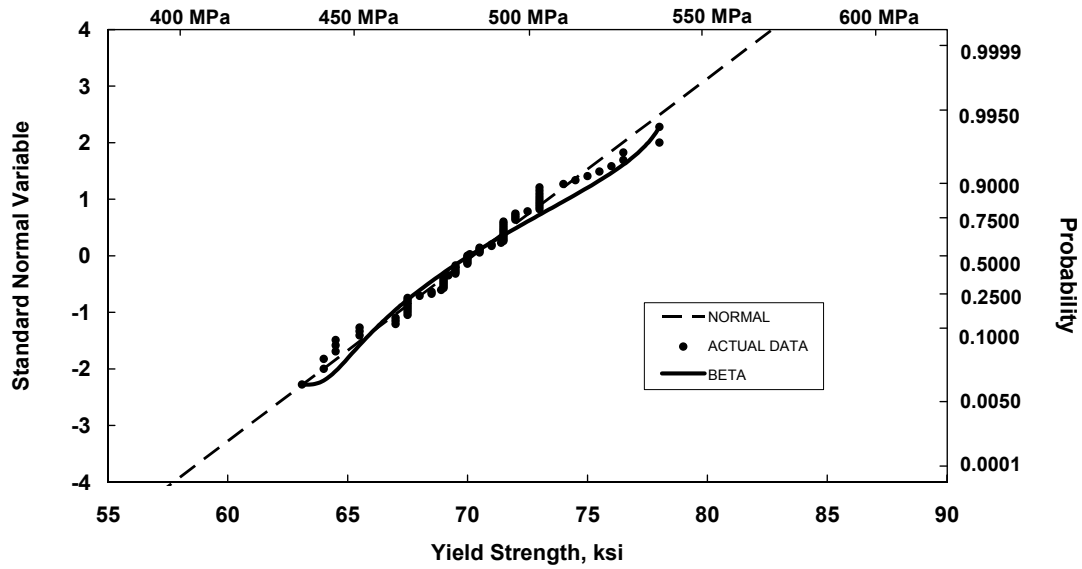


(a)

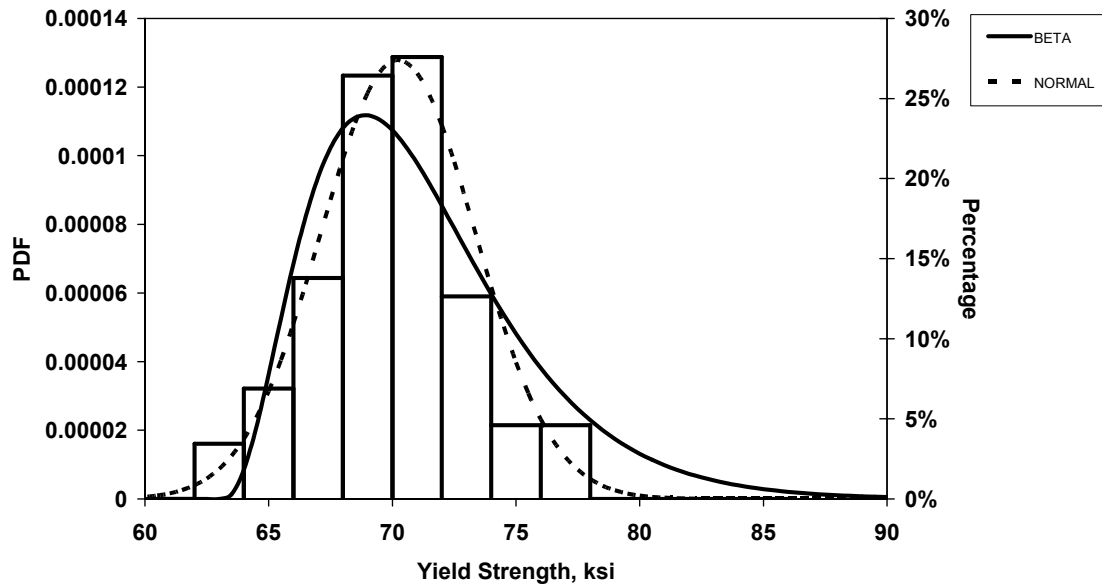


(b)

Figure 48: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 9 bars

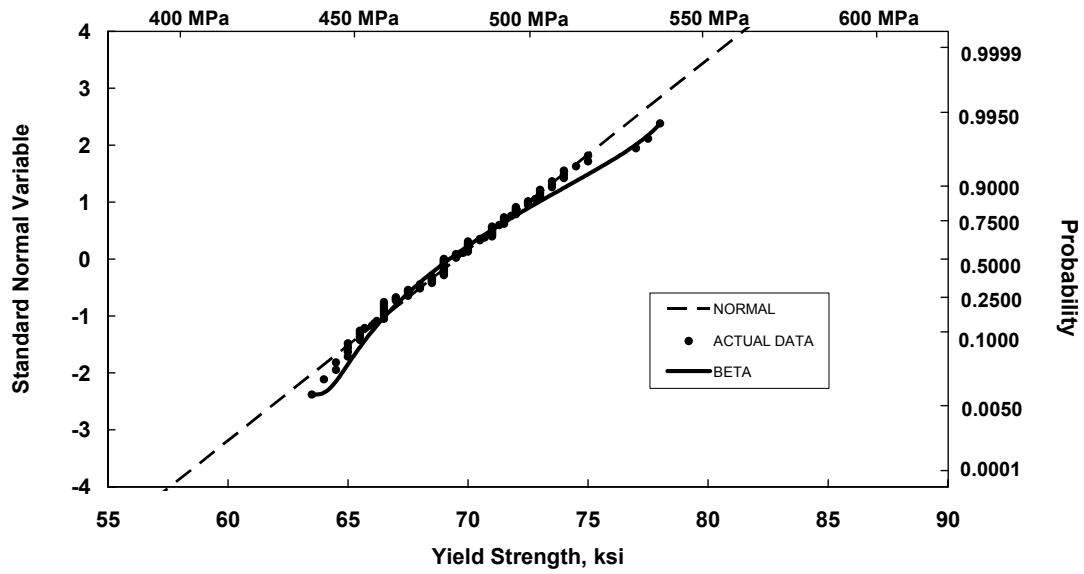


(a)

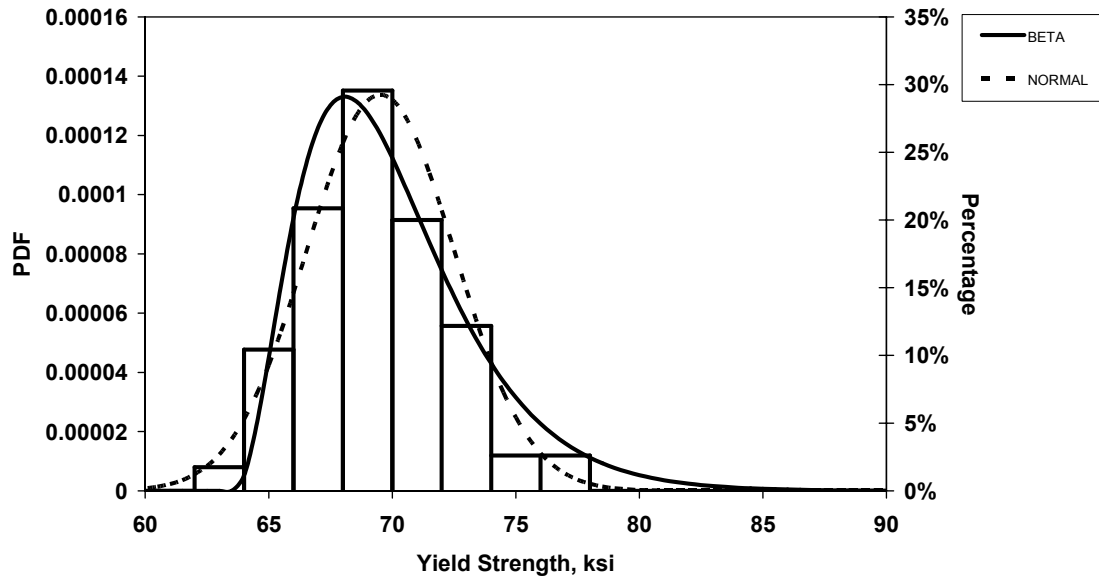


(b)

Figure 49: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 10 bars

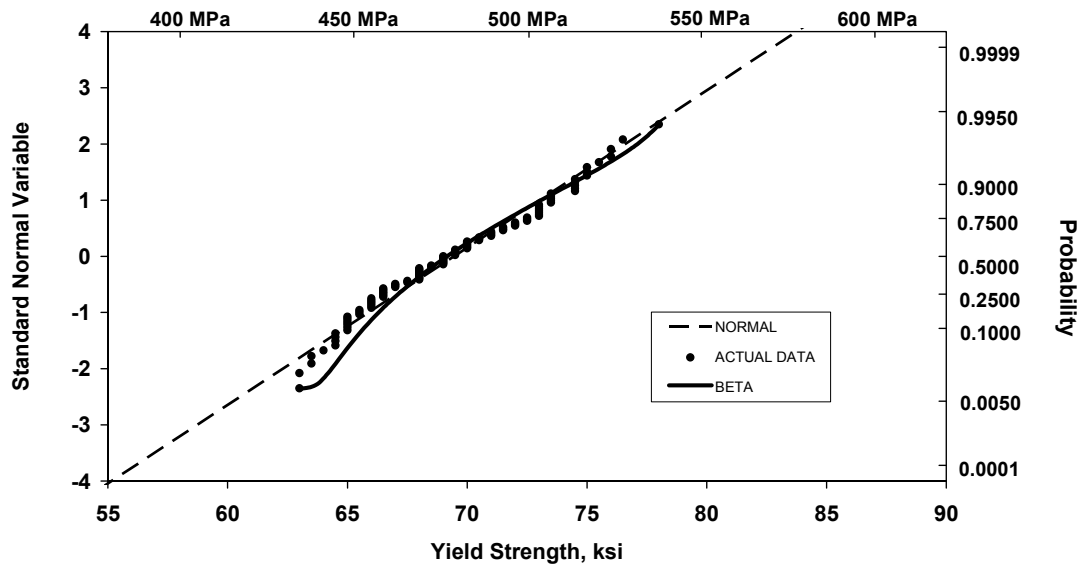


(a)

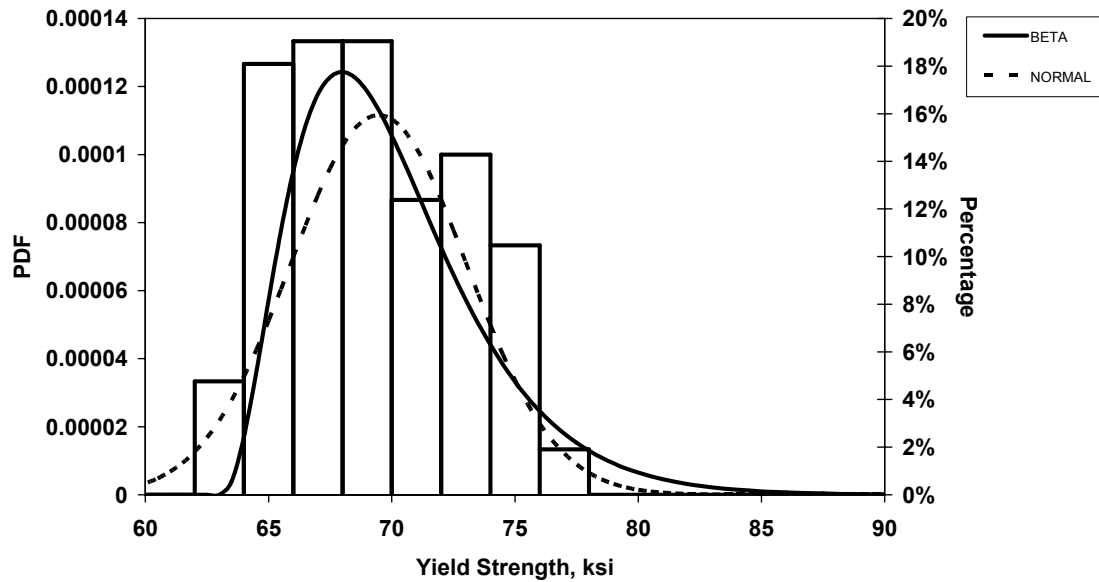


(b)

Figure 50: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 11 bars

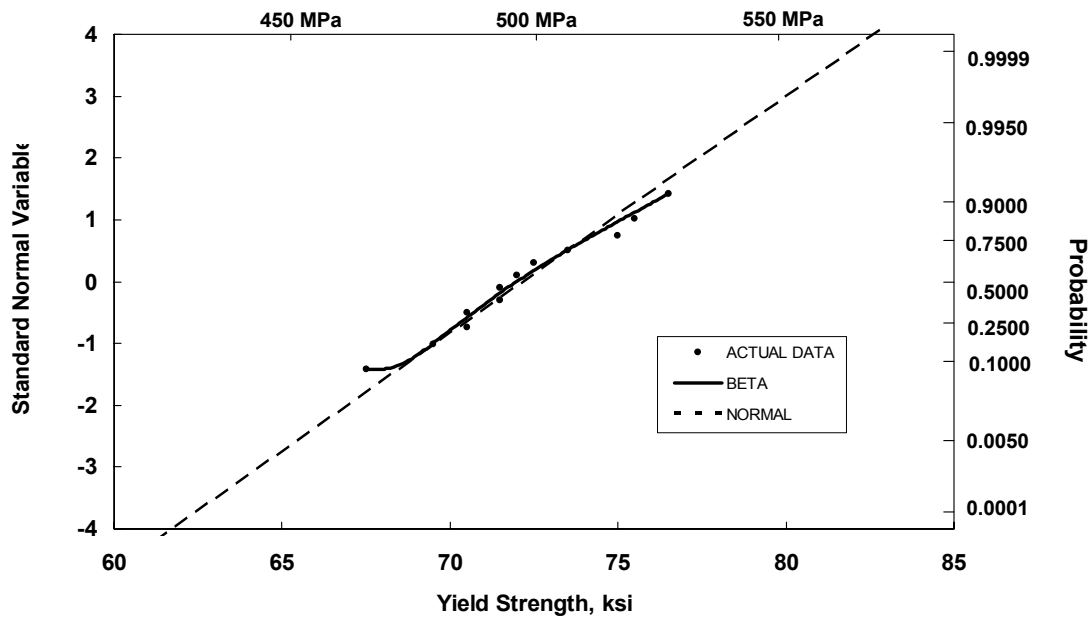


(a)

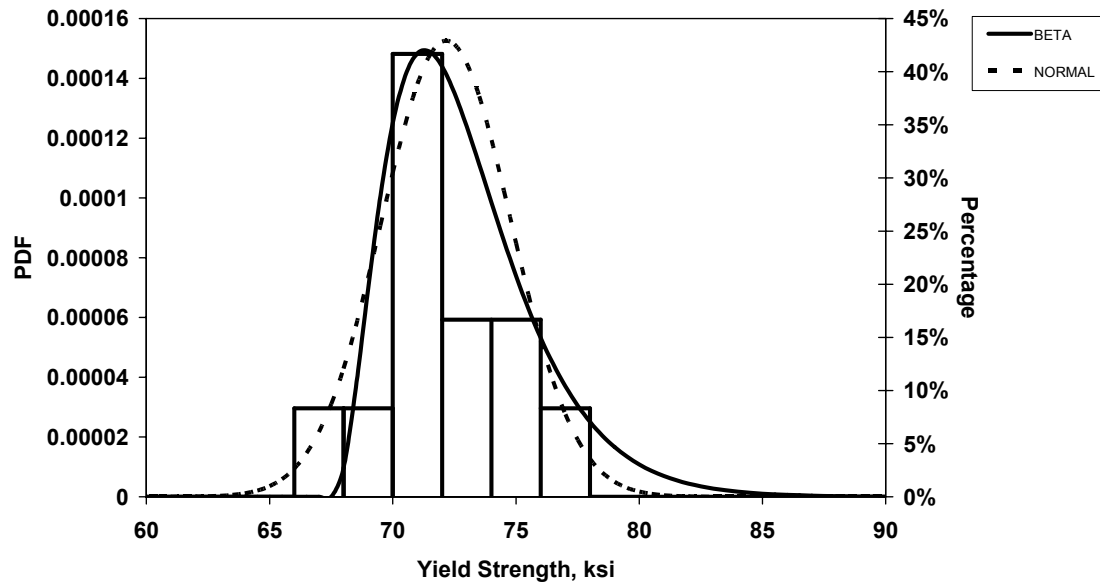


(b)

Figure 51: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 14 bars

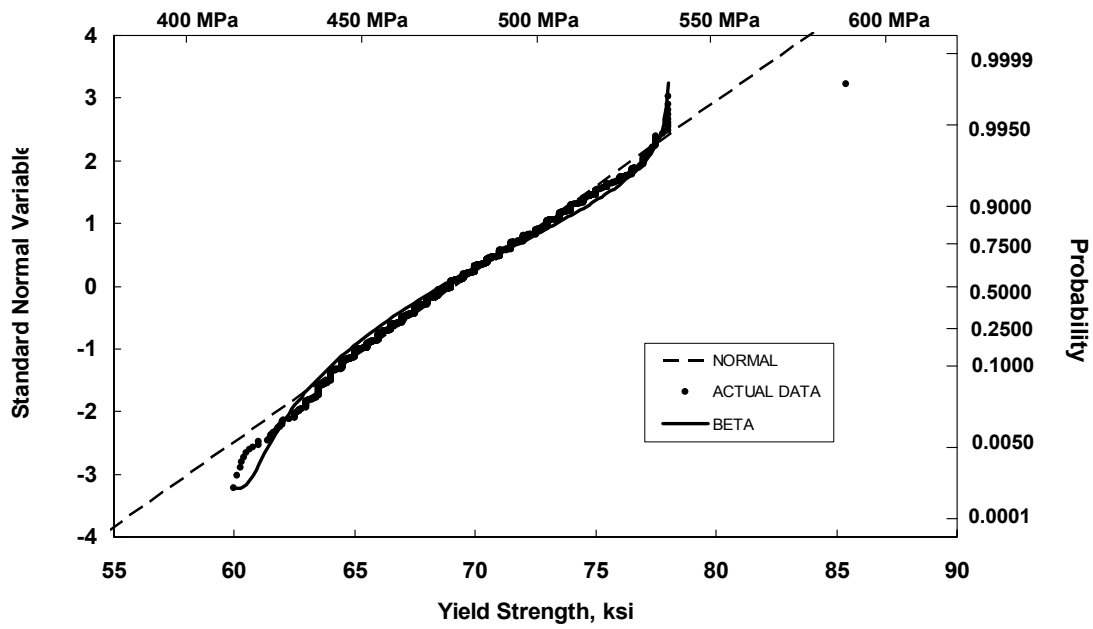


(a)

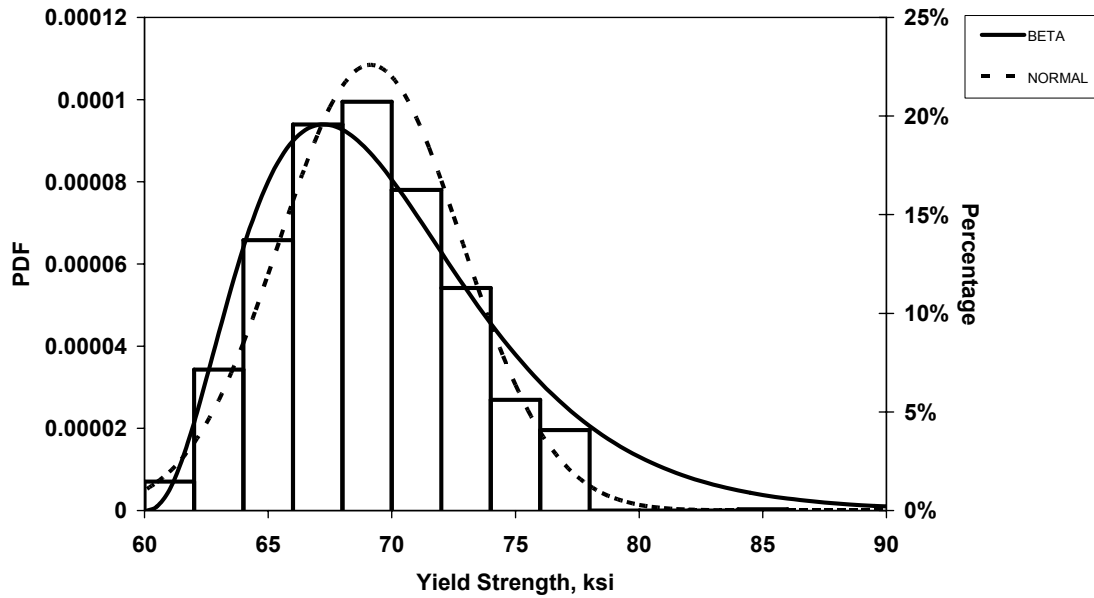


(b)

Figure 52: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of A 706 Grade 60 No. 18 bars

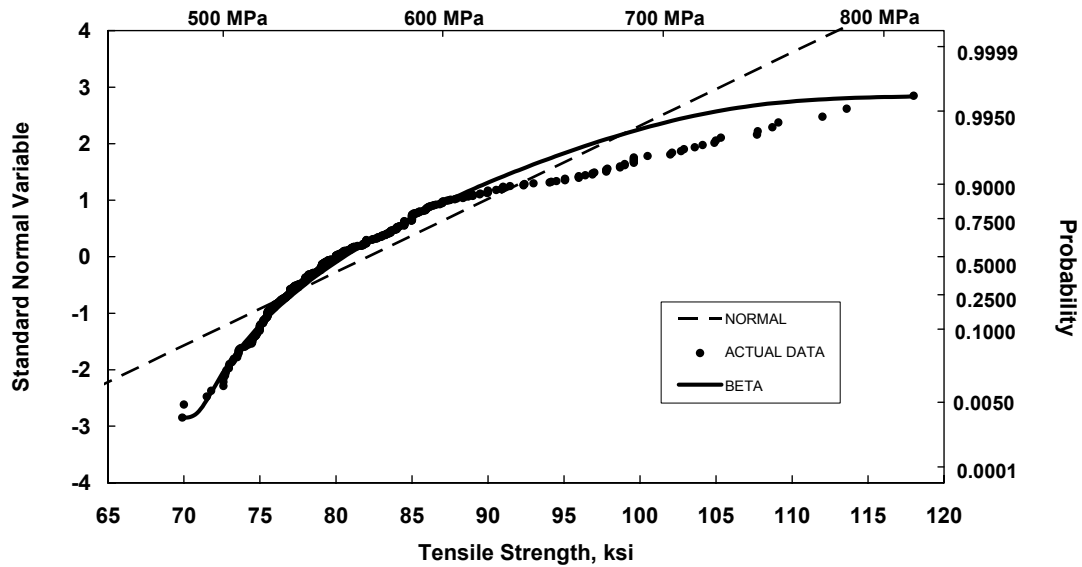


(a)

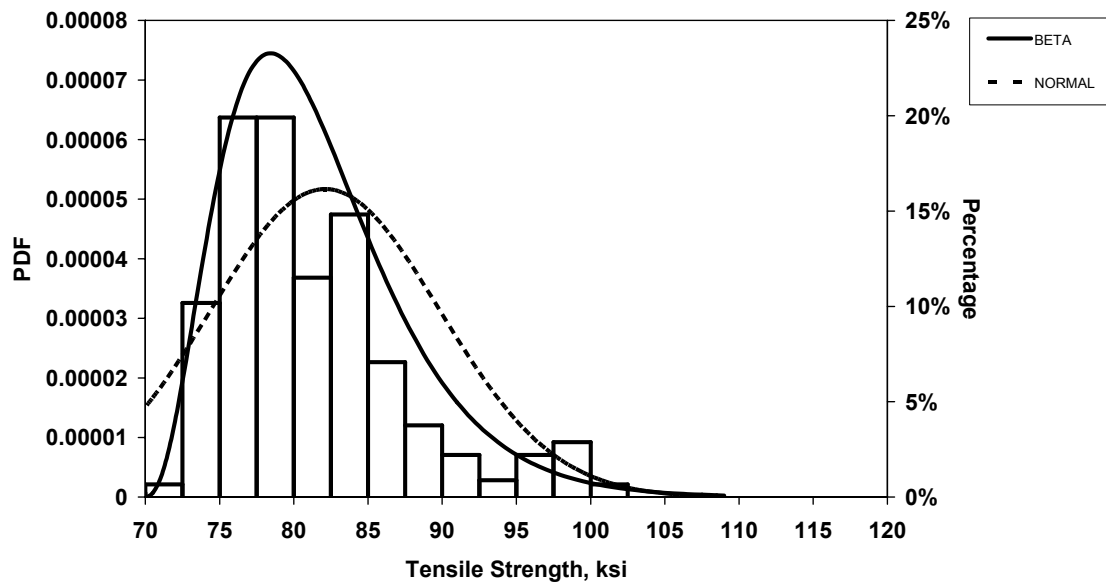


(b)

Figure 53: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Yield Strength of All A 706 Grade 60 bars

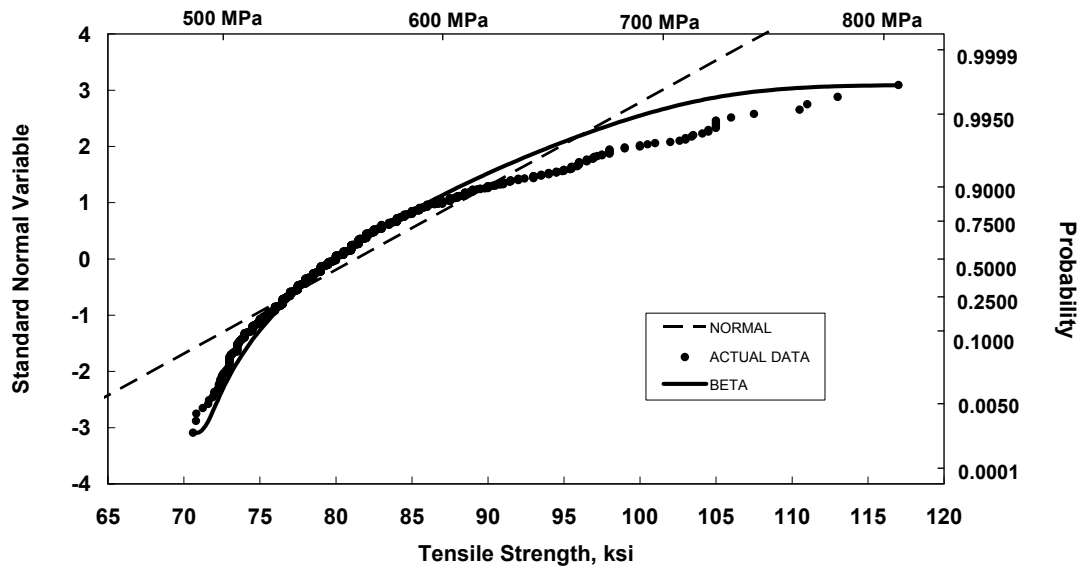


(a)

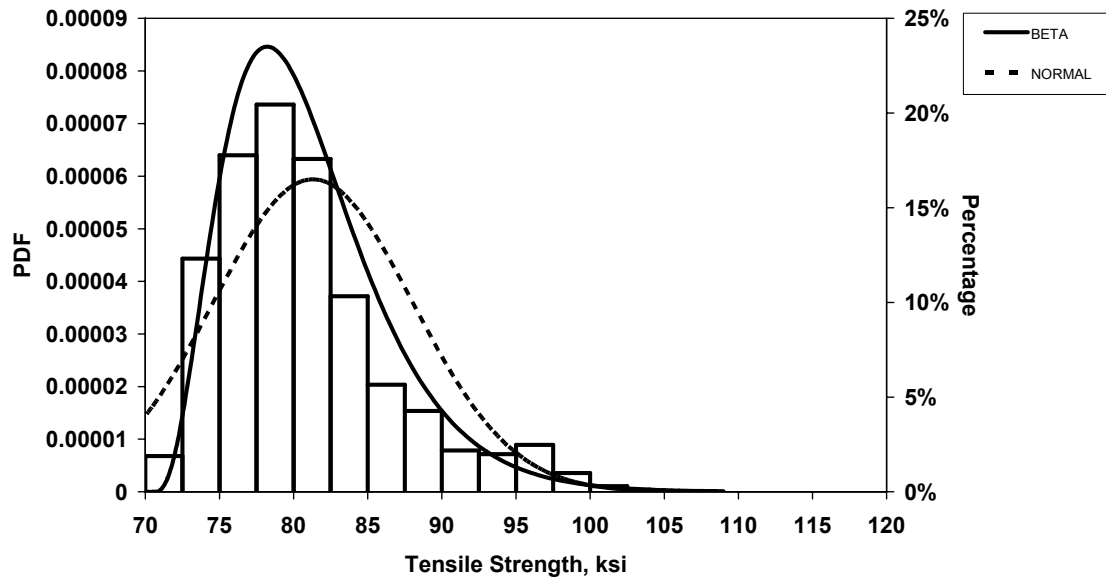


(b)

Figure 54: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 40 No. 3 bars

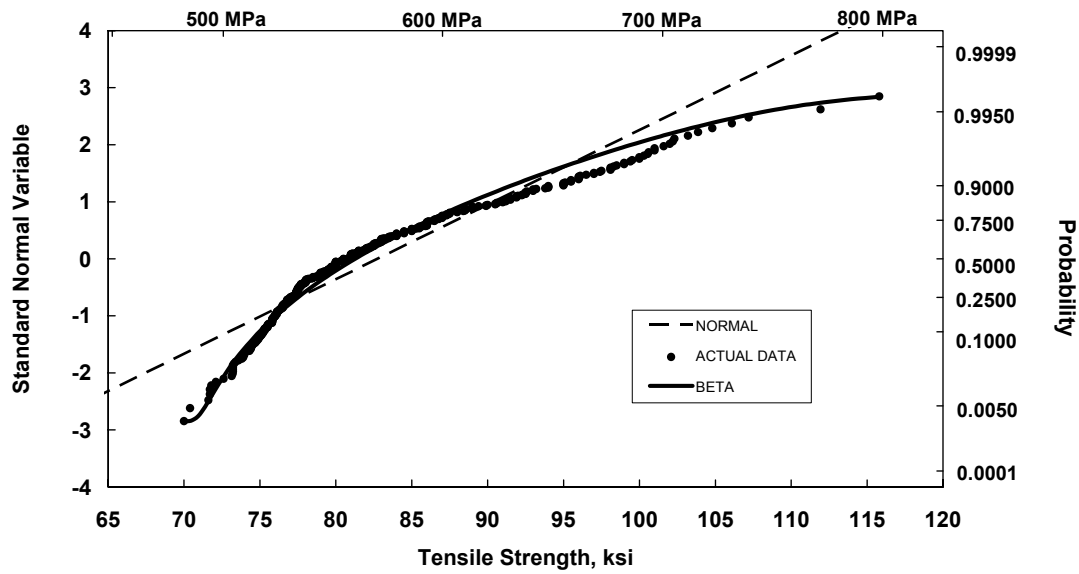


(a)

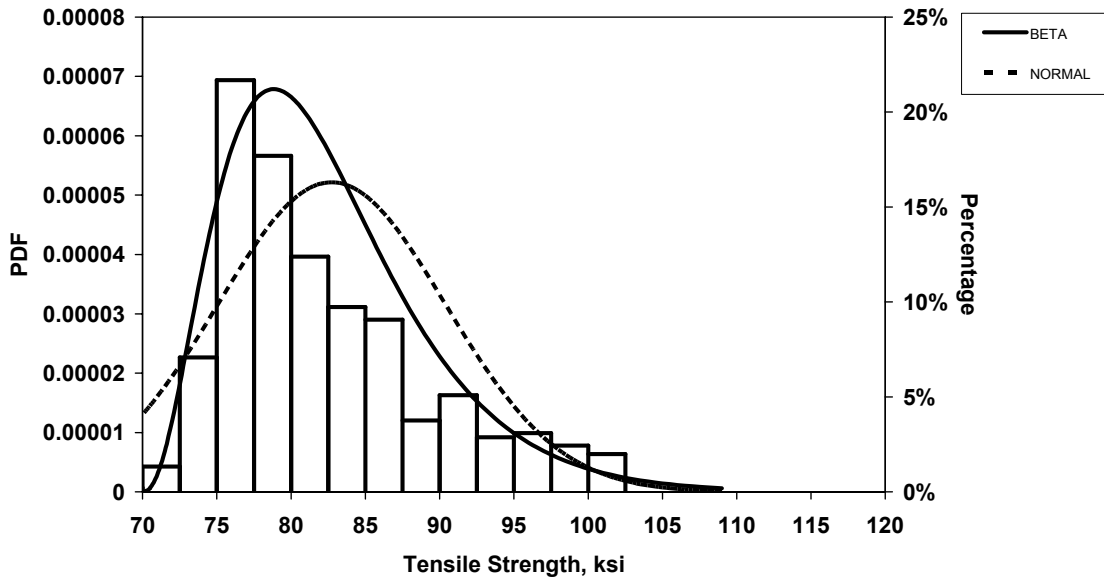


(b)

Figure 55: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 40 No. 4 bars

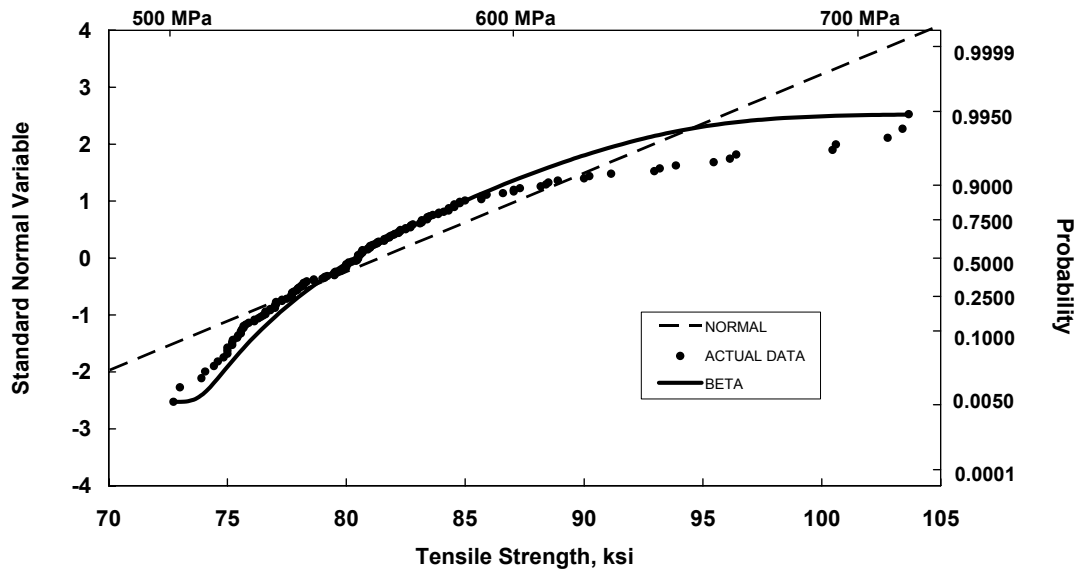


(a)

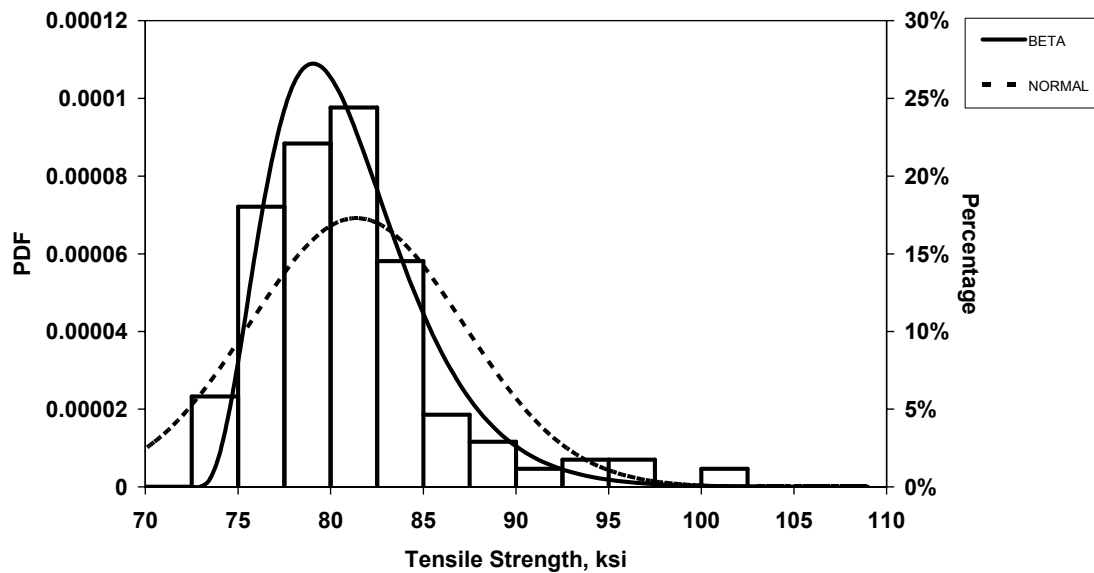


(b)

Figure 56: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 40 No. 5 bars

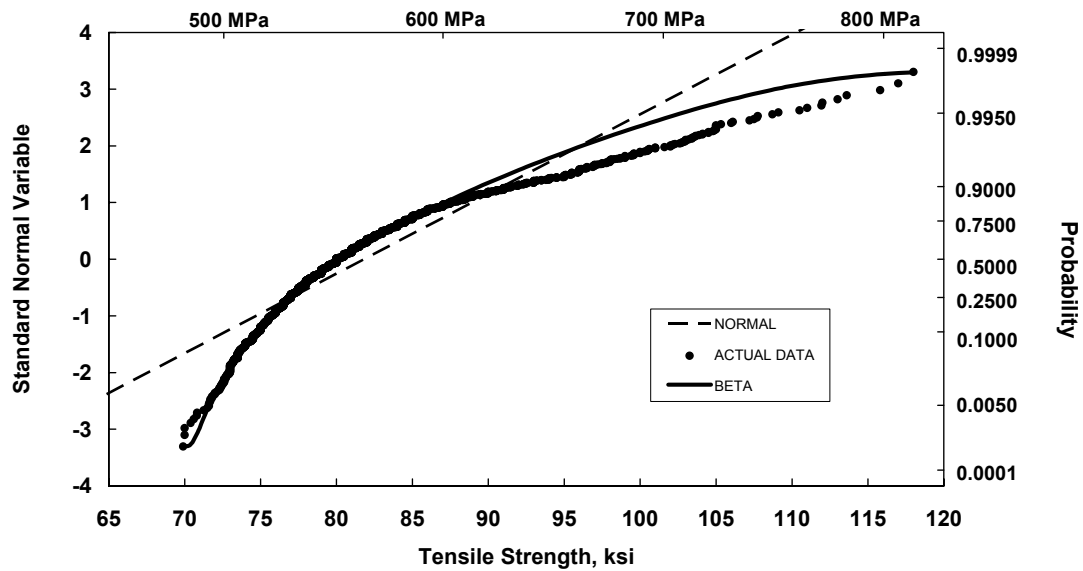


(a)

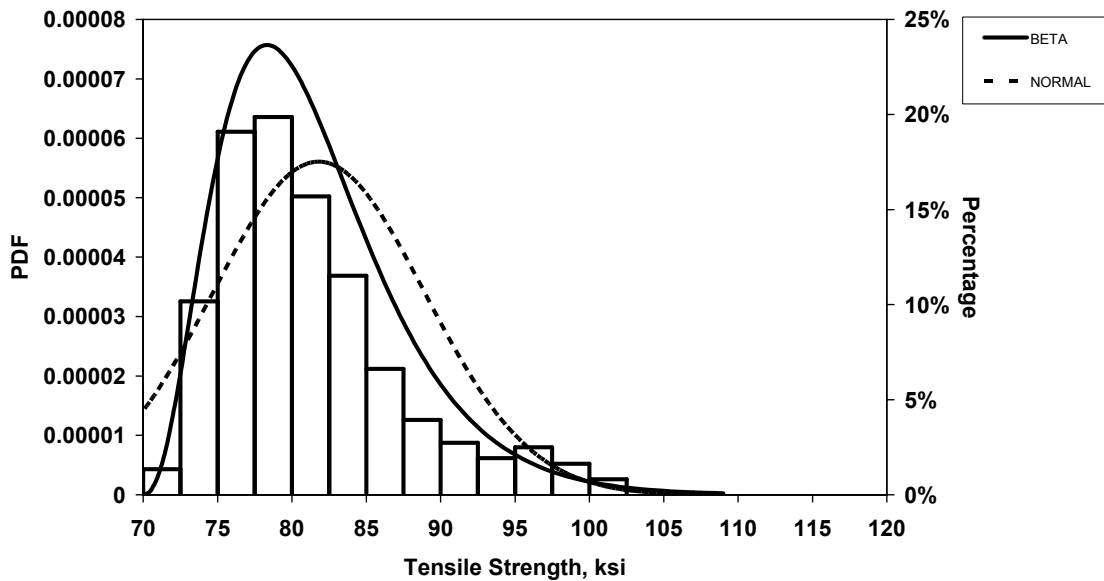


(b)

Figure 57: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 40 No. 6 bars

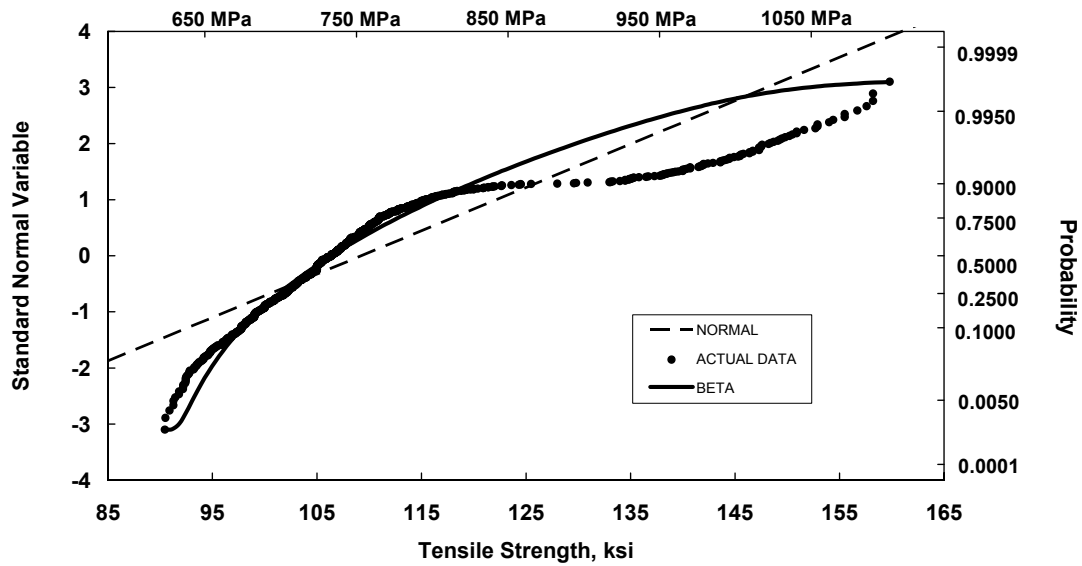


(a)

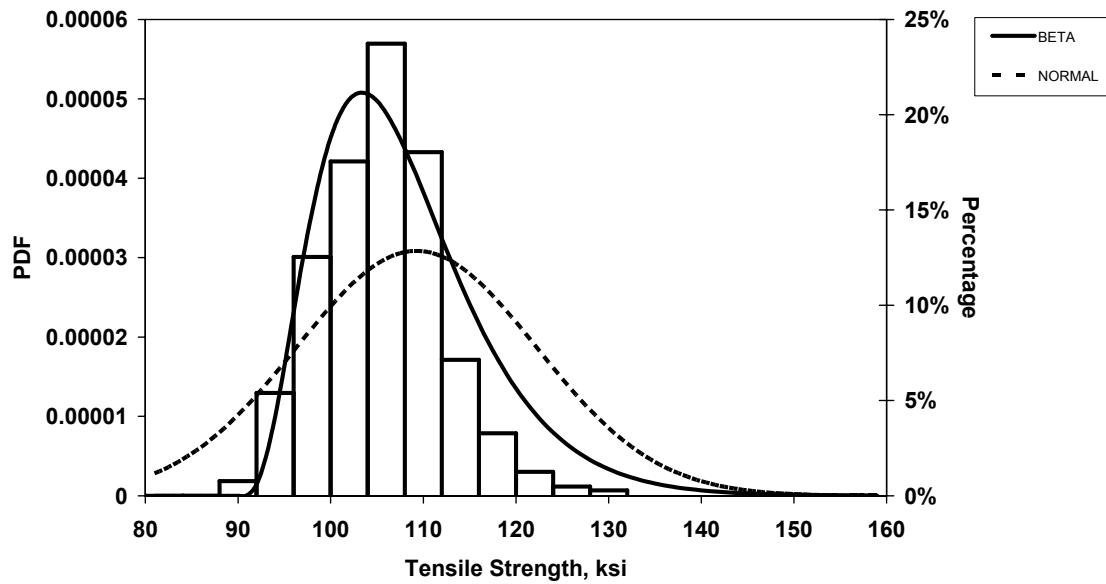


(b)

Figure 58: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of All A 615 Grade 40 bars

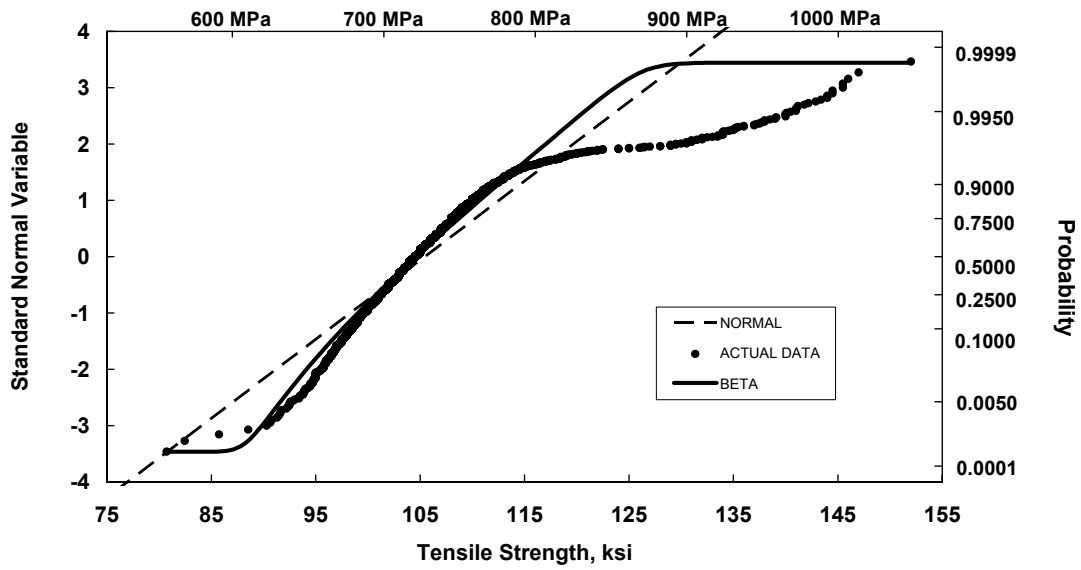


(a)

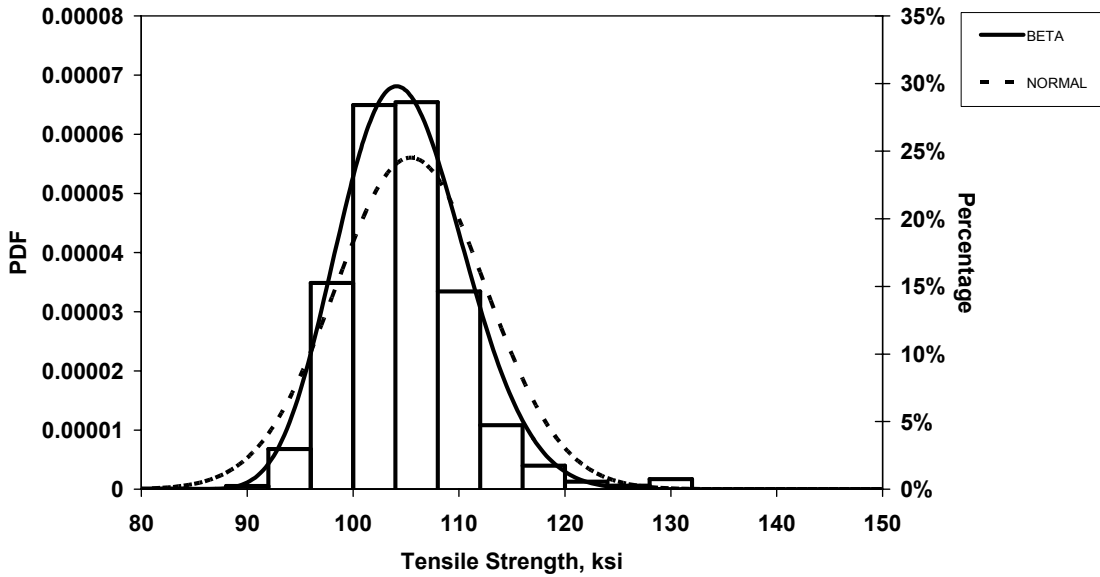


(b)

Figure 59: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 3 bars

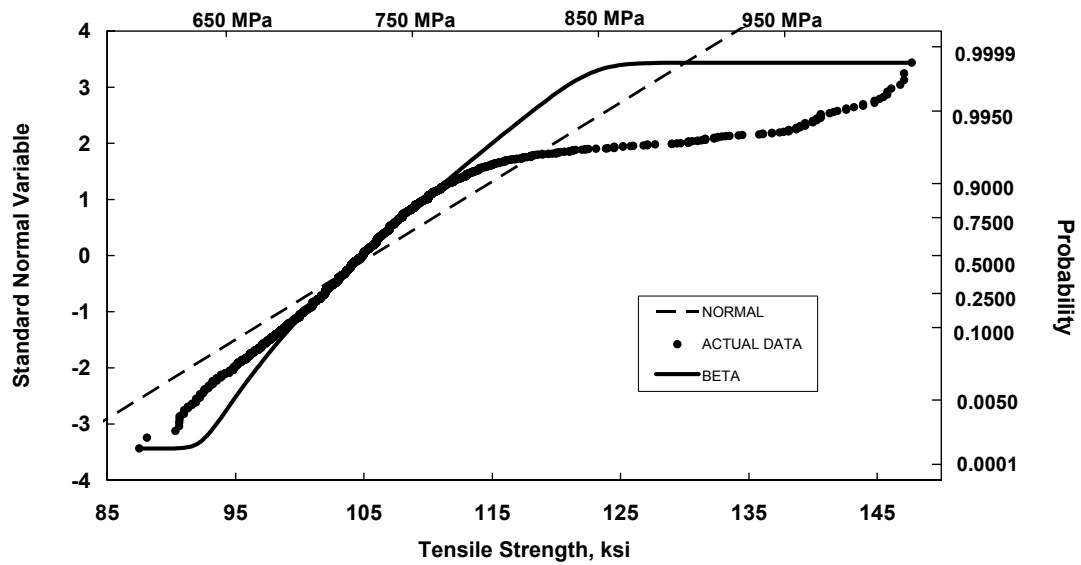


(a)

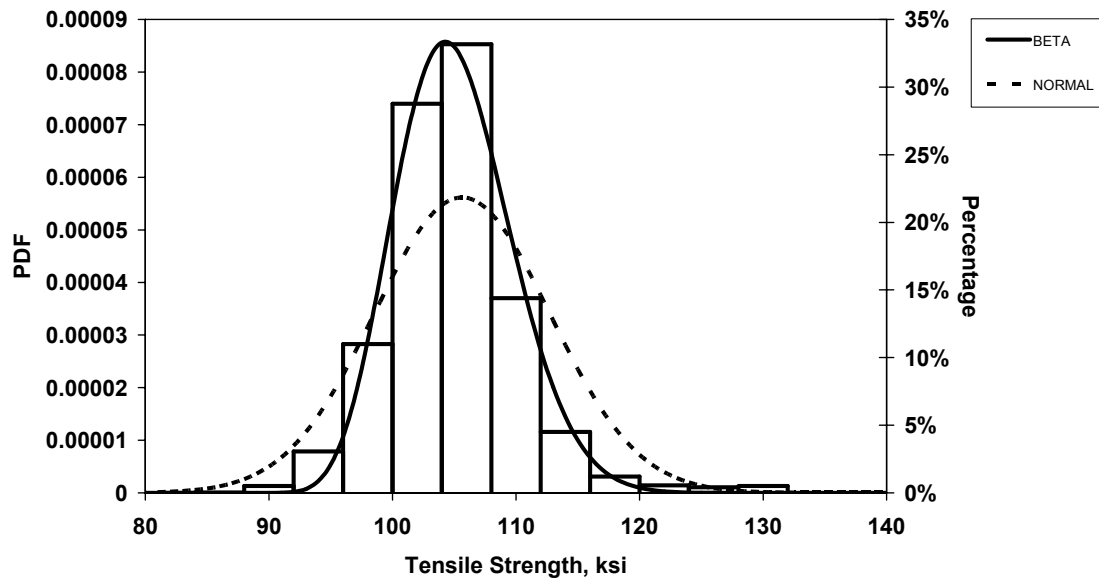


(b)

Figure 60: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 4 bars

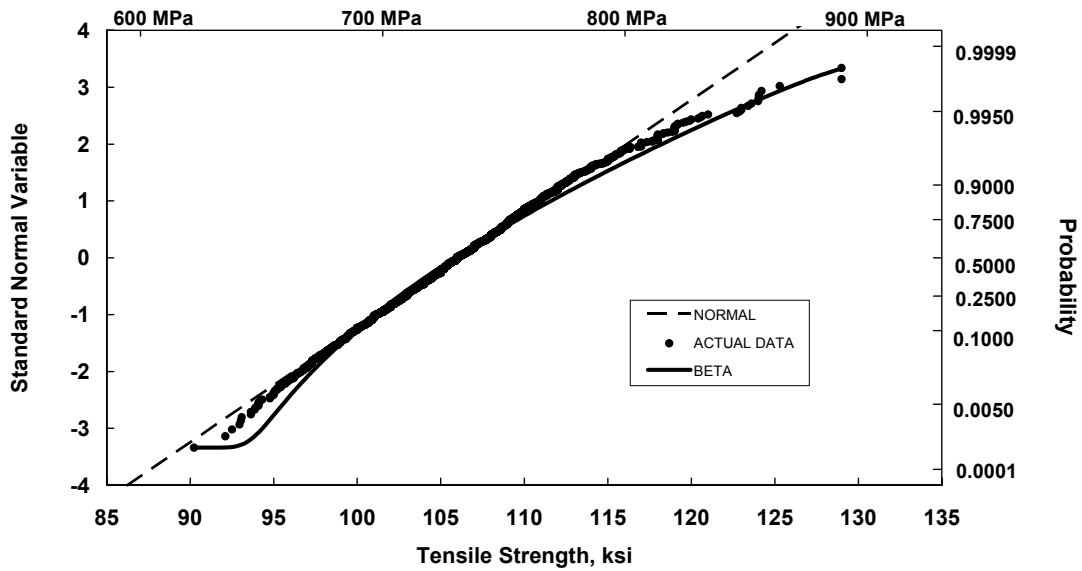


(a)

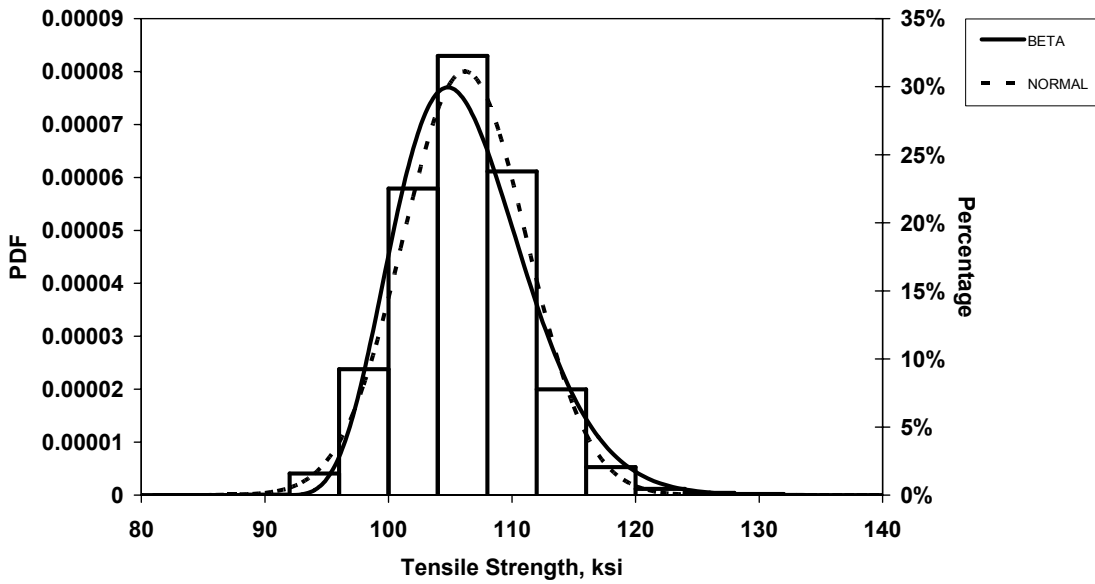


(b)

Figure 61: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 5 bars

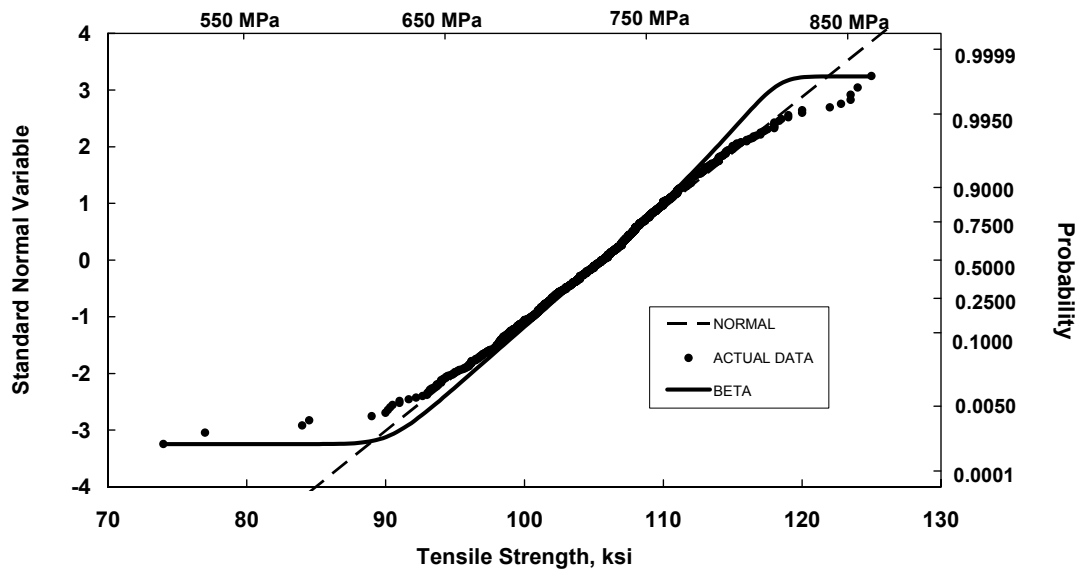


(a)

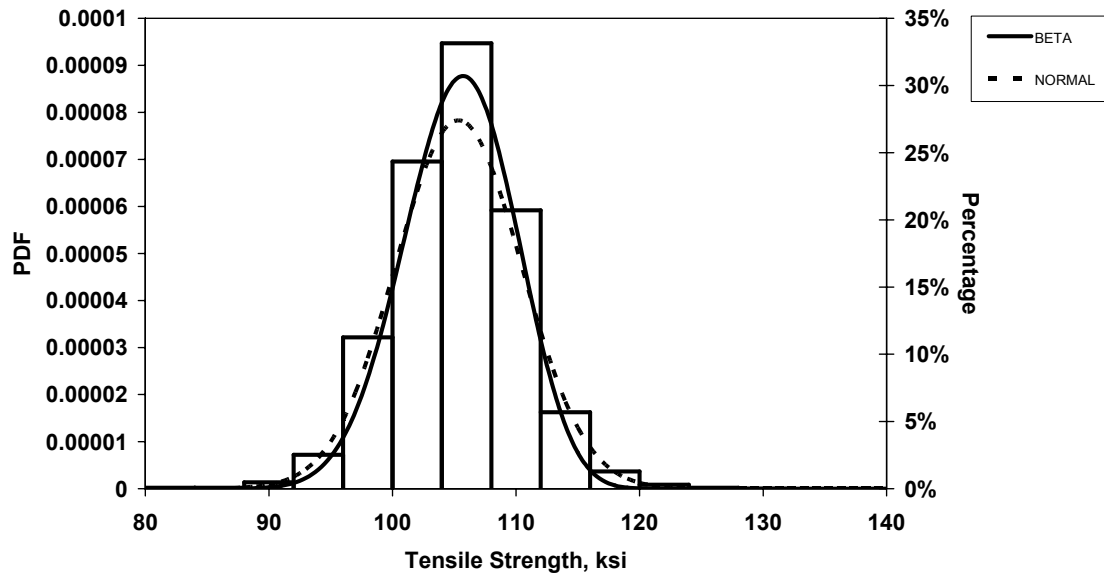


(b)

Figure 62: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 6 bars

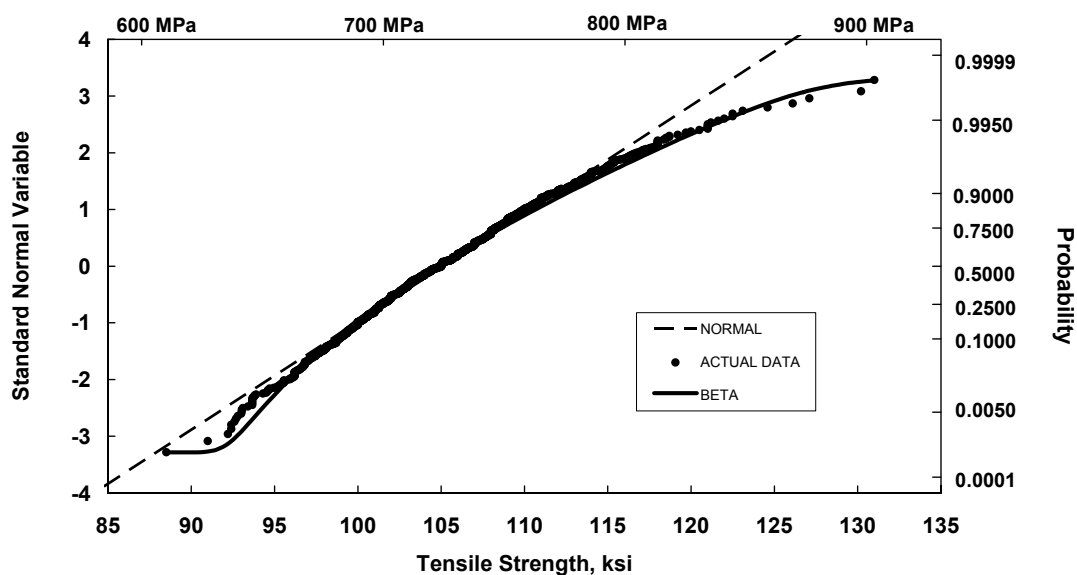


(a)

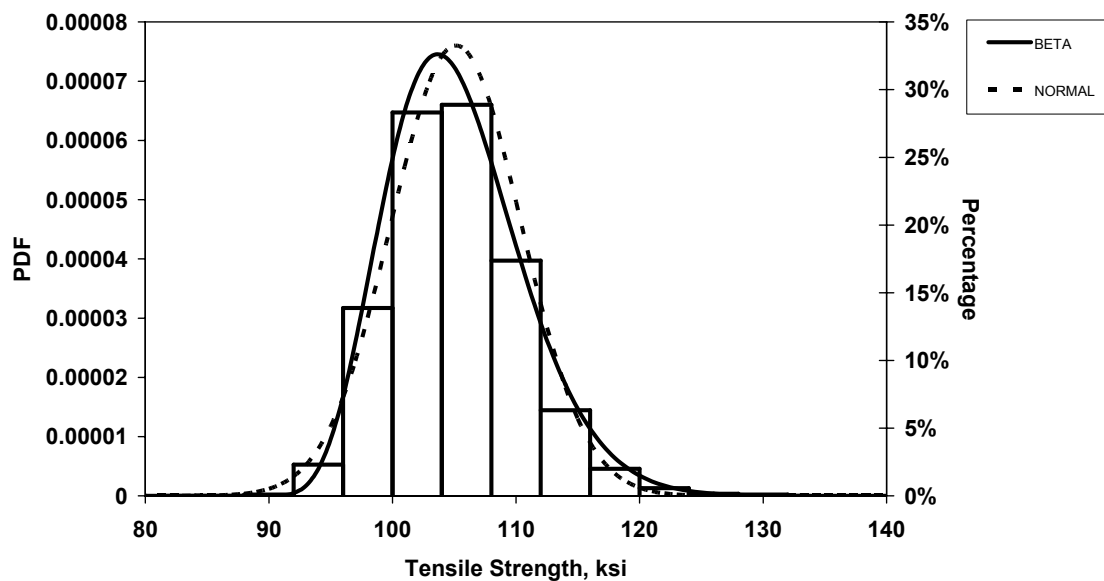


(b)

Figure 63: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 7 bars

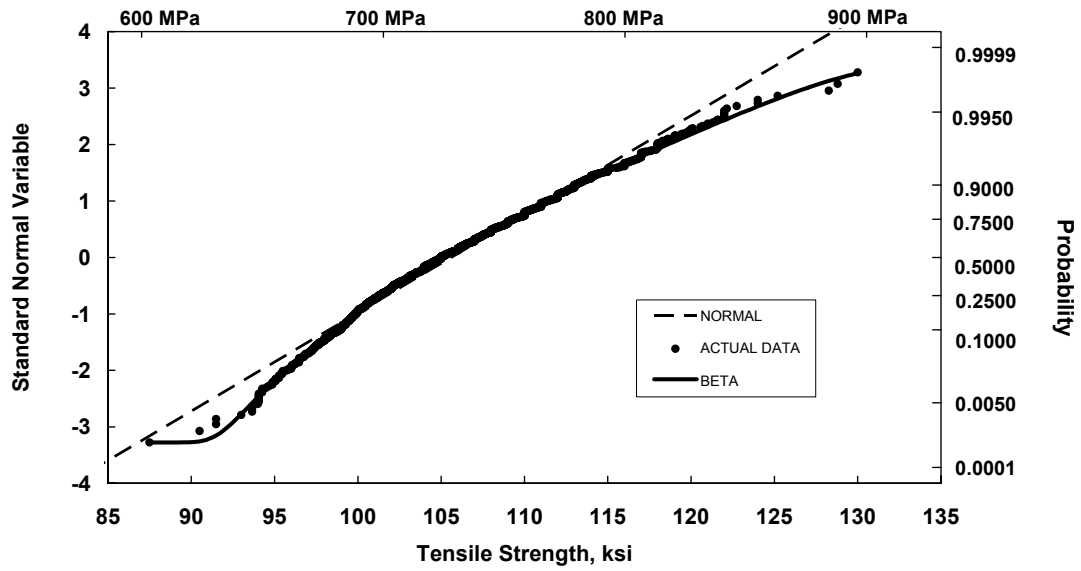


(a)

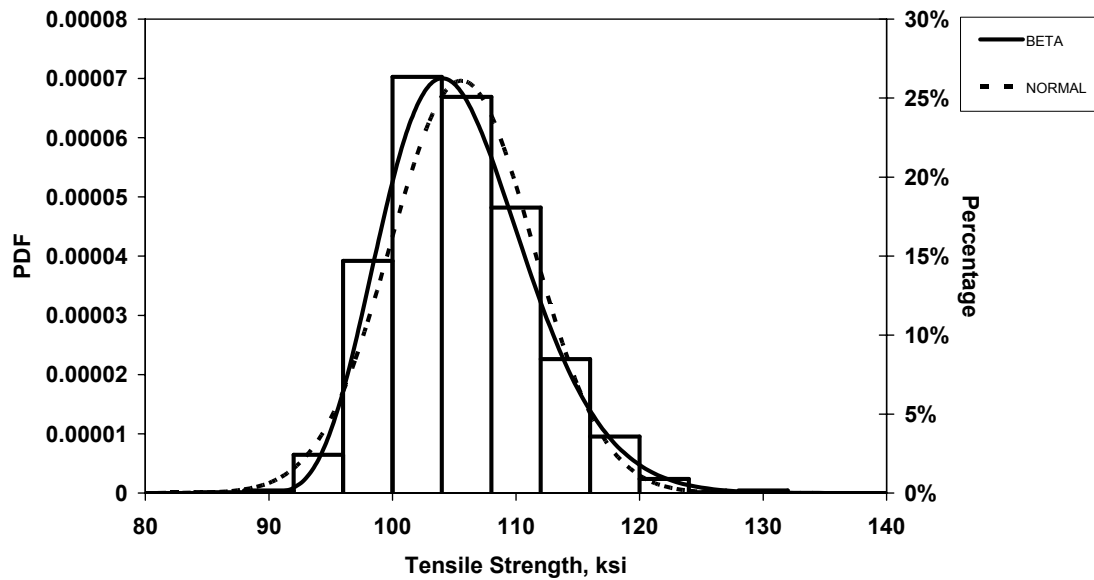


(b)

Figure 64: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 8 bars

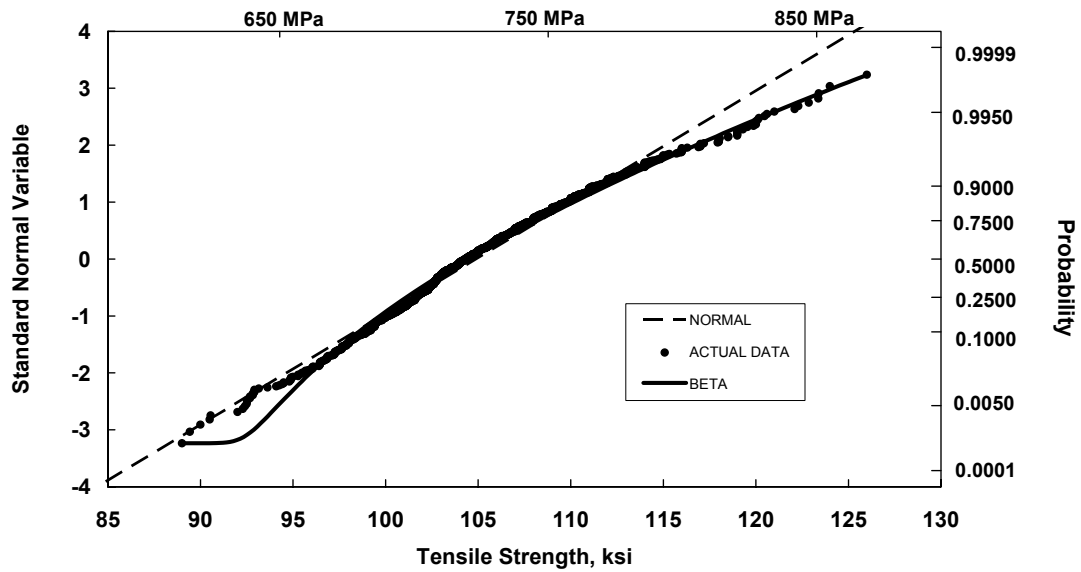


(a)

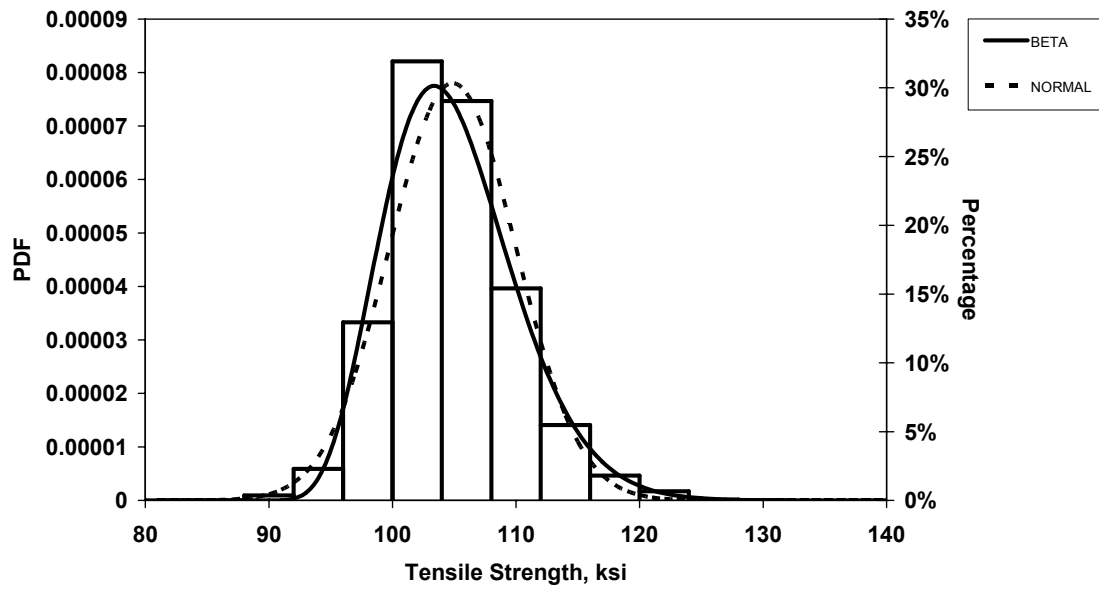


(b)

Figure 65: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 9 bars

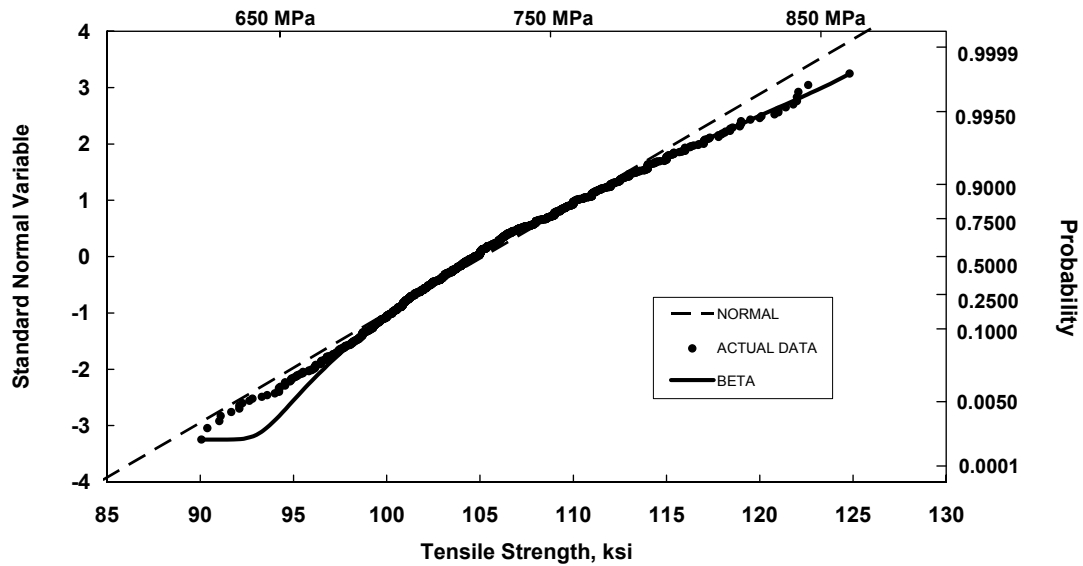


(a)

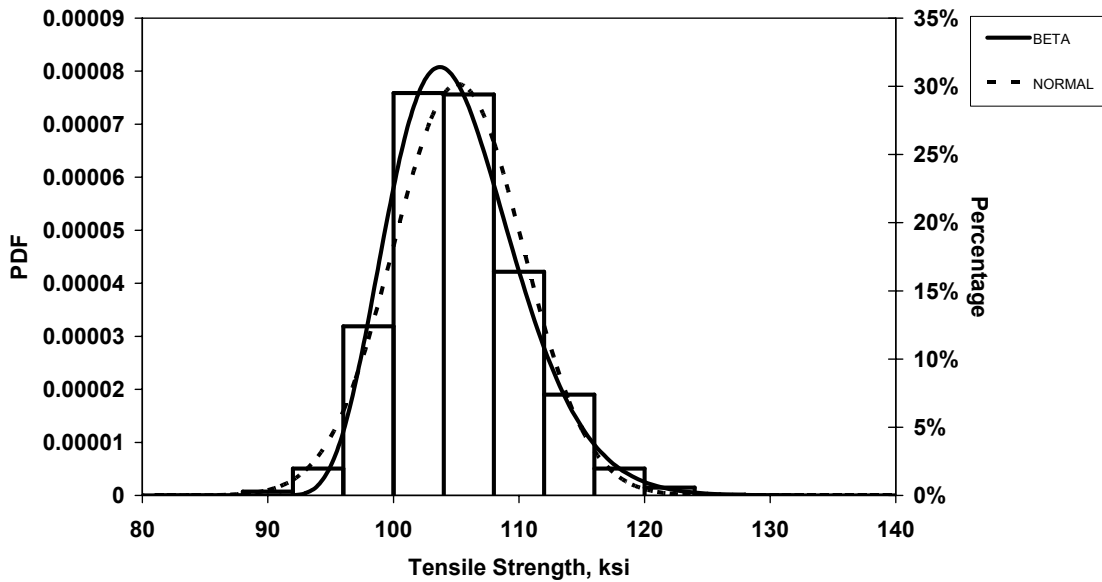


(b)

Figure 66: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 10 bars

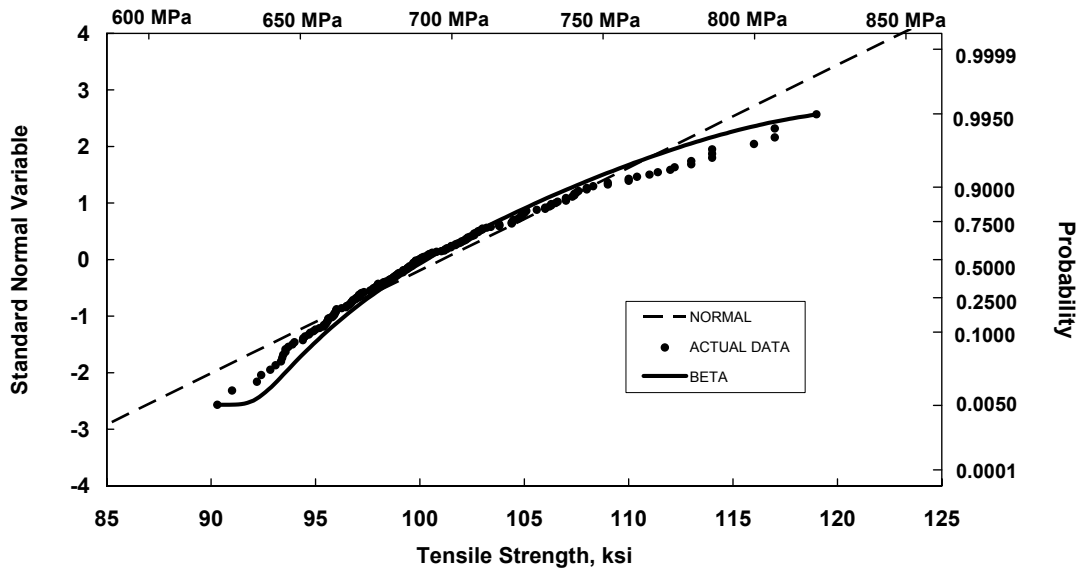


(a)

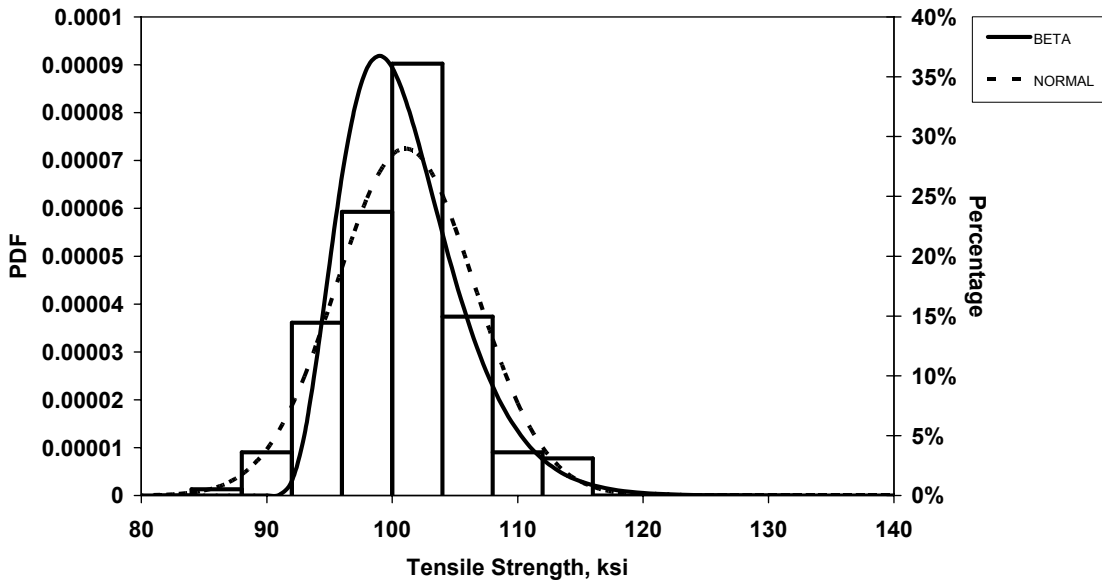


(b)

Figure 67: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 11 bars

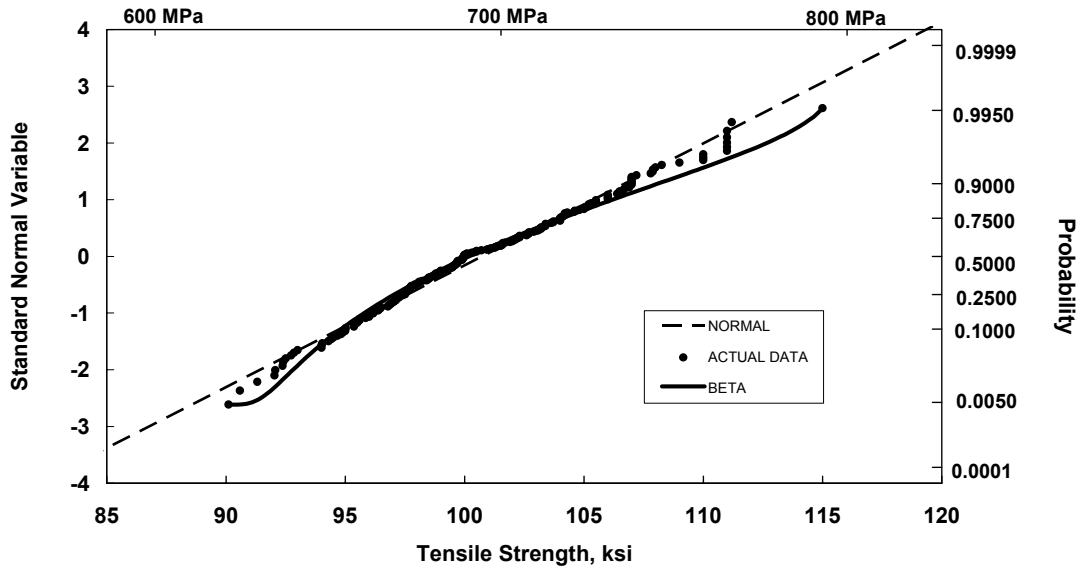


(a)

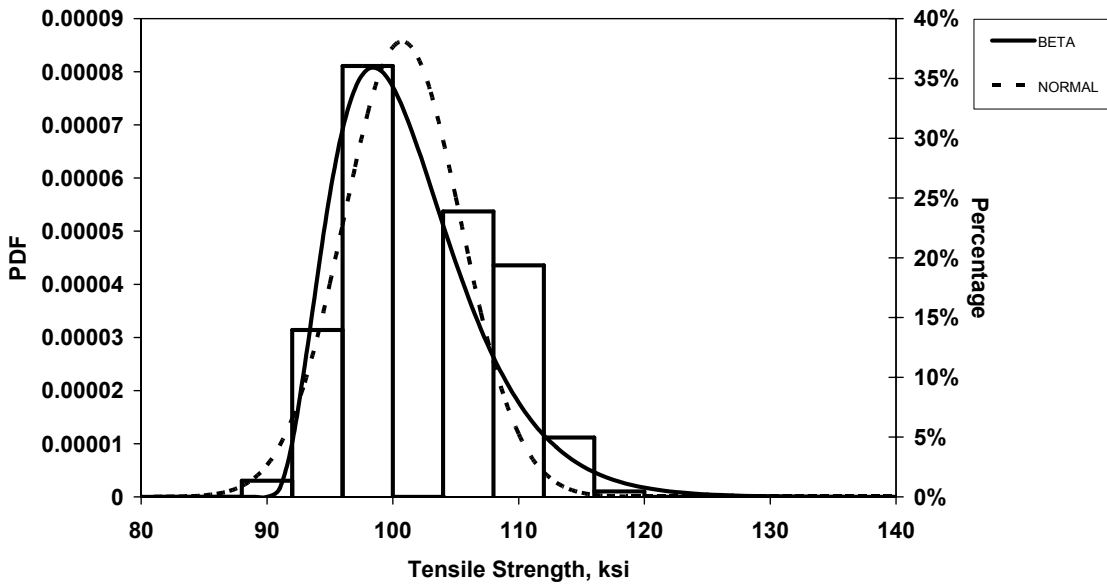


(b)

Figure 68: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 14 bars

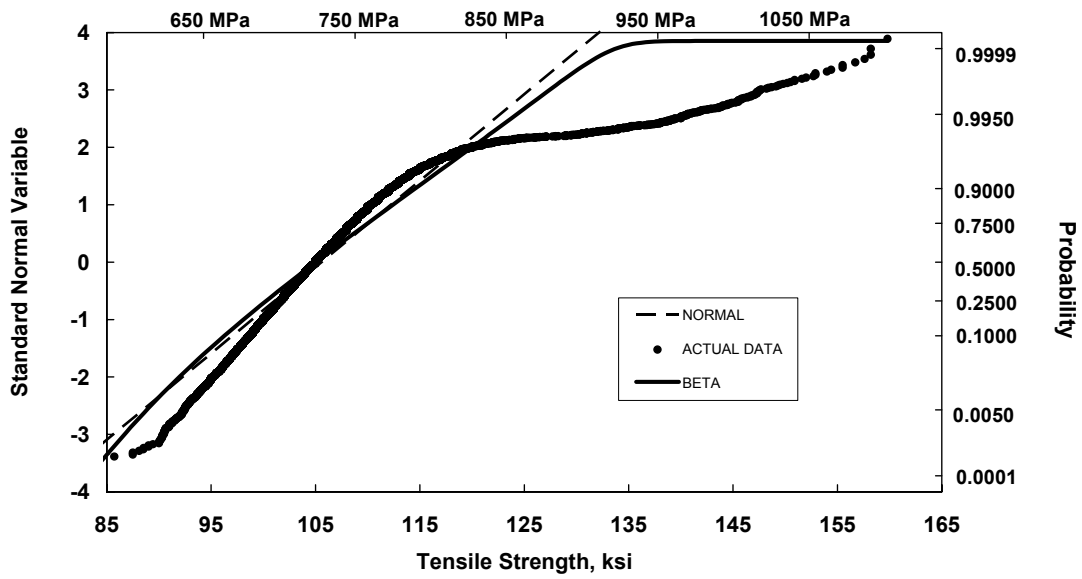


(a)

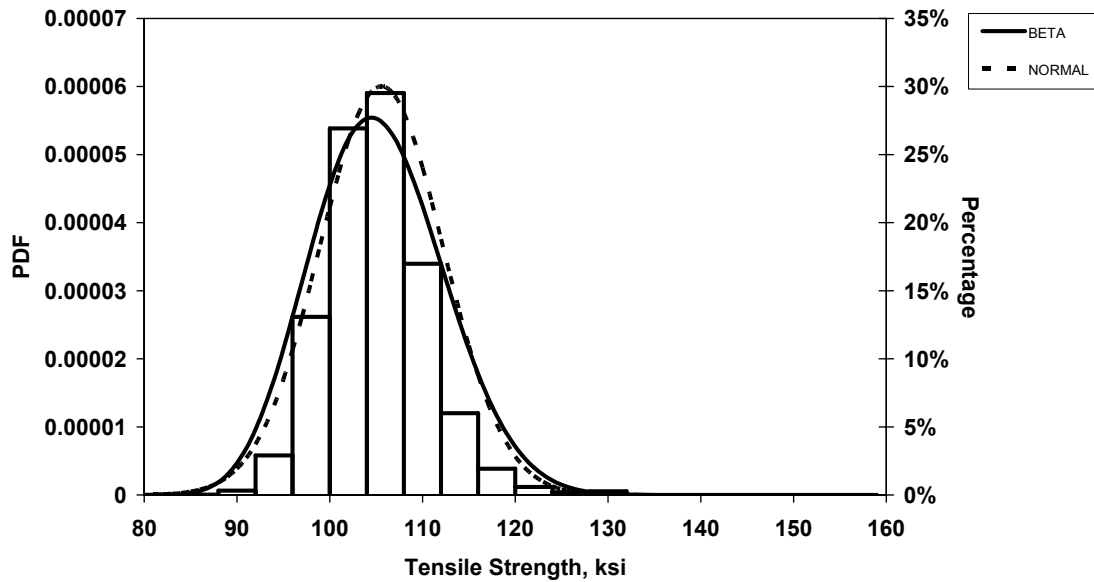


(b)

Figure 69: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 60 No. 18 bars

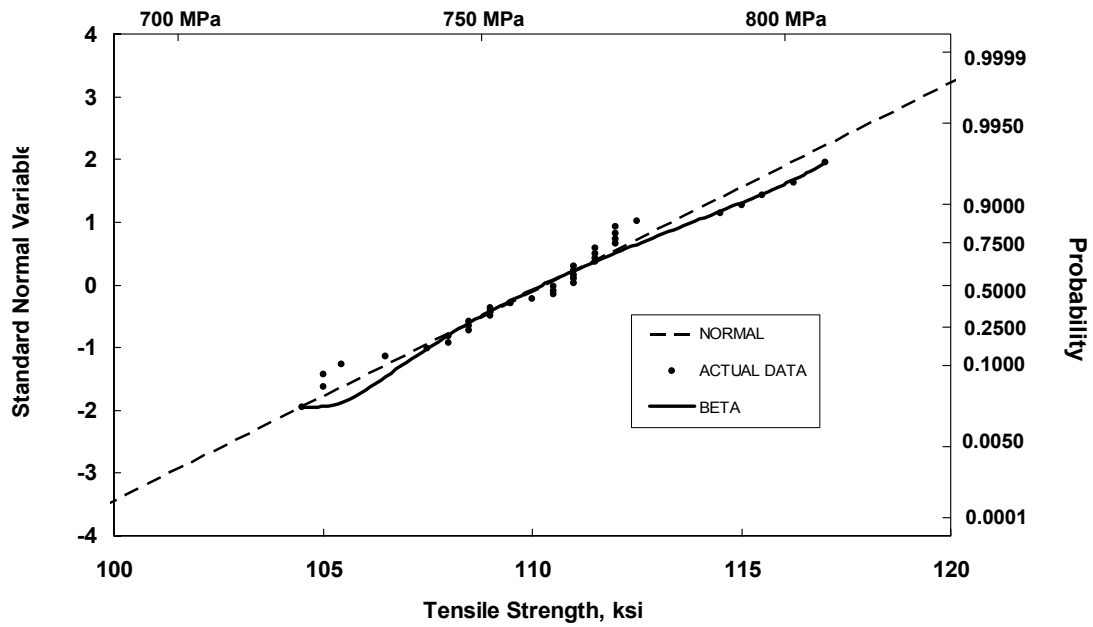


(a)

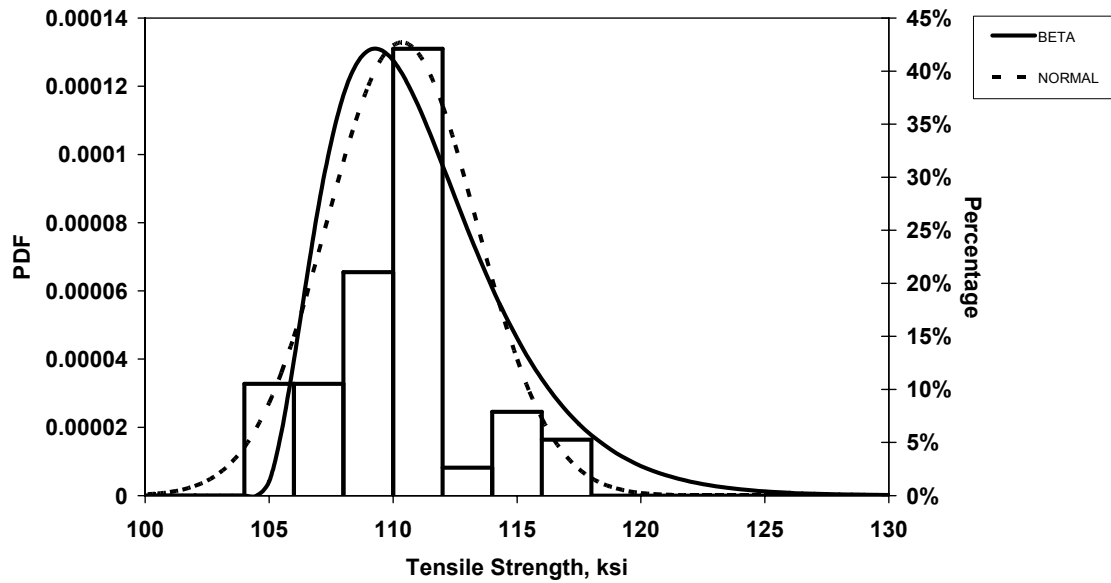


(b)

Figure 70: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of All A 615 Grade 60 bars

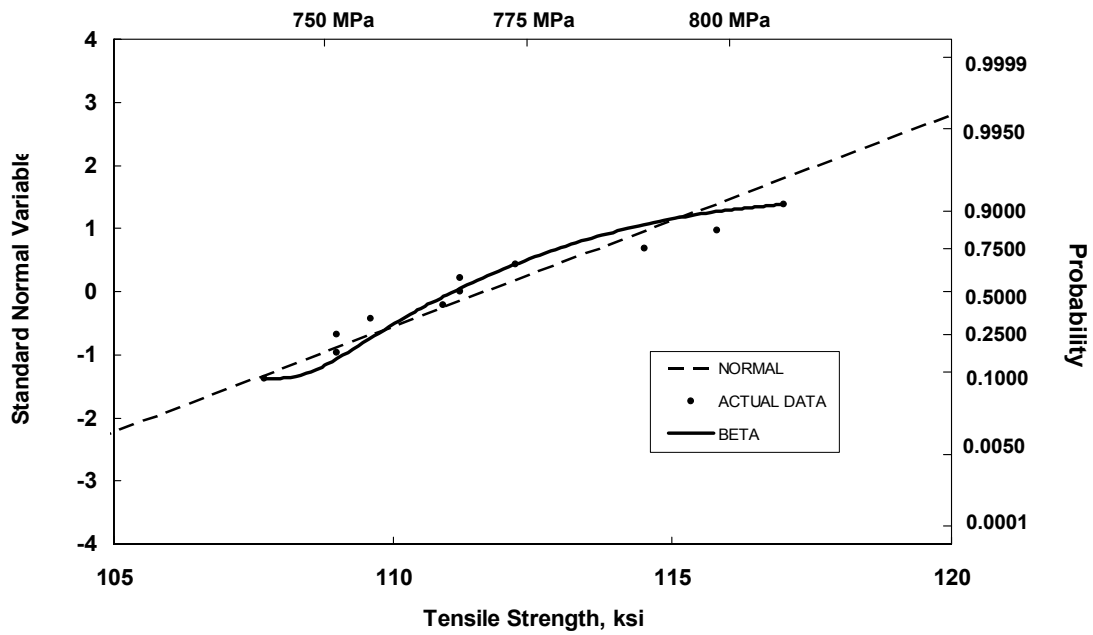


(a)

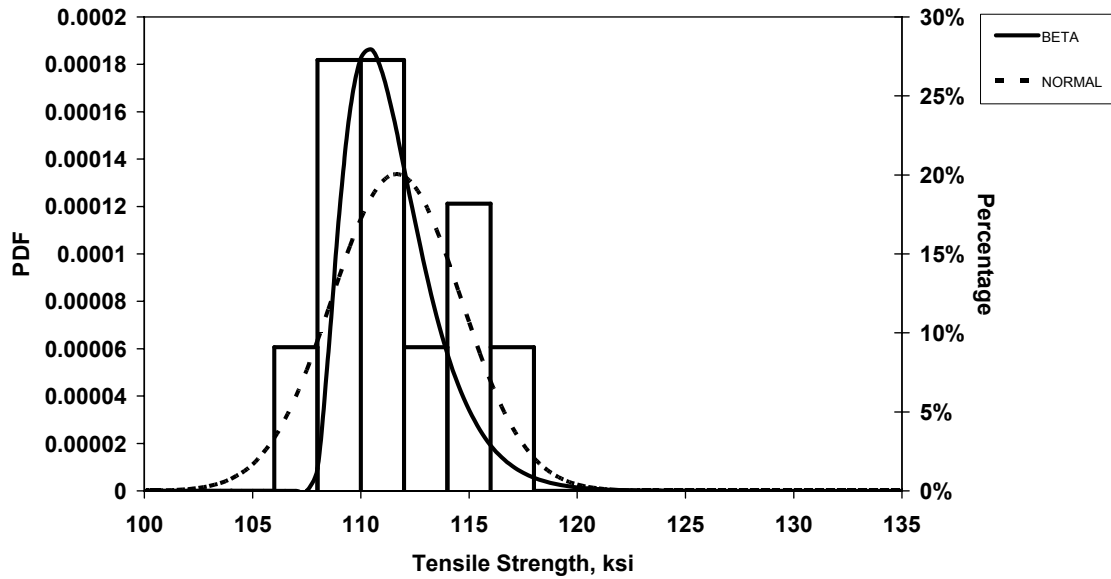


(b)

Figure 71: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 4 bars

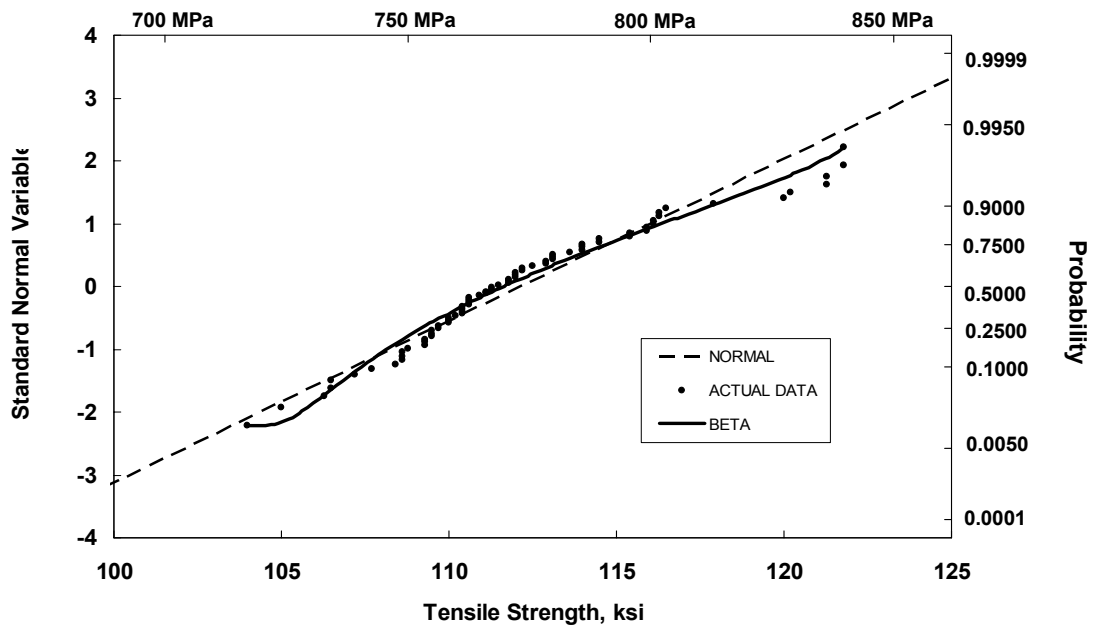


(a)

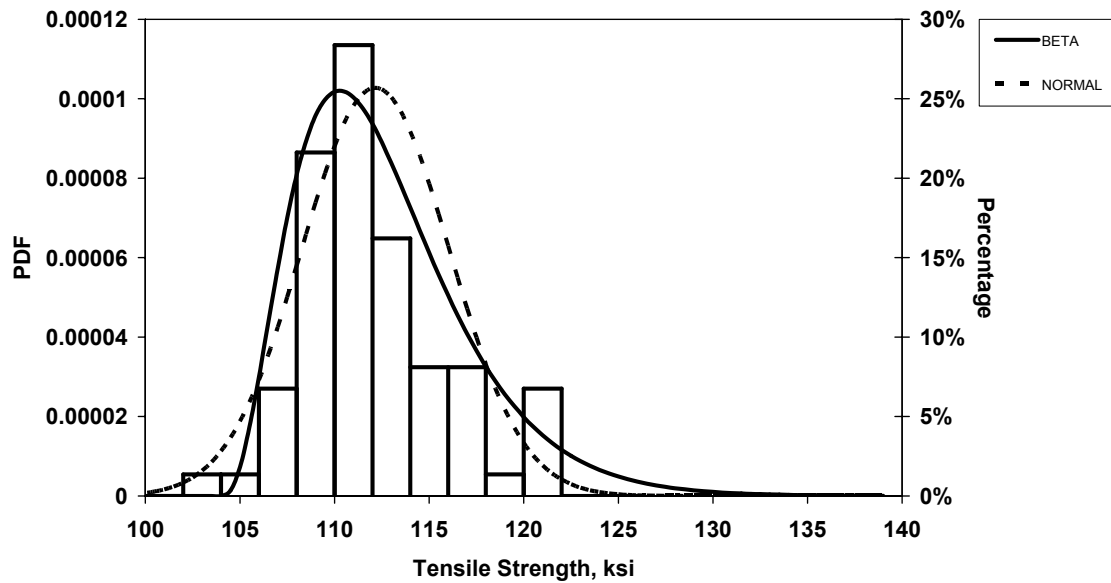


(b)

Figure 72: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 5 bars

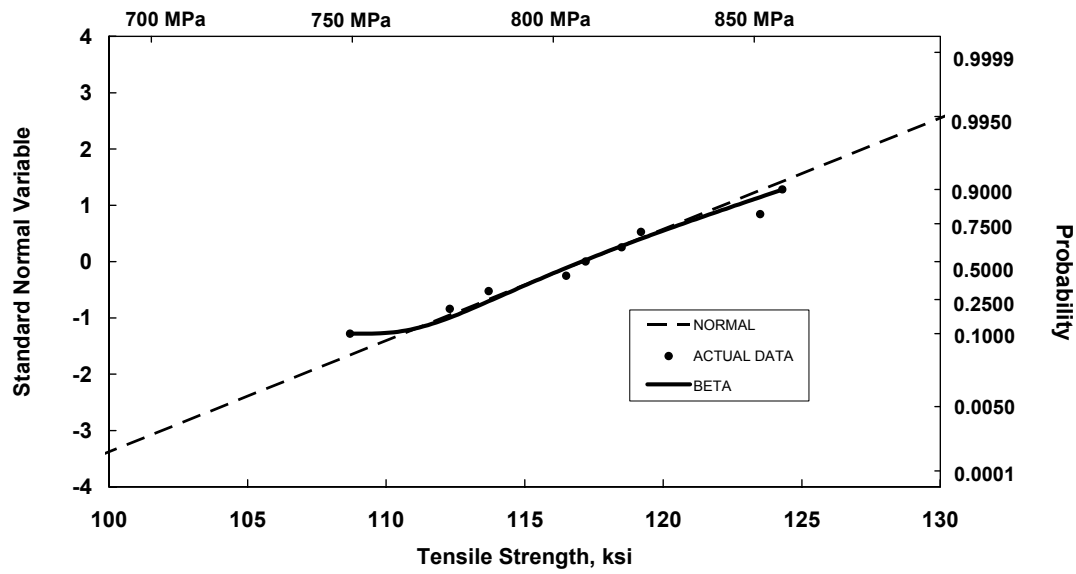


(a)

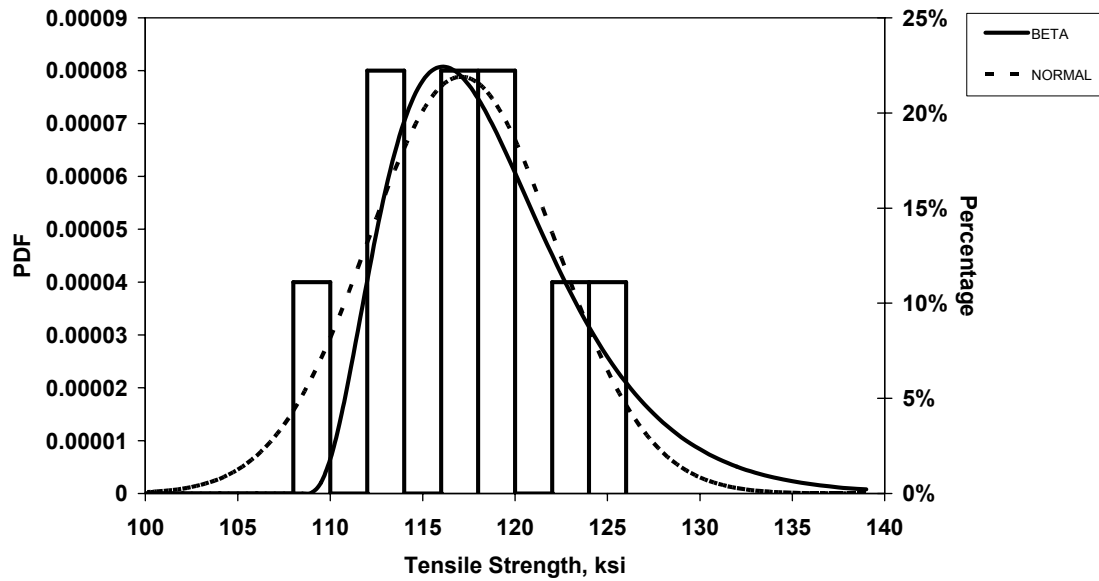


(b)

Figure 73: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 6 bars

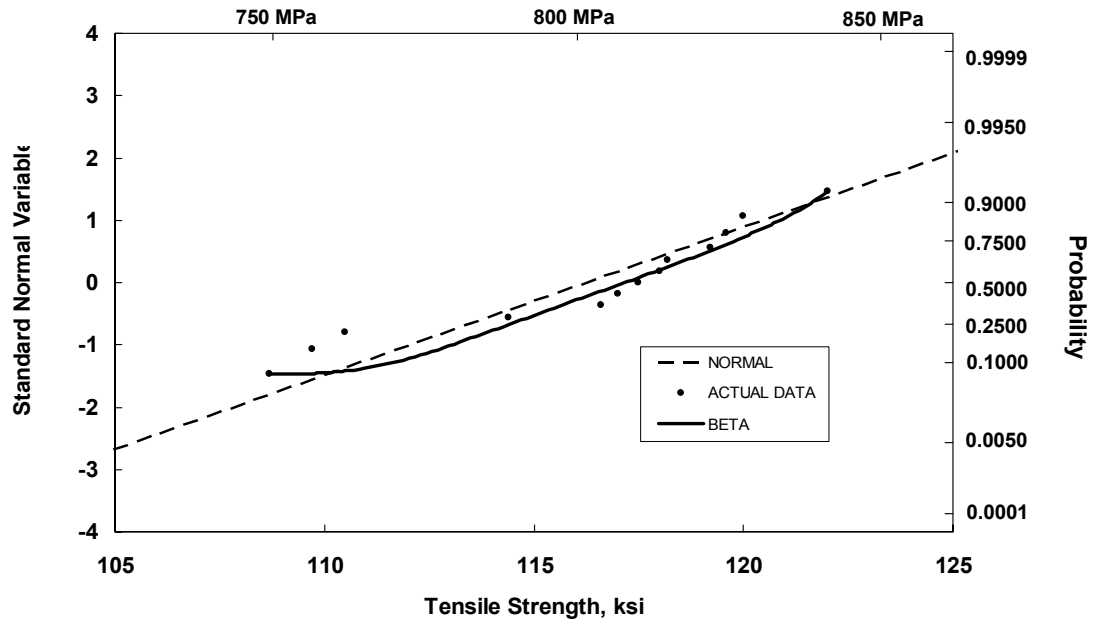


(a)

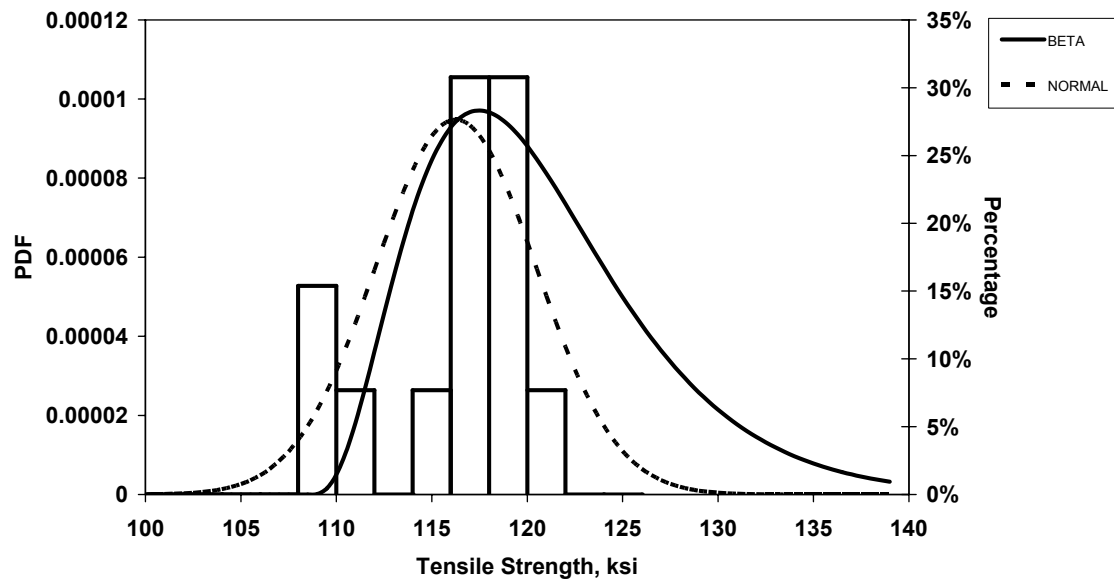


(b)

Figure 74: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 7 bars

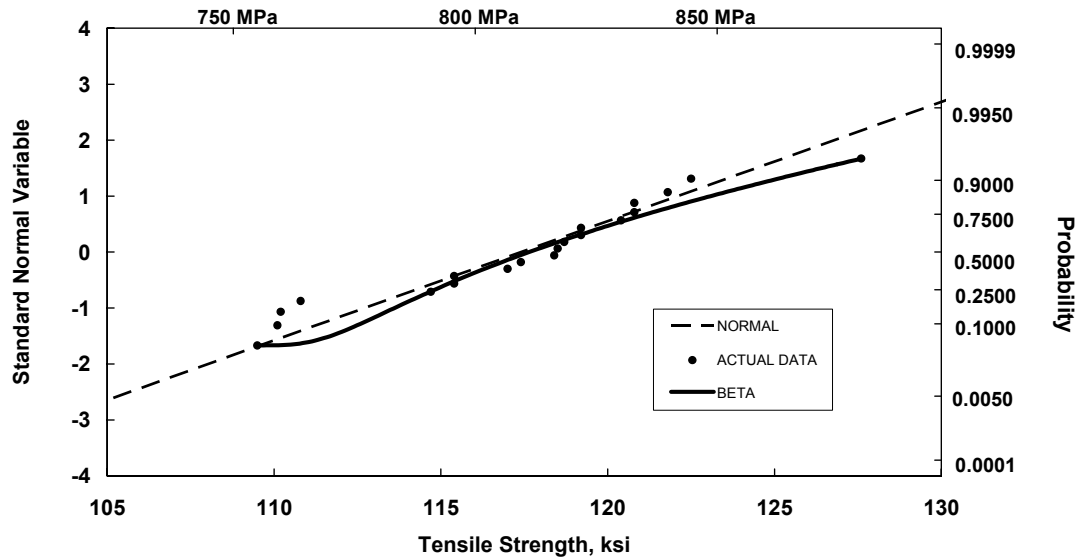


(a)

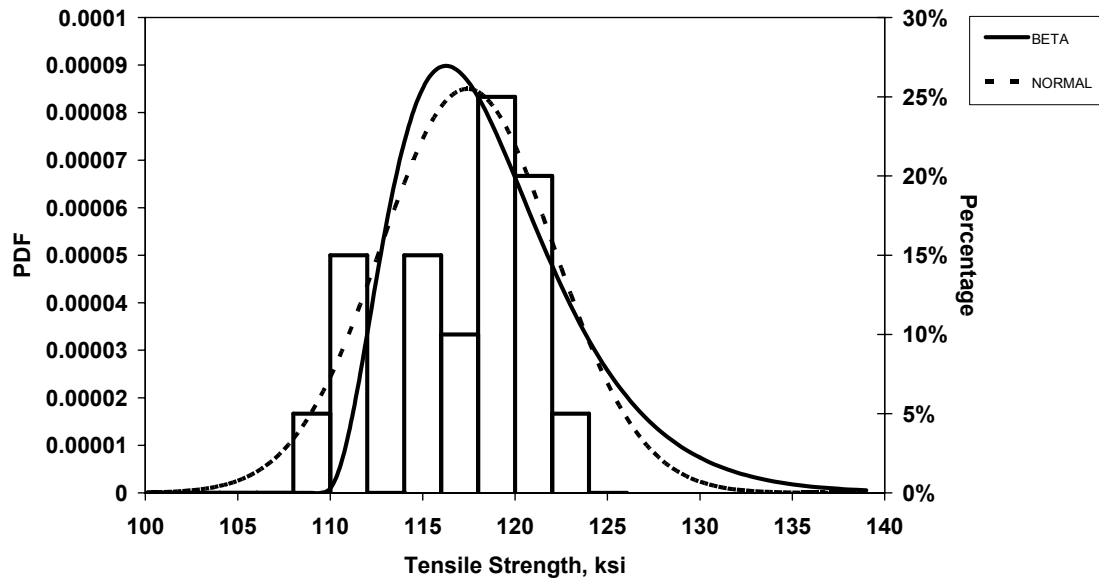


(b)

Figure 75: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 8 bars

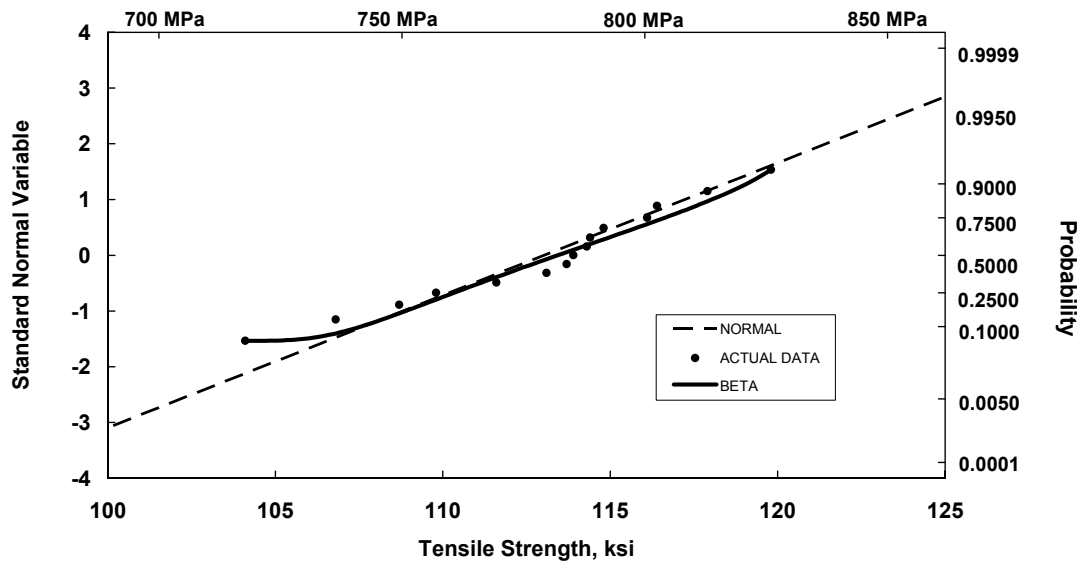


(a)

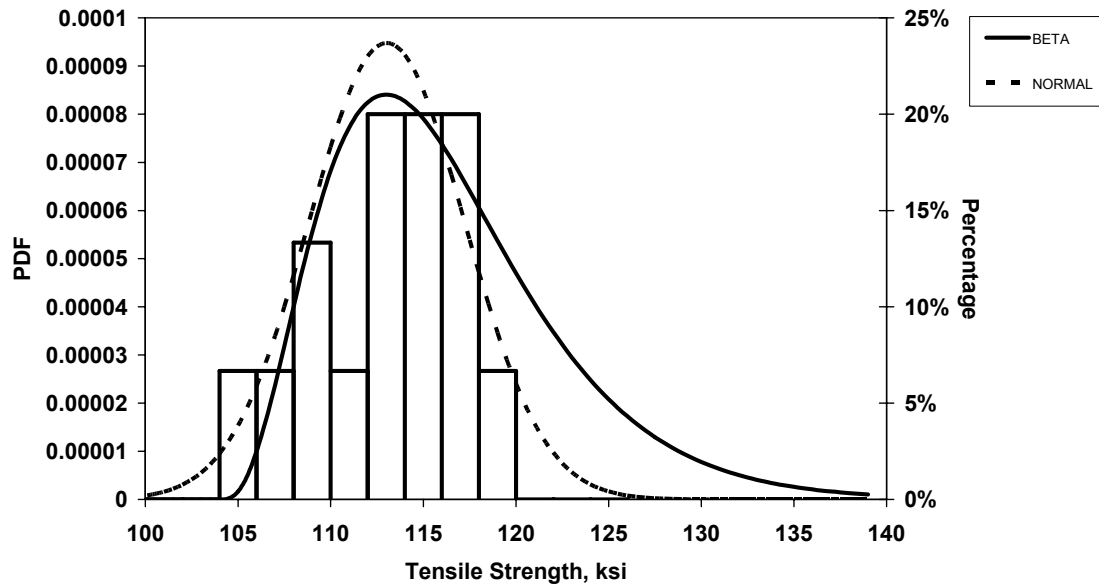


(b)

Figure 76: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 9 bars

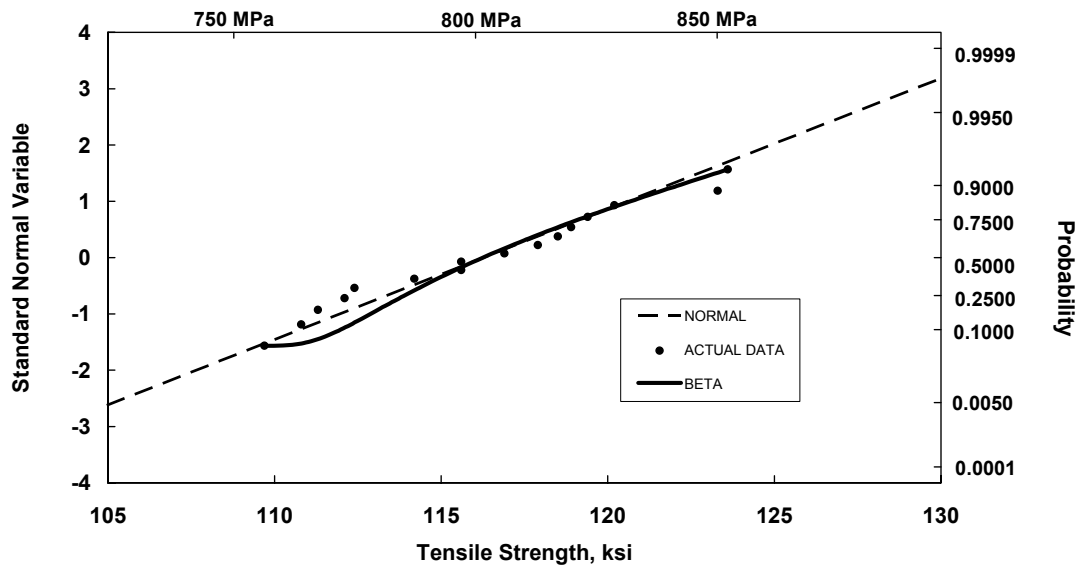


(a)

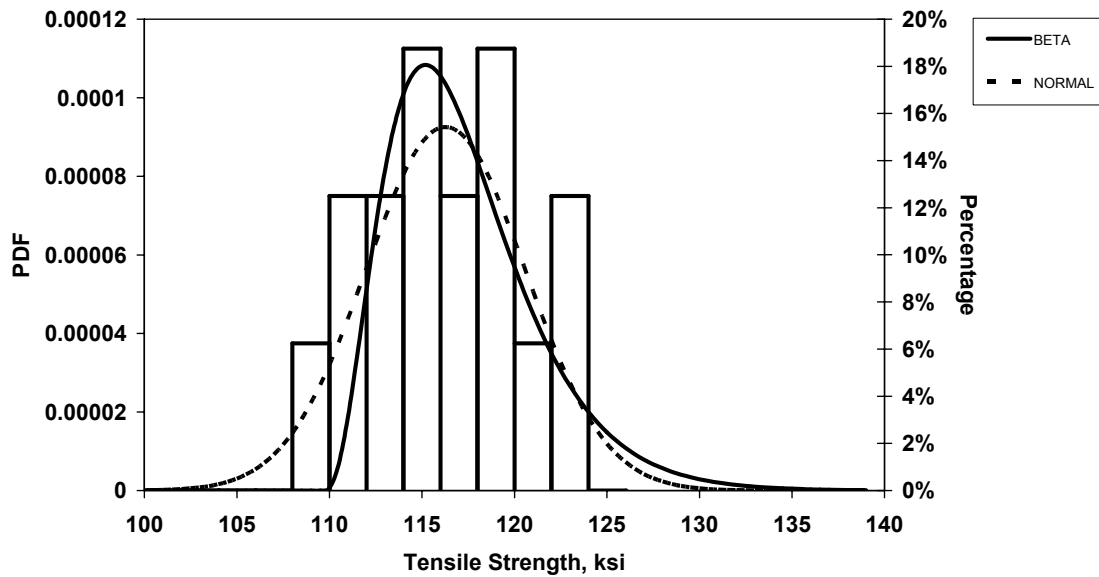


(b)

Figure 77: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 10 bars

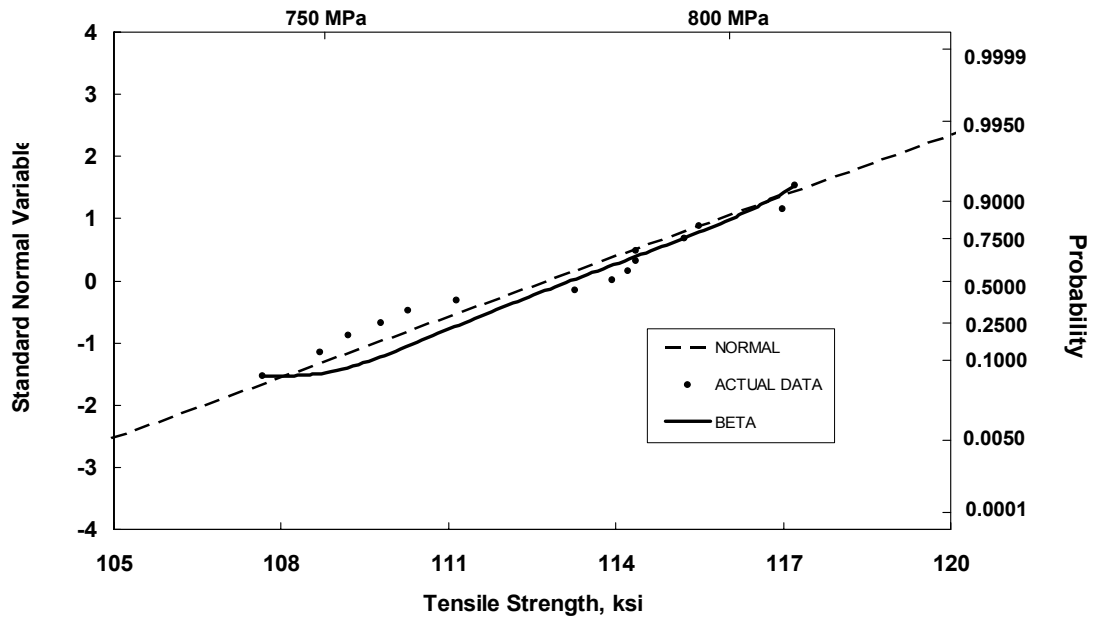


(a)

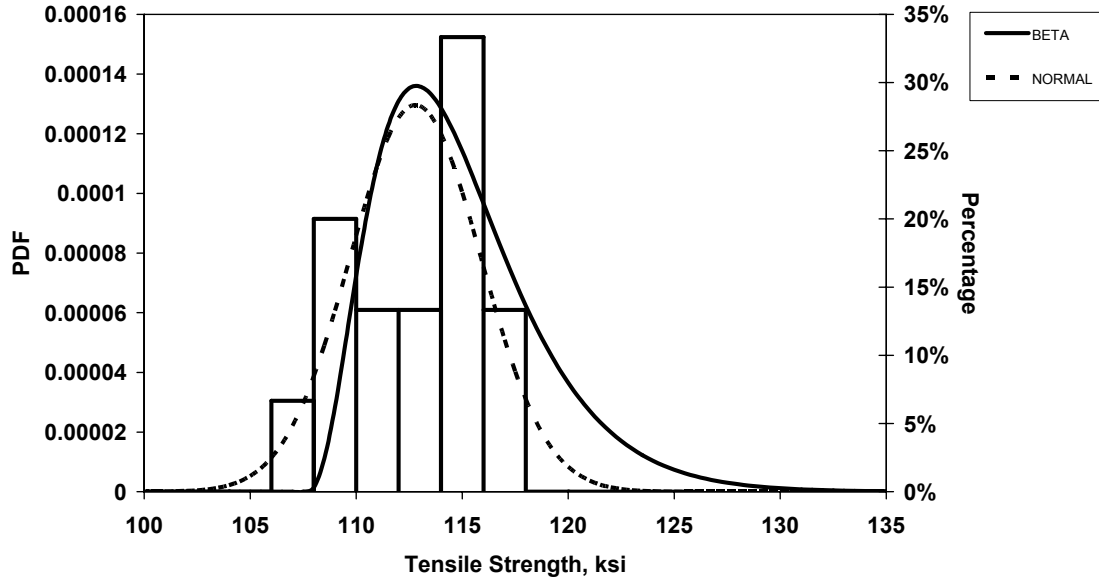


(b)

Figure 78: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 11 bars

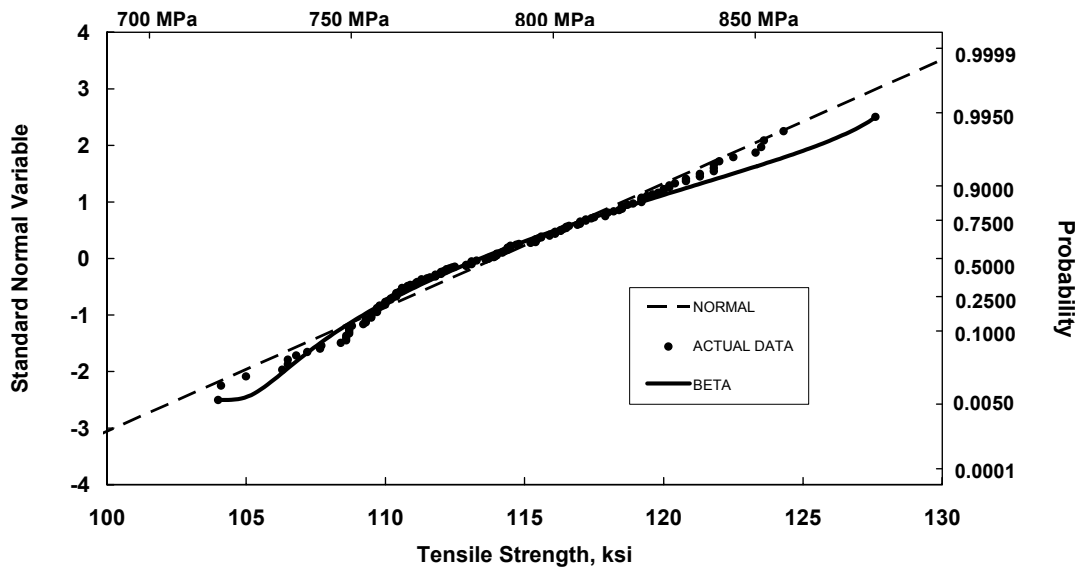


(a)

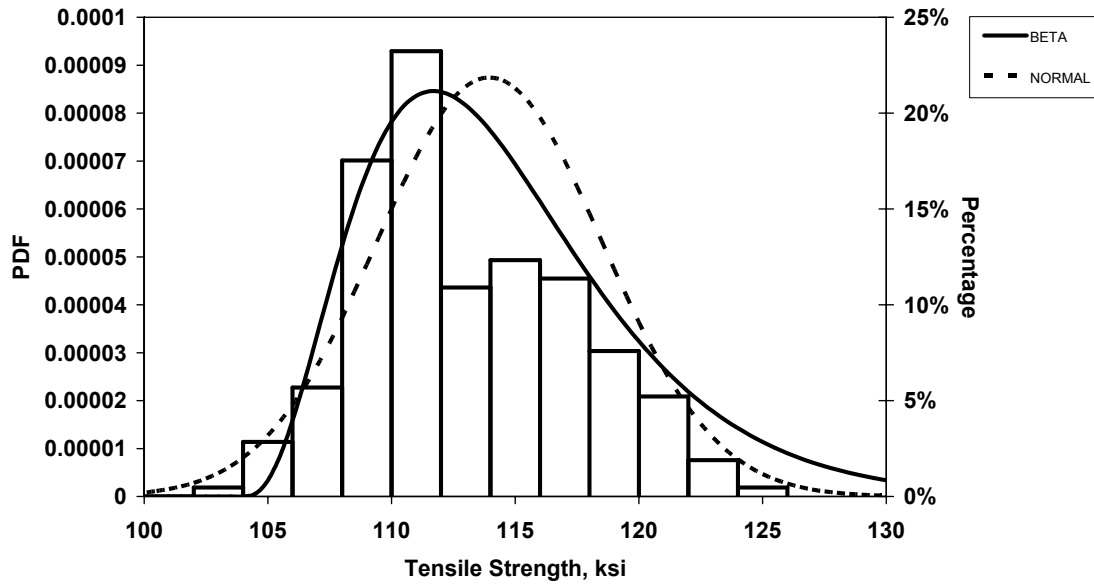


(b)

Figure 79: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 615 Grade 75 No. 14 bars

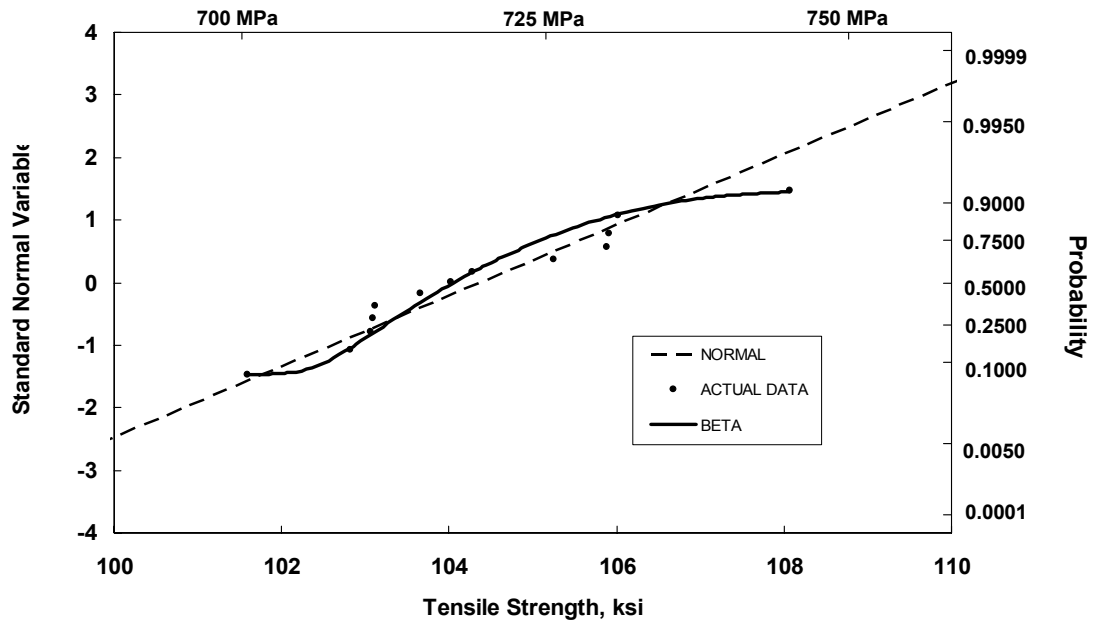


(a)

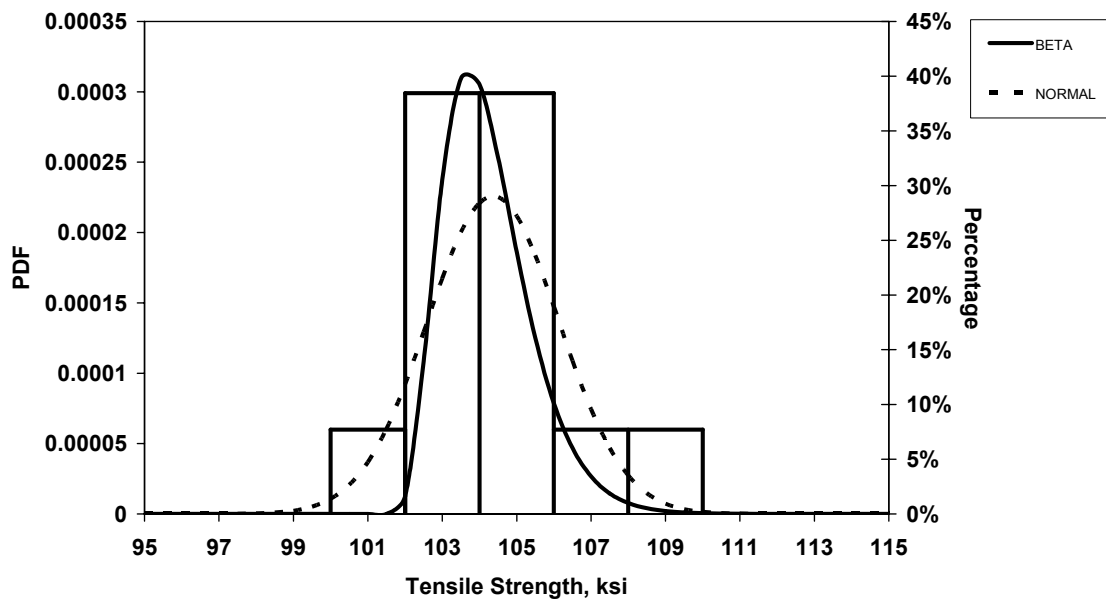


(b)

Figure 80: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of All A 615 Grade 75 bars

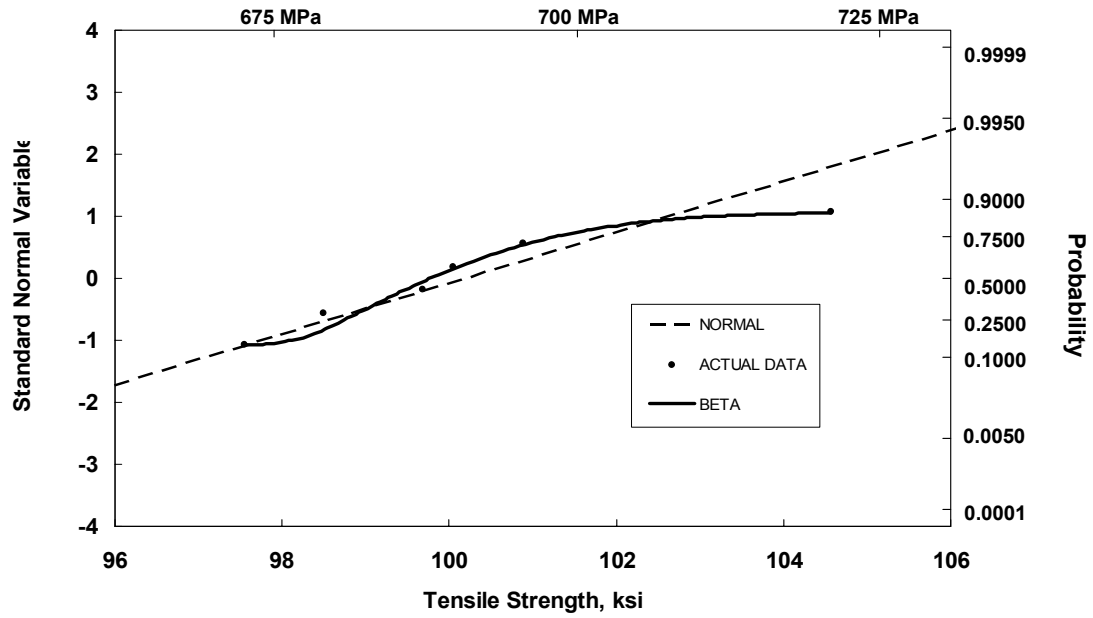


(a)

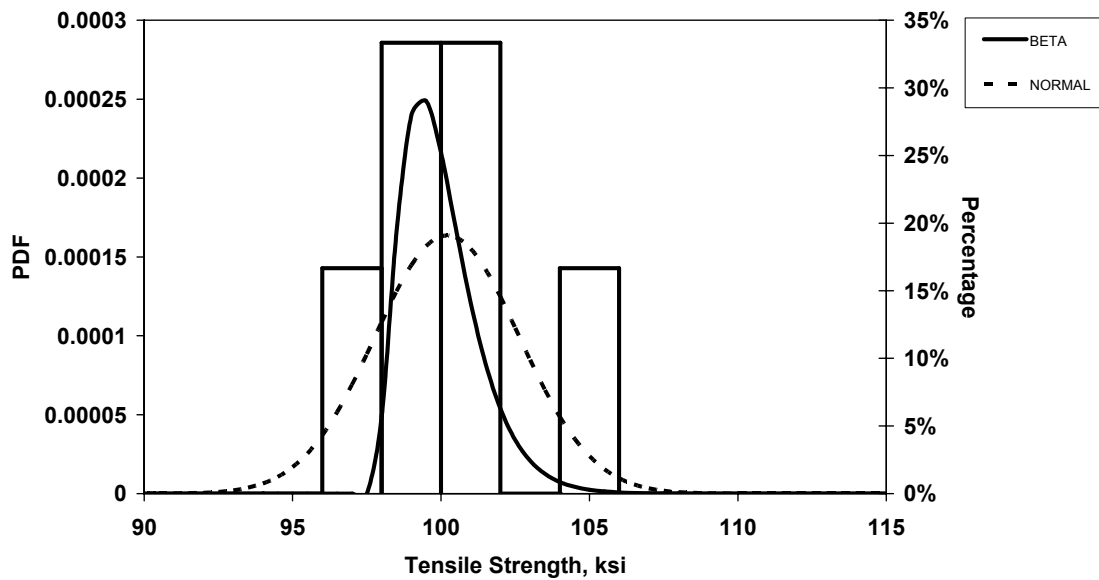


(b)

Figure 81: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 616 Grade 60 No. 8 bars

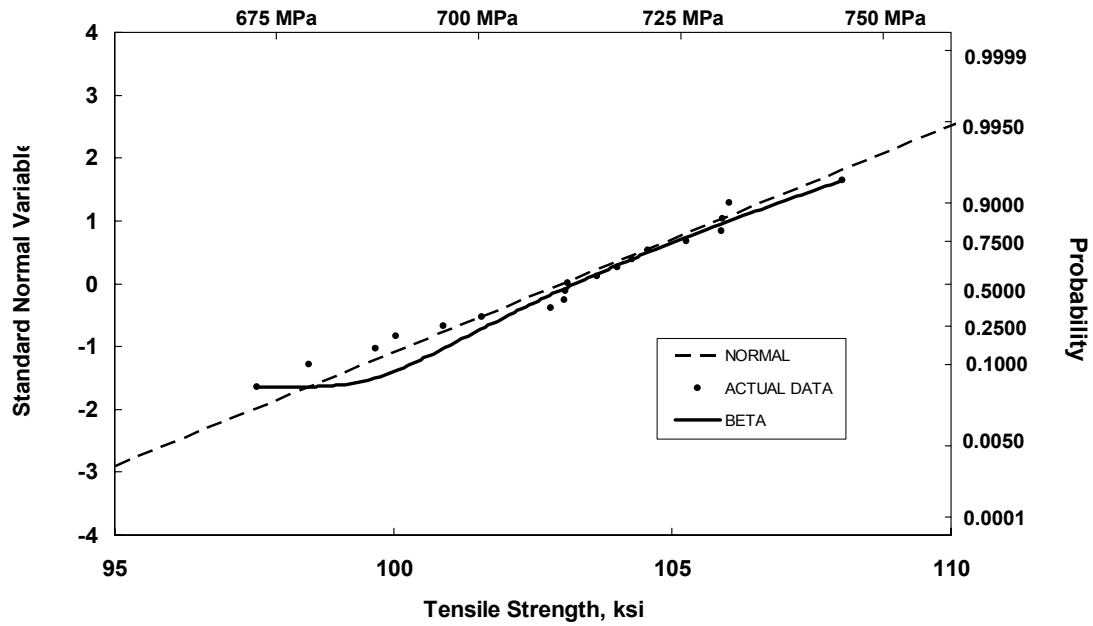


(a)

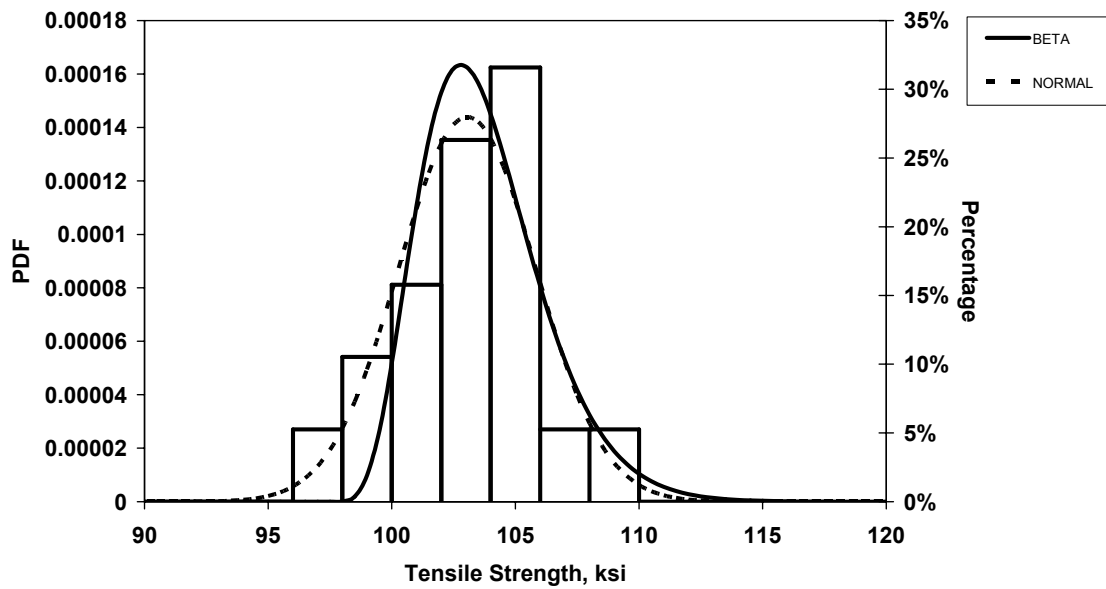


(b)

Figure 82: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 616 Grade 60 No. 10 bars



(a)



(b)

Figure 83: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of All A 616 Grade 60 bars

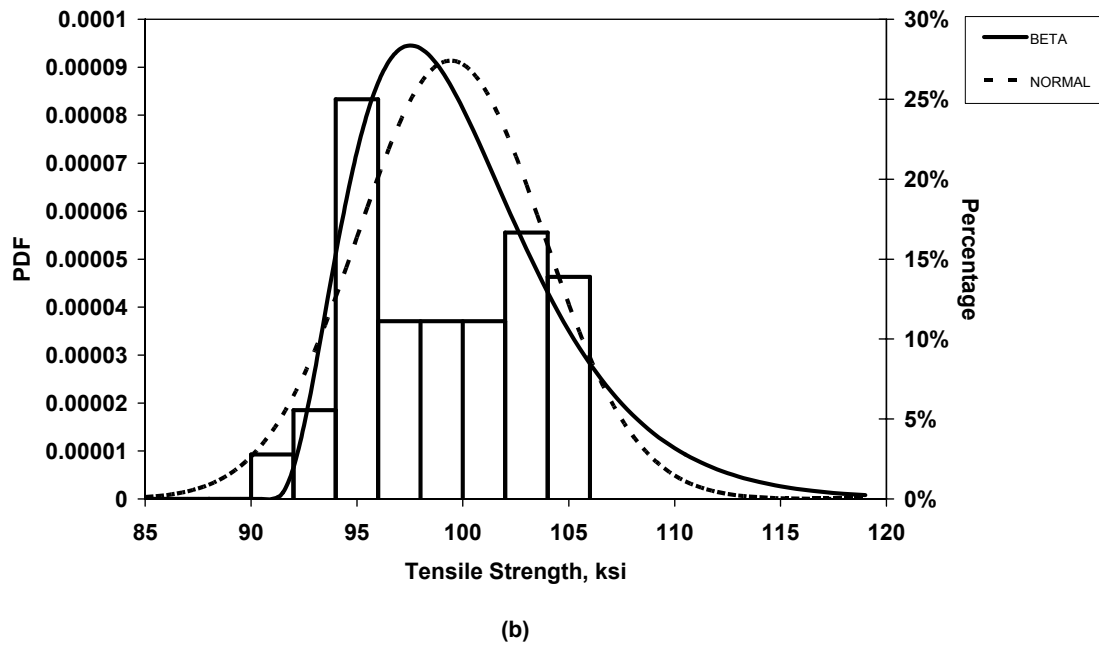
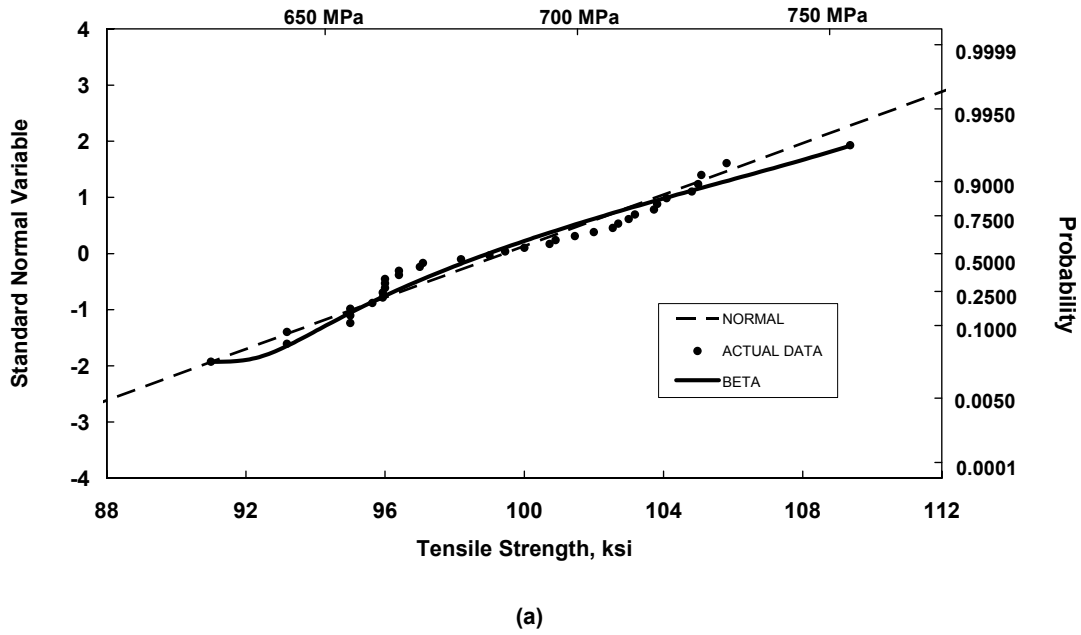
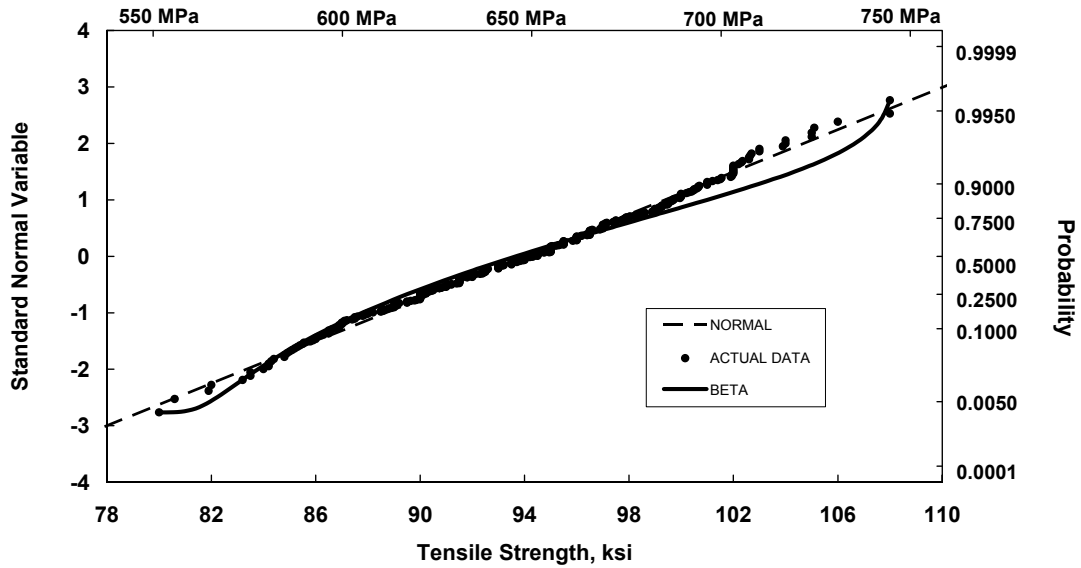
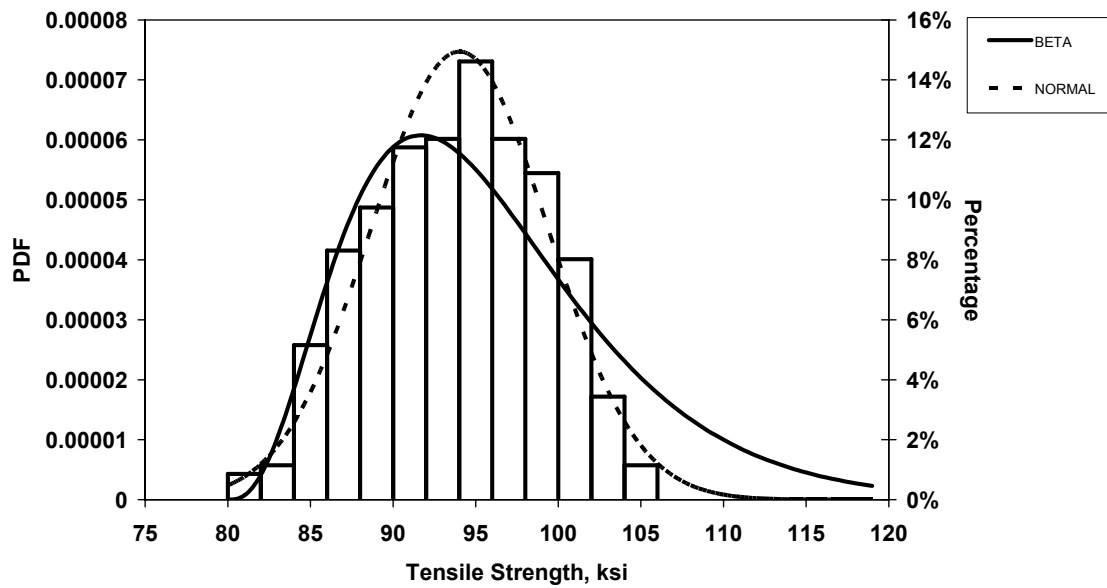


Figure 84: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 3 bars

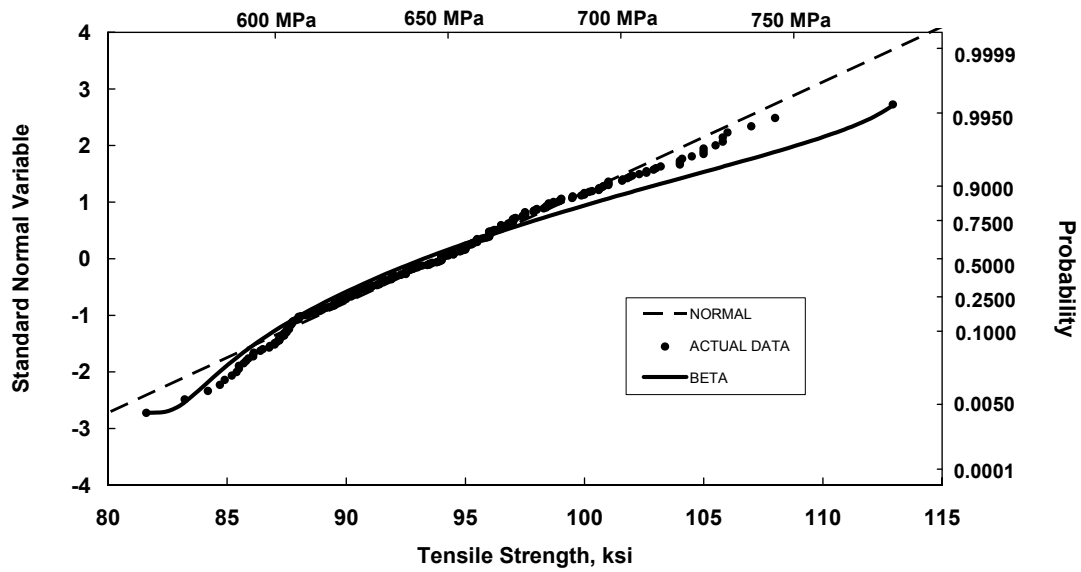


(a)

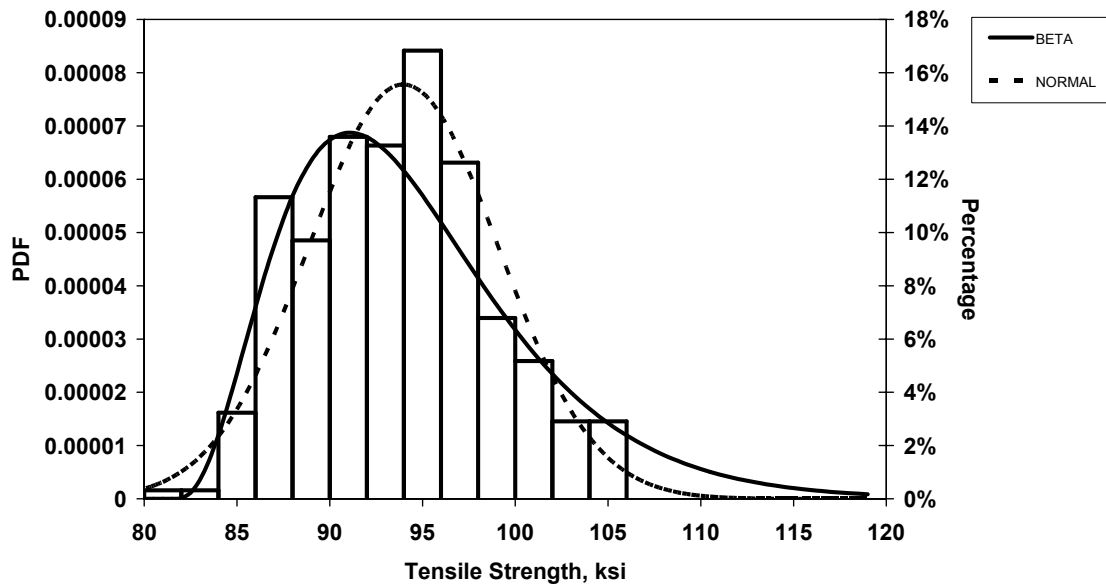


(b)

Figure 85: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 4 bars

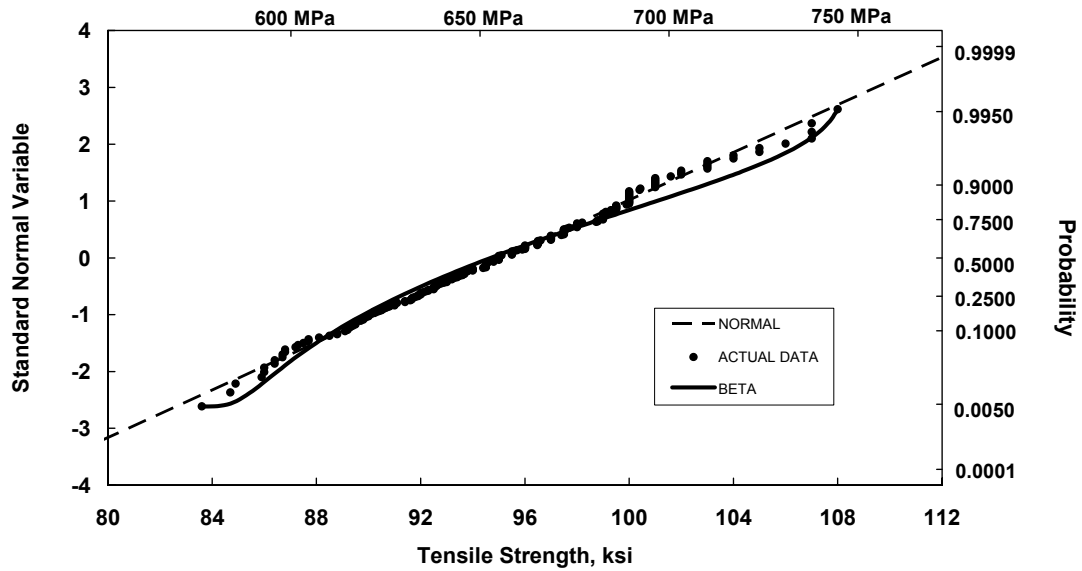


(a)

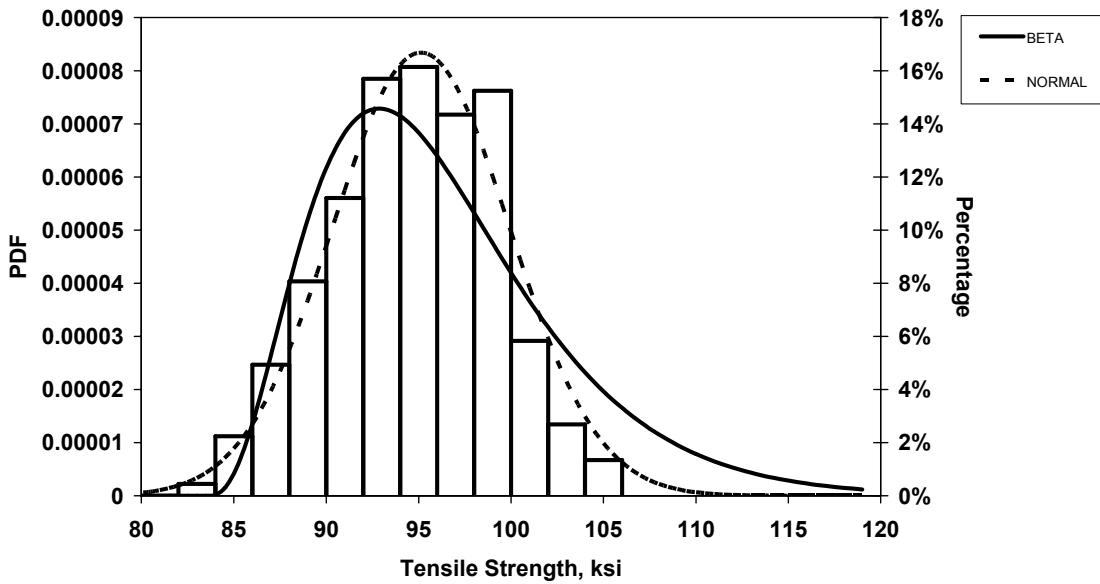


(b)

Figure 86: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 5 bars

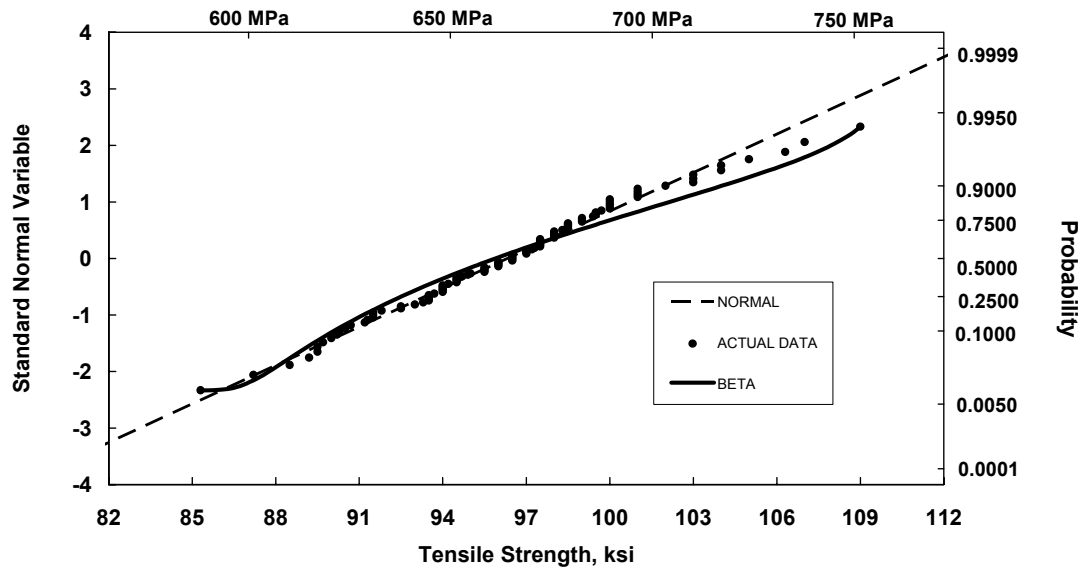


(a)

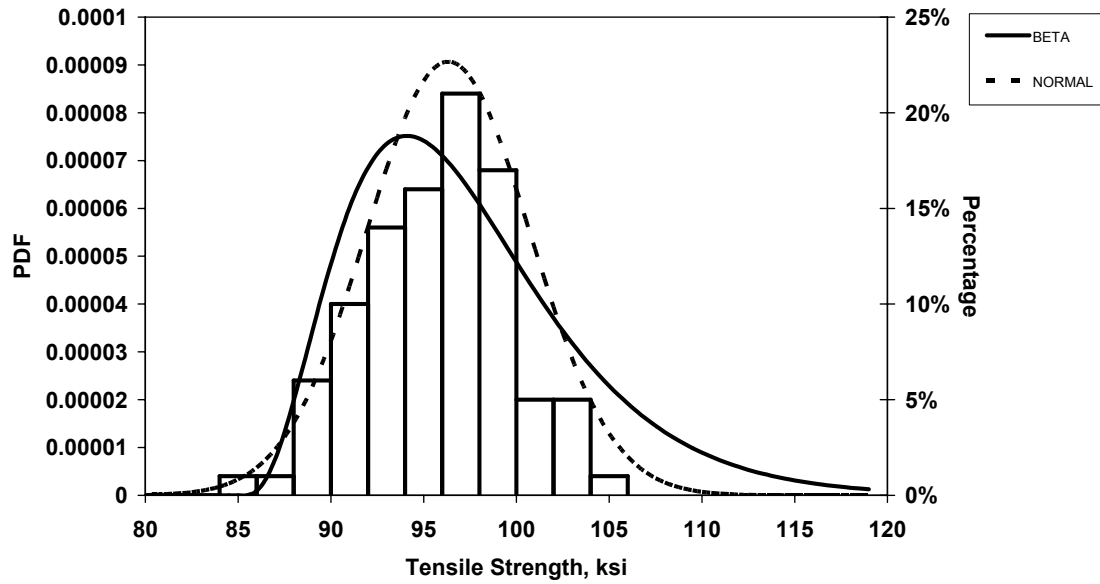


(b)

Figure 87: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 6 bars

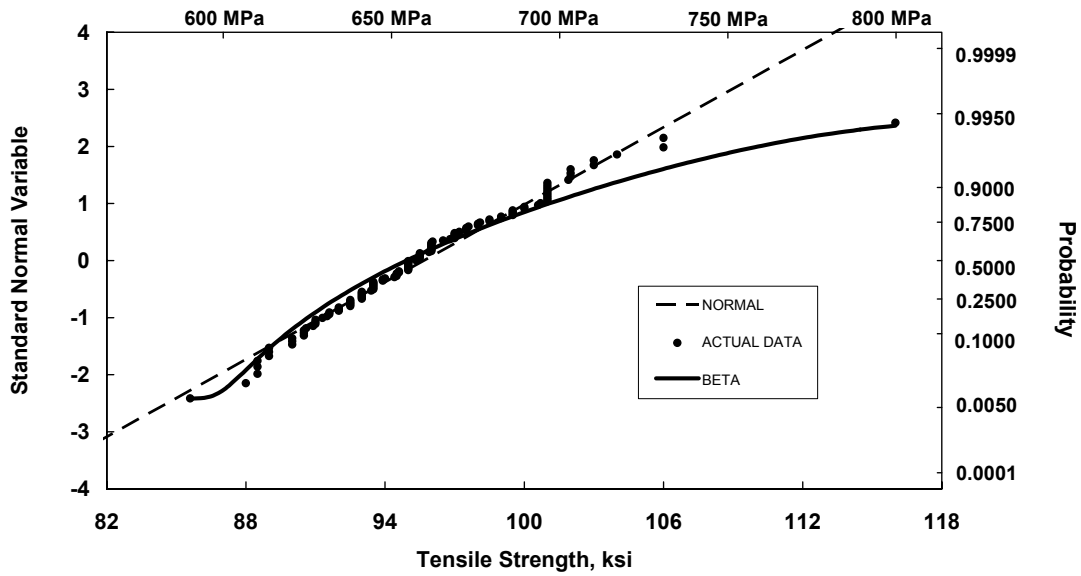


(a)

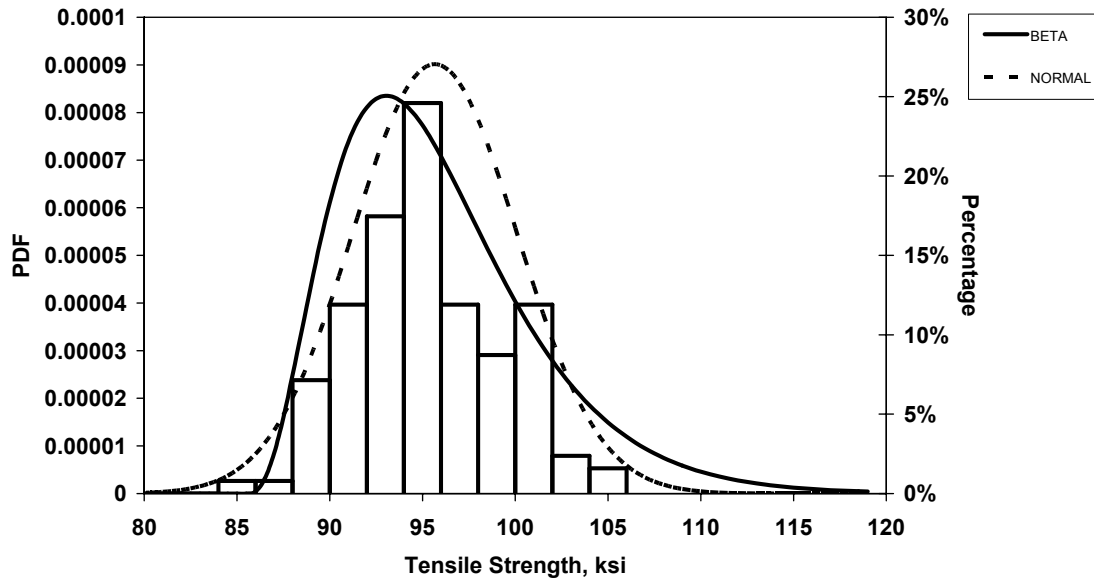


(b)

Figure 88: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 7 bars

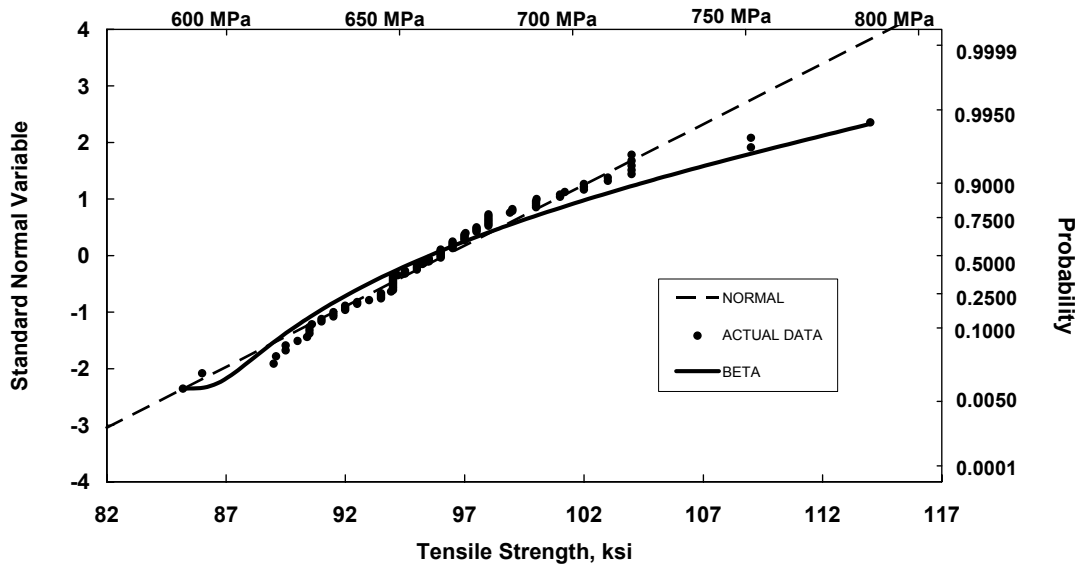


(a)

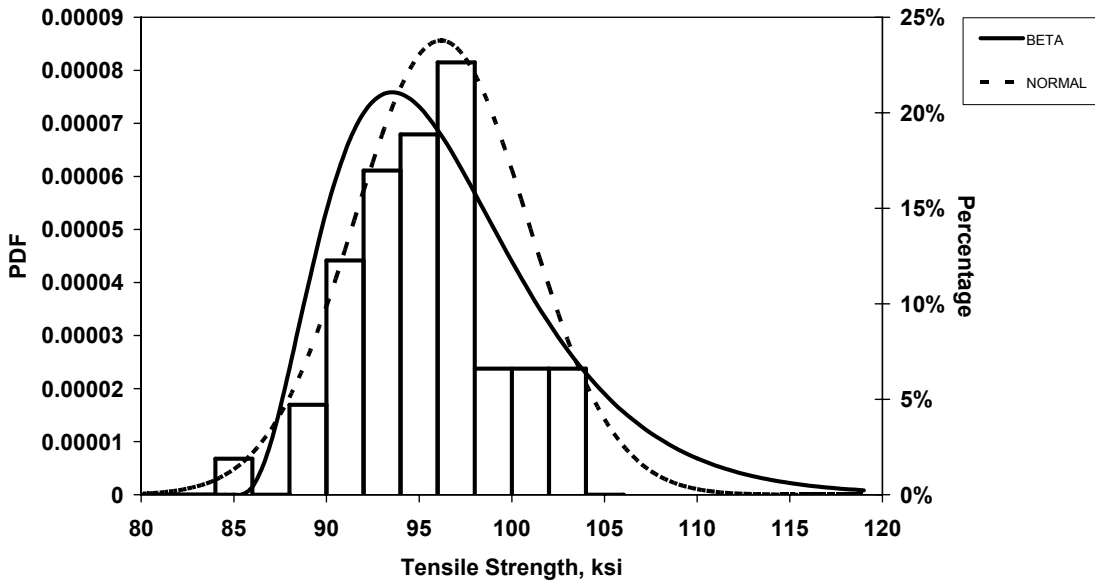


(b)

Figure 89: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 8 bars

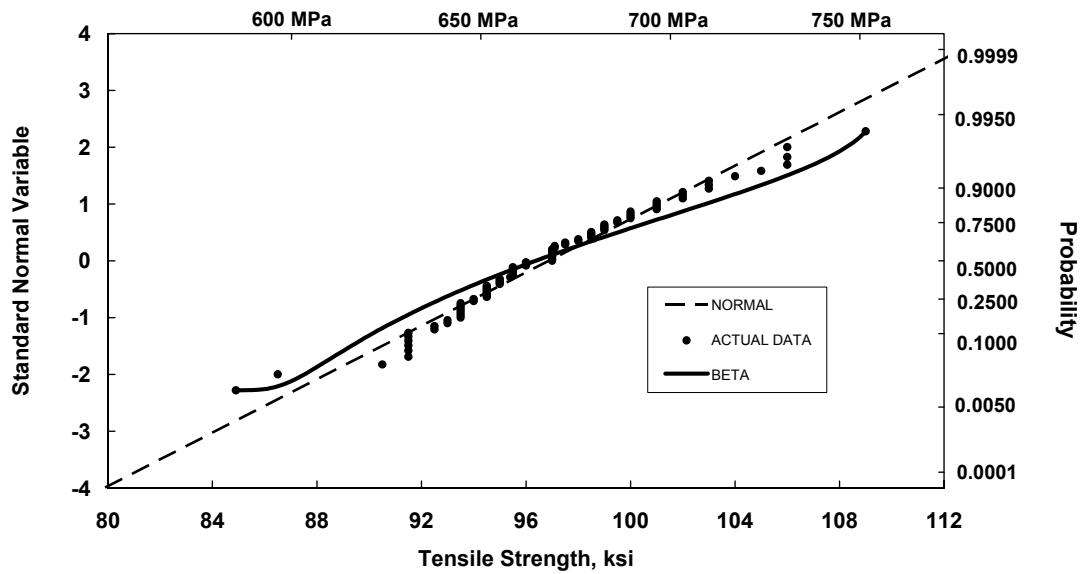


(a)

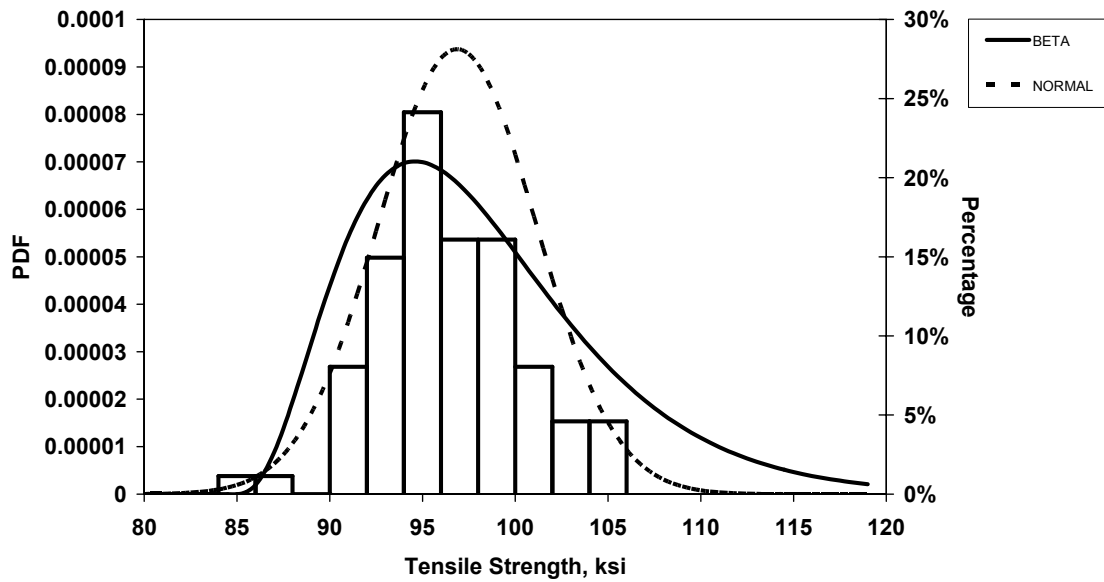


(b)

Figure 90: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 9 bars

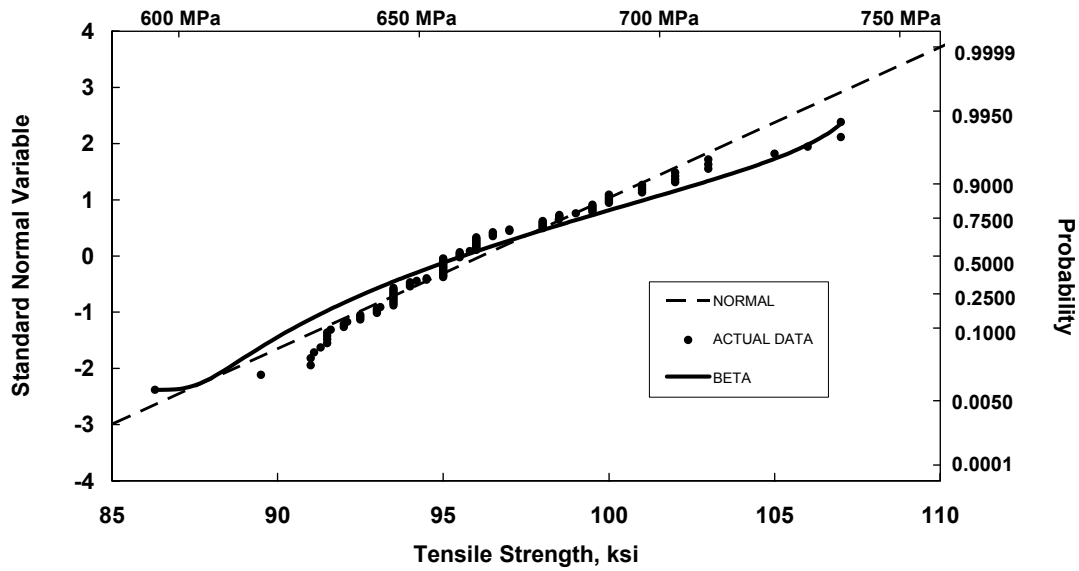


(a)

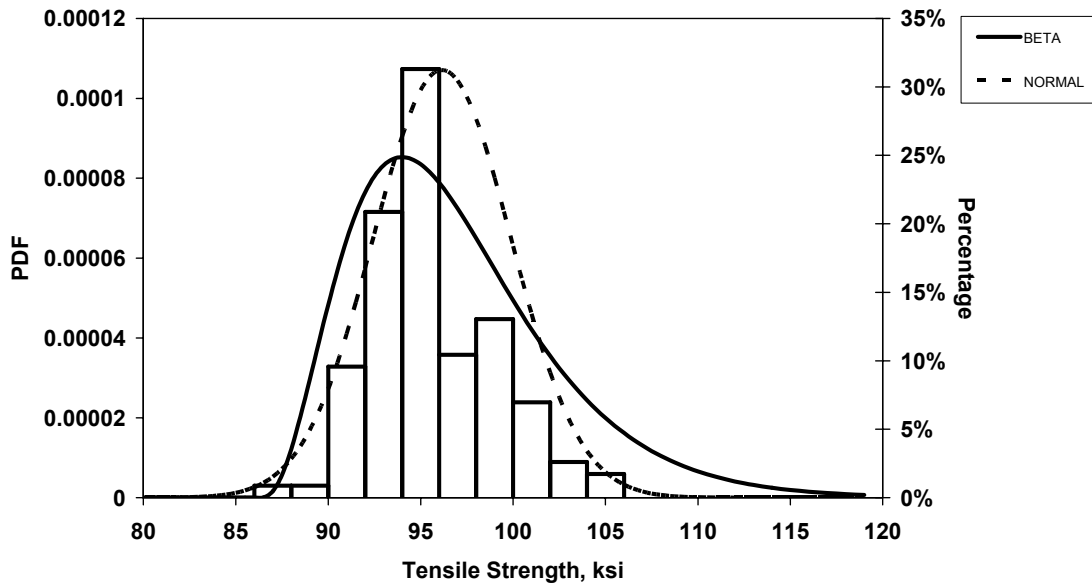


(b)

Figure 91: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 10 bars

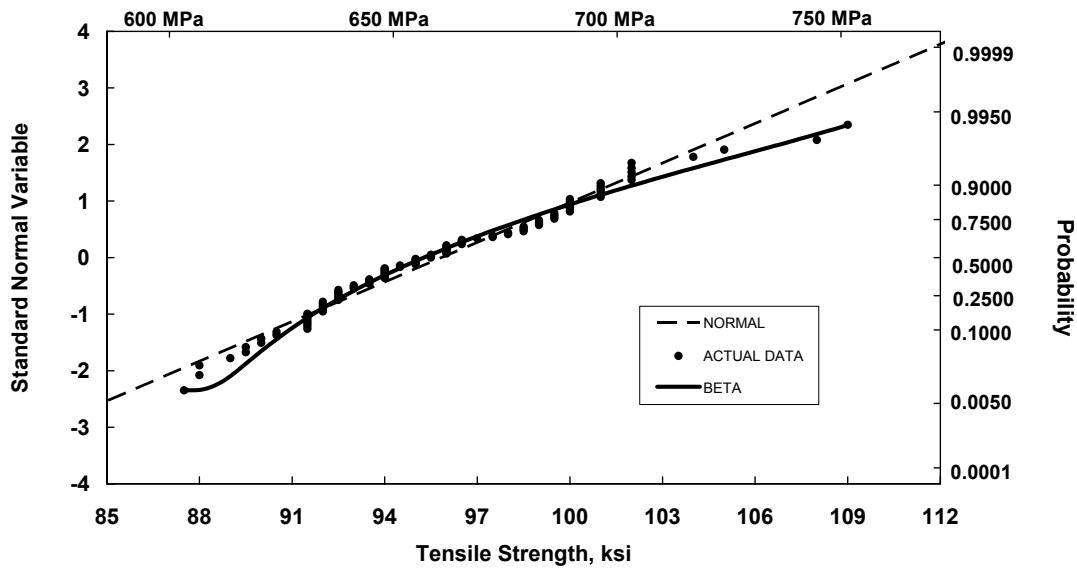


(a)

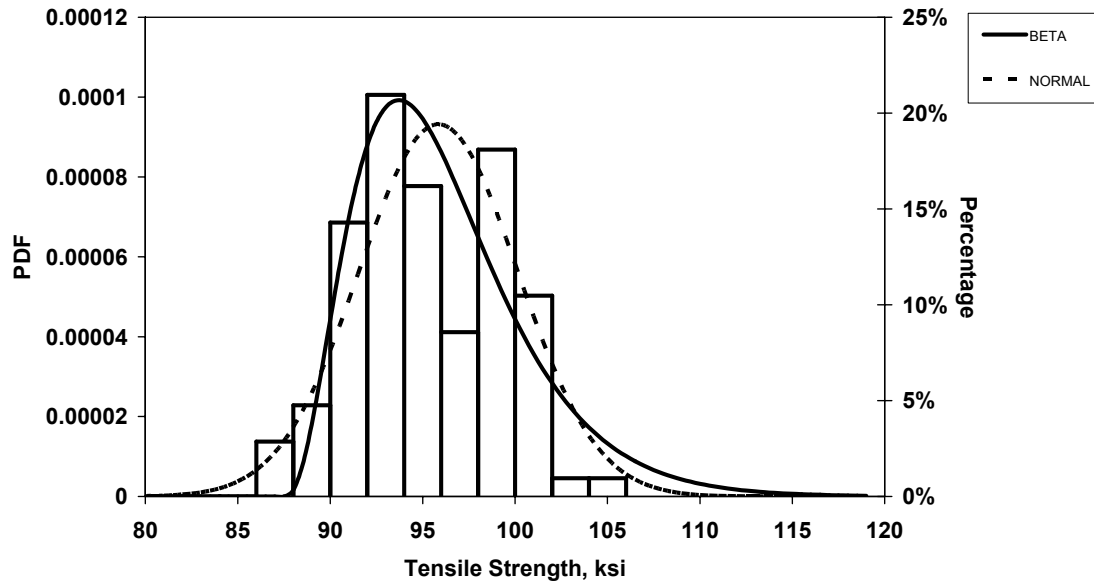


(b)

Figure 92: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 11 bars

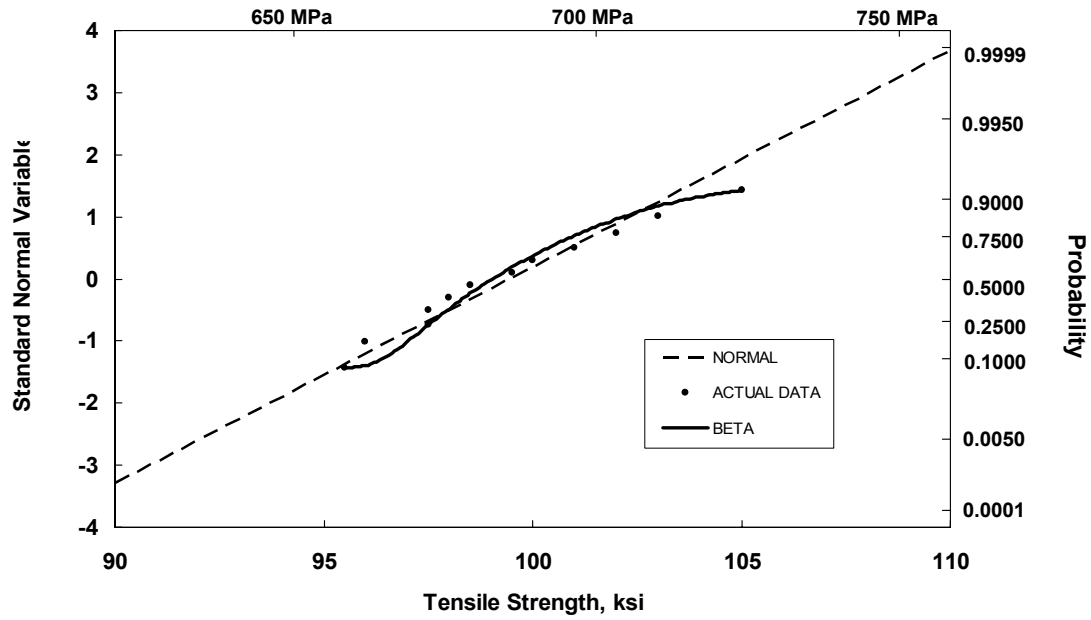


(a)

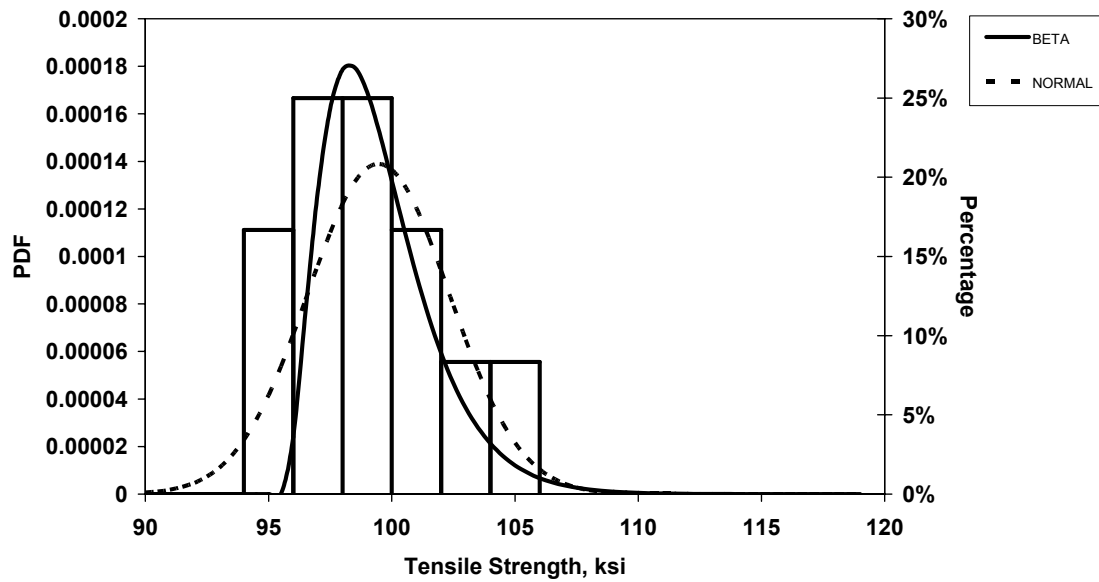


(b)

Figure 93: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 14 bars

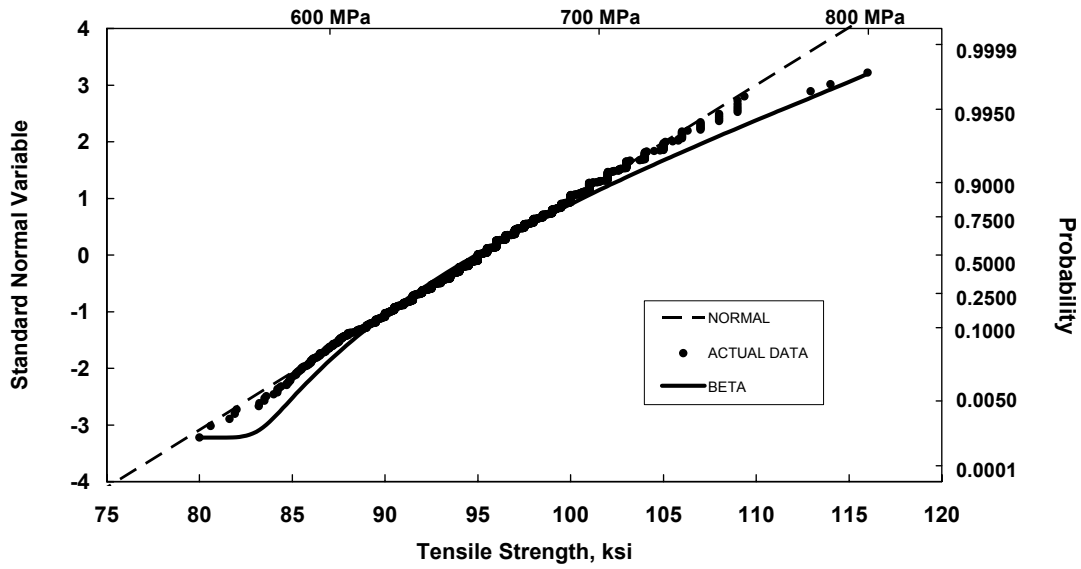


(a)

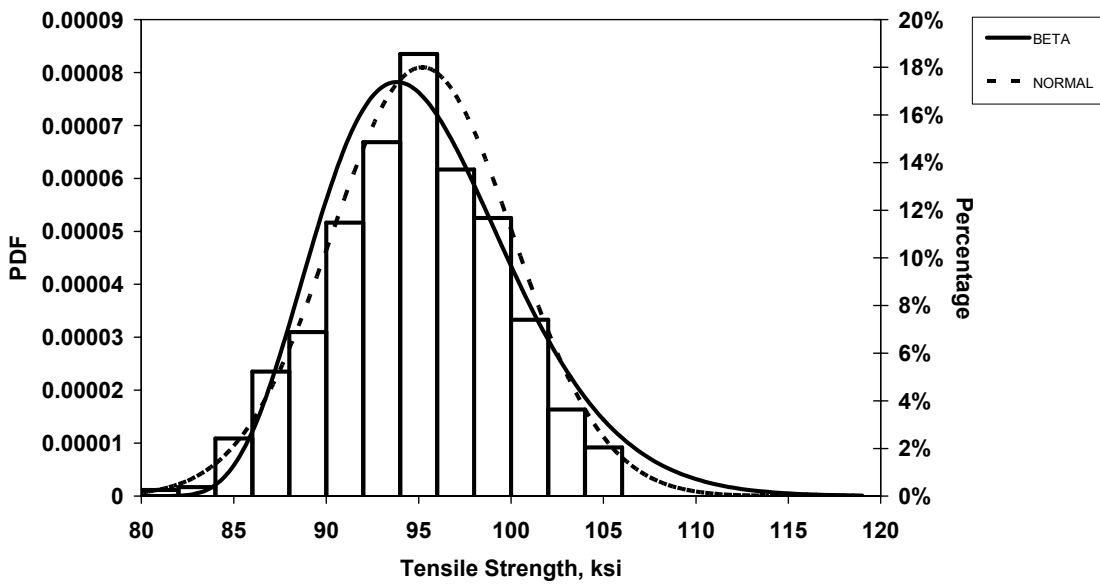


(b)

Figure 94: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of A 706 Grade 60 No. 18 bars

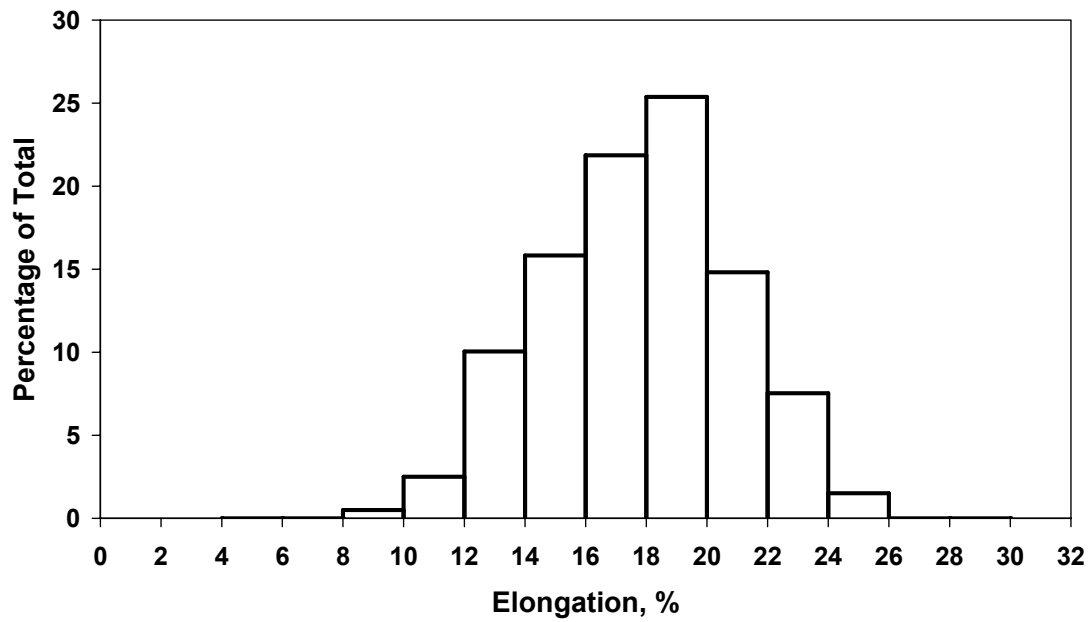


(a)

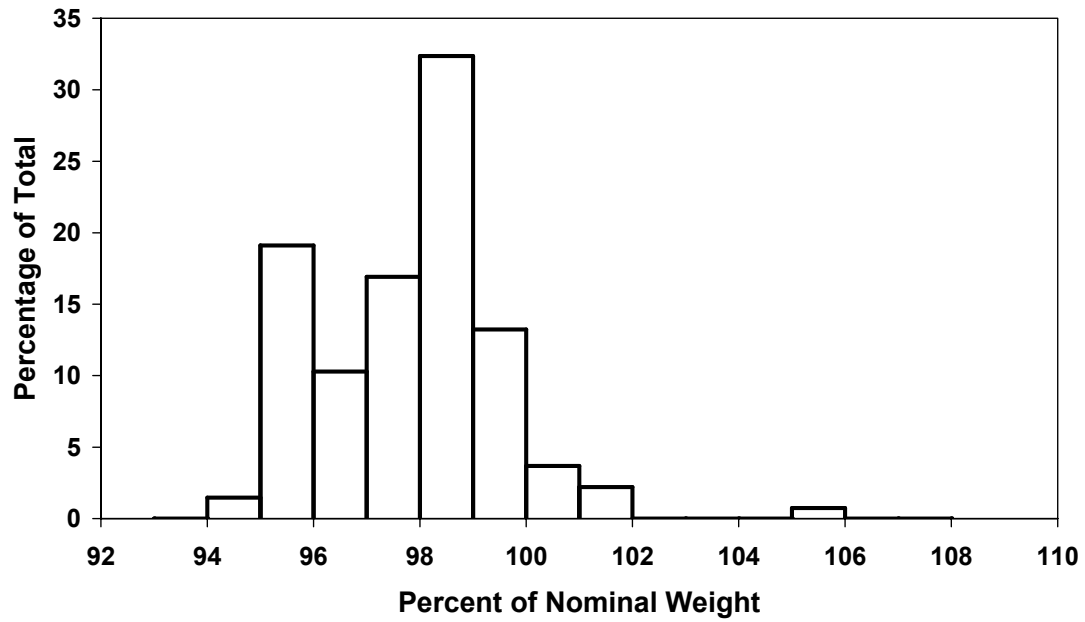


(b)

Figure 95: (a) Cumulative Density Function and (b) Histogram with Probability Density Function for Tensile Strength of All A 706 Grade 60 bars

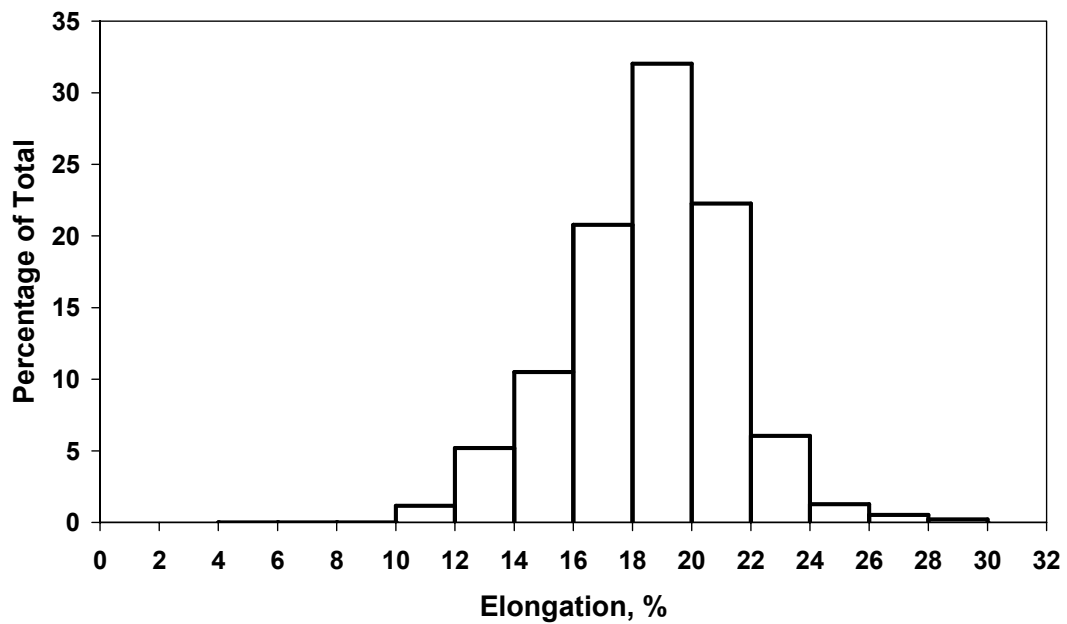


(a)

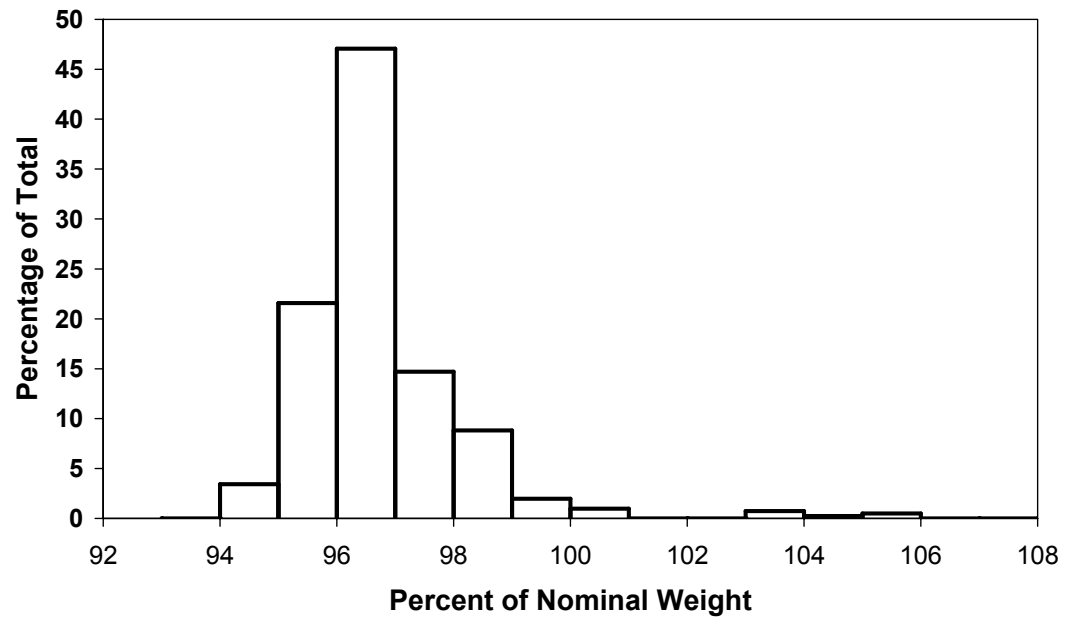


(b)

Figure 96: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 40 No. 3 bars

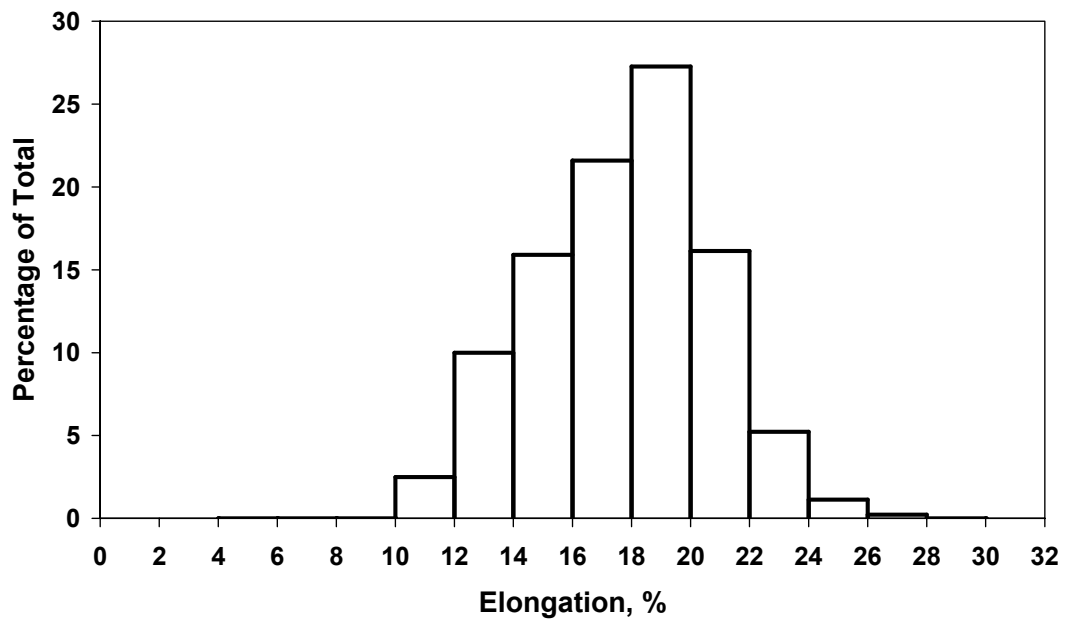


(a)

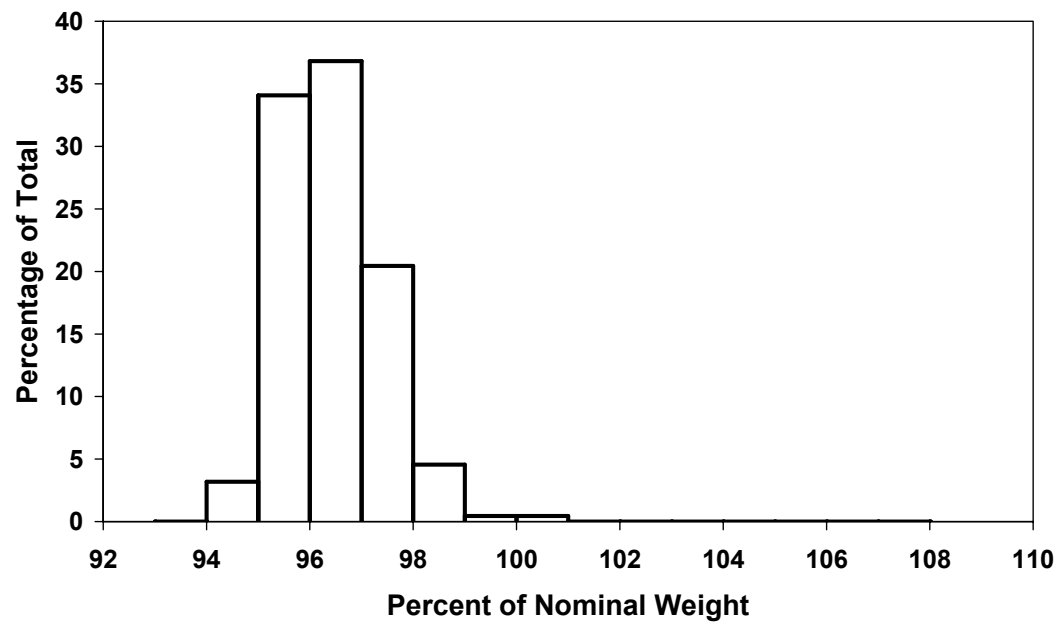


(b)

Figure 97: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 40 No. 4 bars

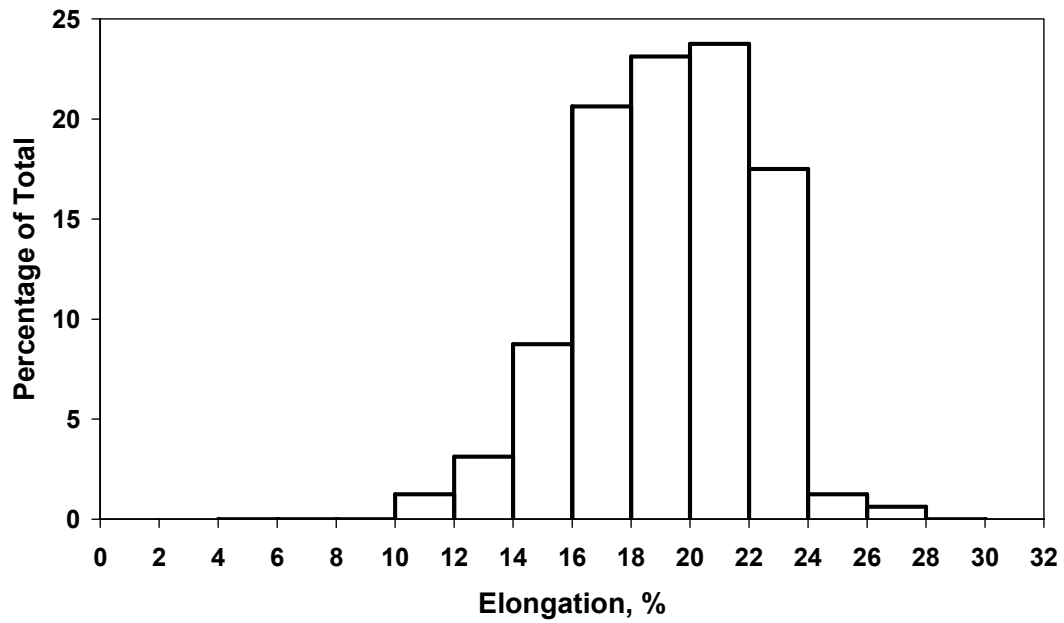


(a)

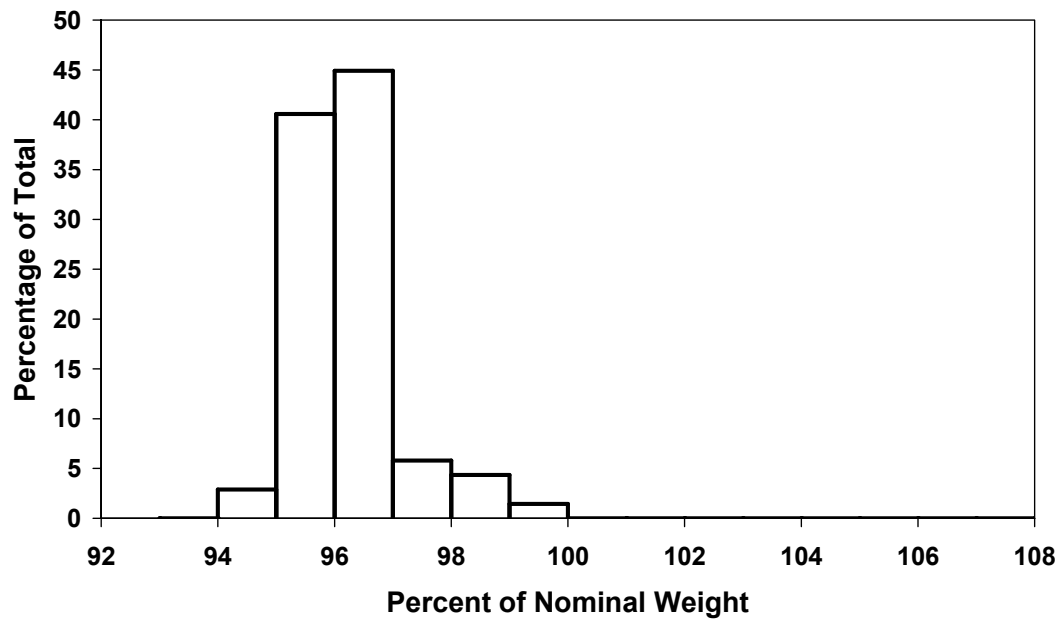


(b)

Figure 98: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 40 No. 5 bars

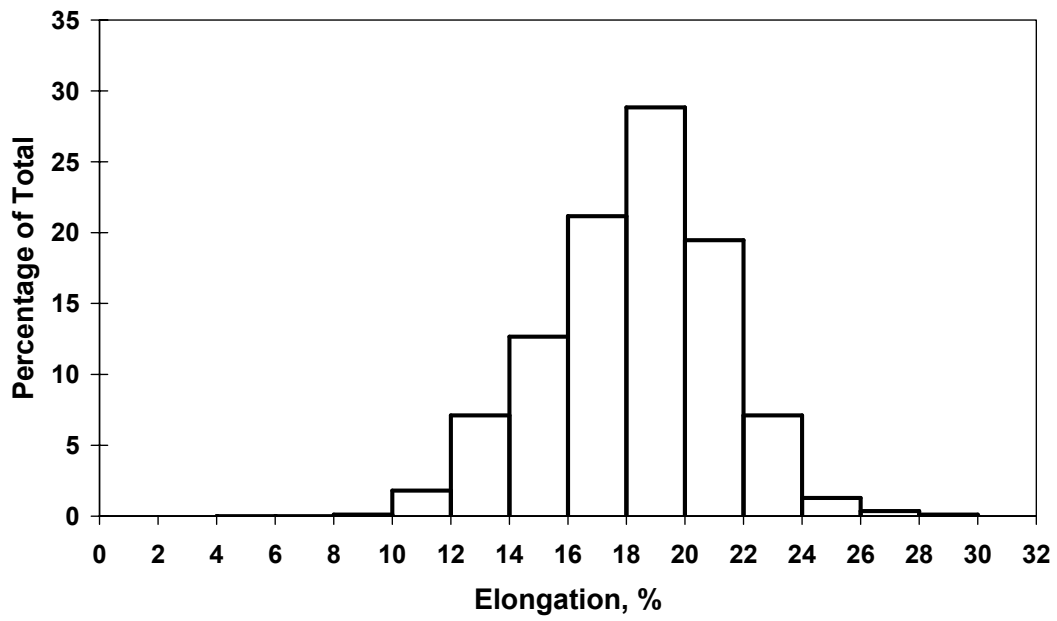


(a)

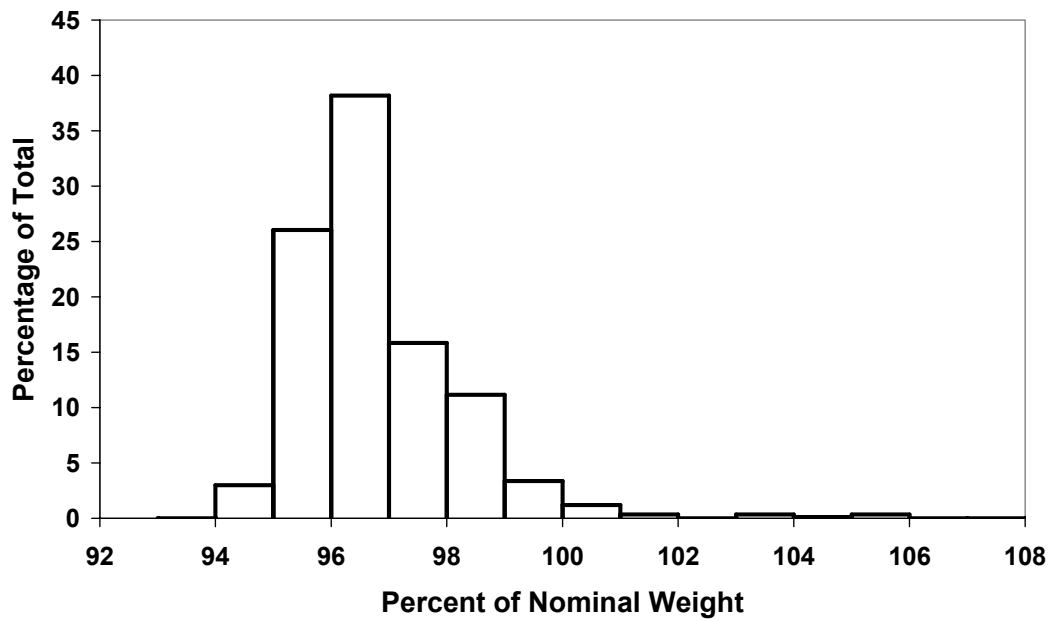


(b)

Figure 99: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 40 No. 6 bars

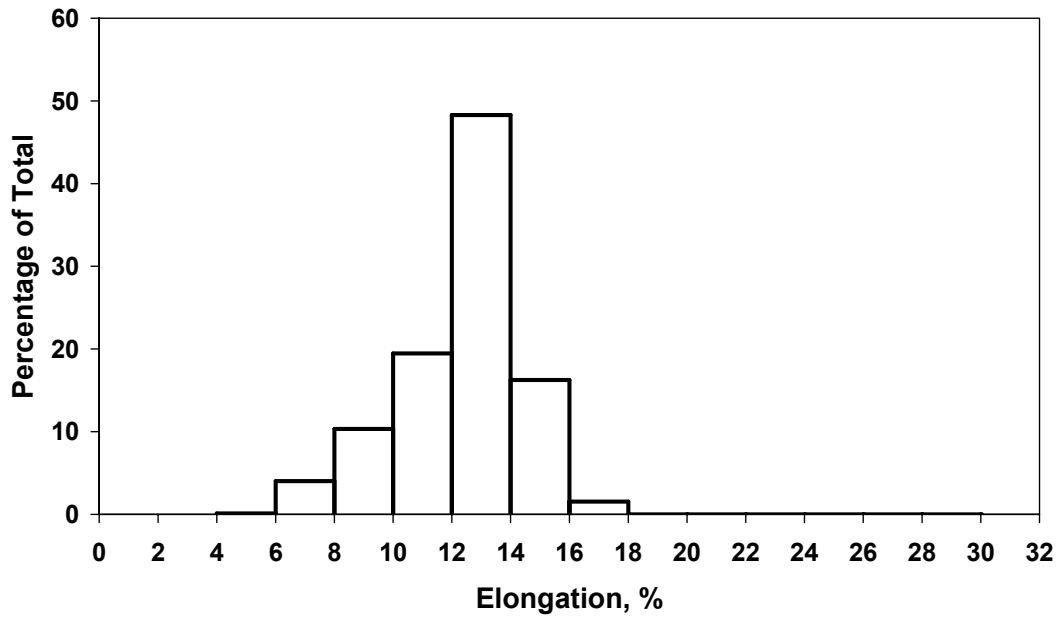


(a)

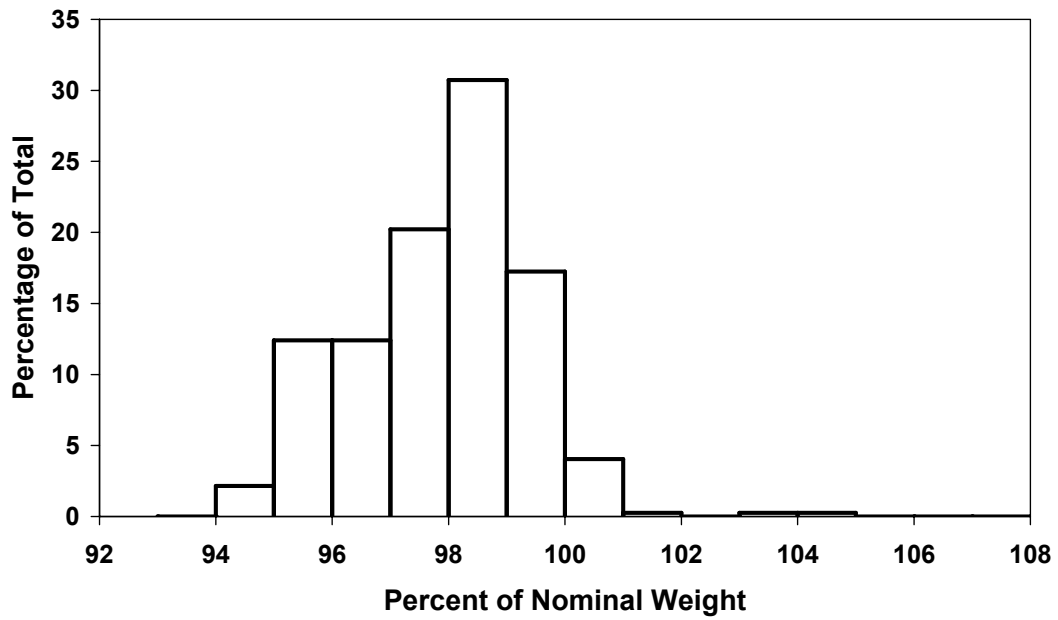


(b)

Figure 100: (a) Elongation and (b) Percent of Nominal Weight for All A 615 Grade 40 bars

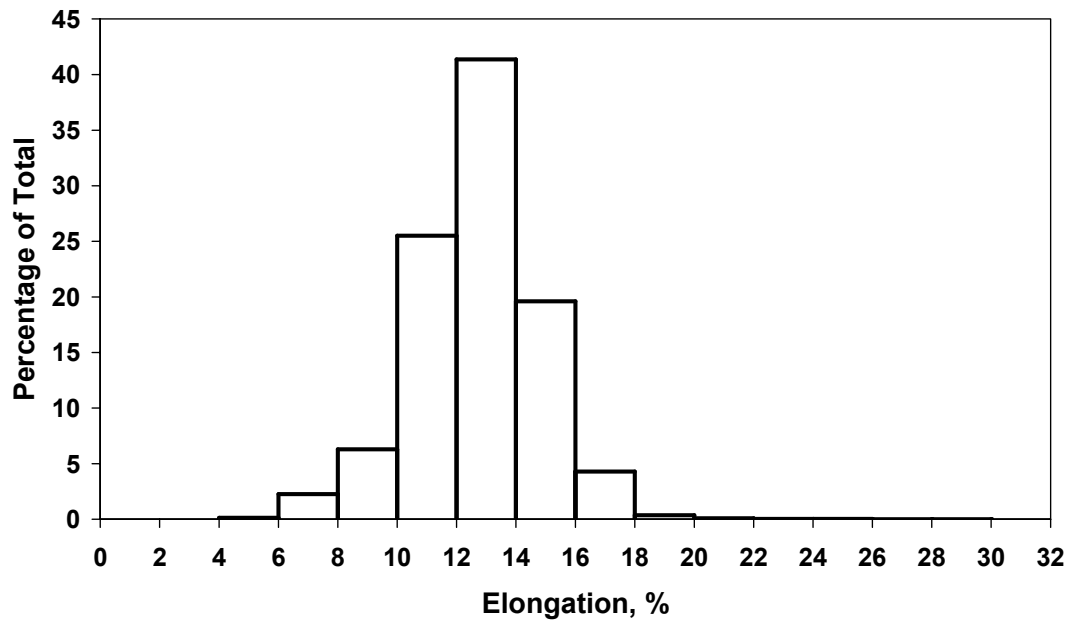


(a)

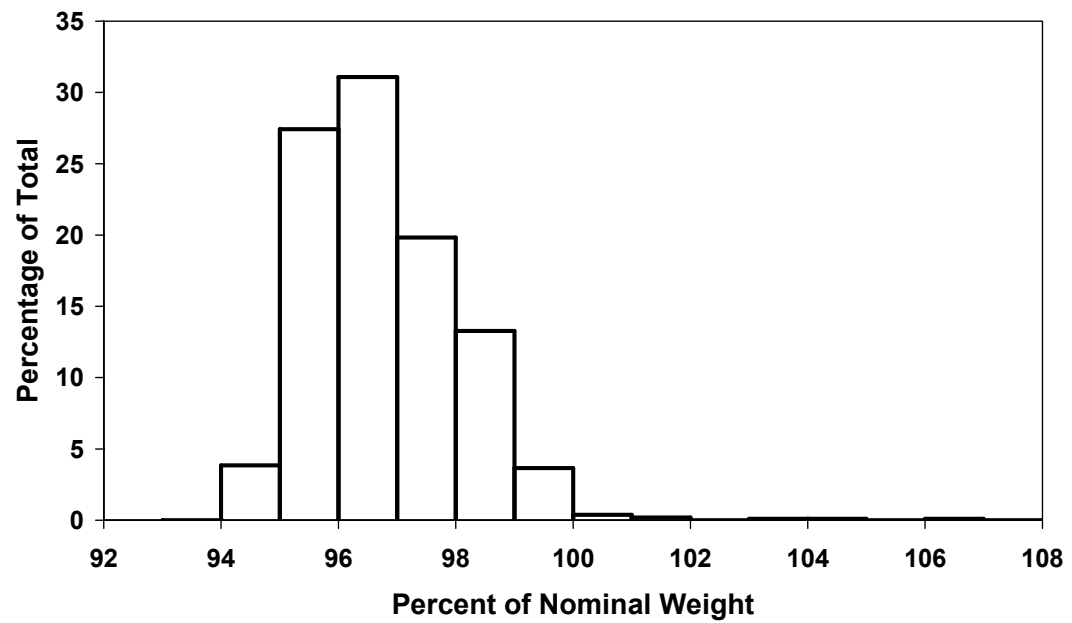


(b)

Figure 101: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 3 bars

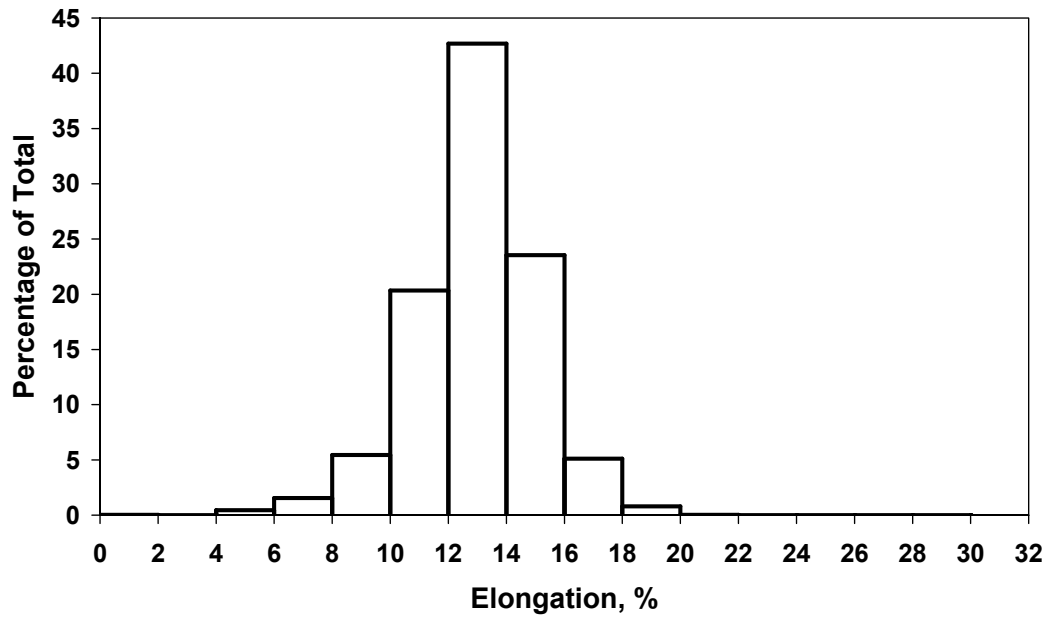


(a)

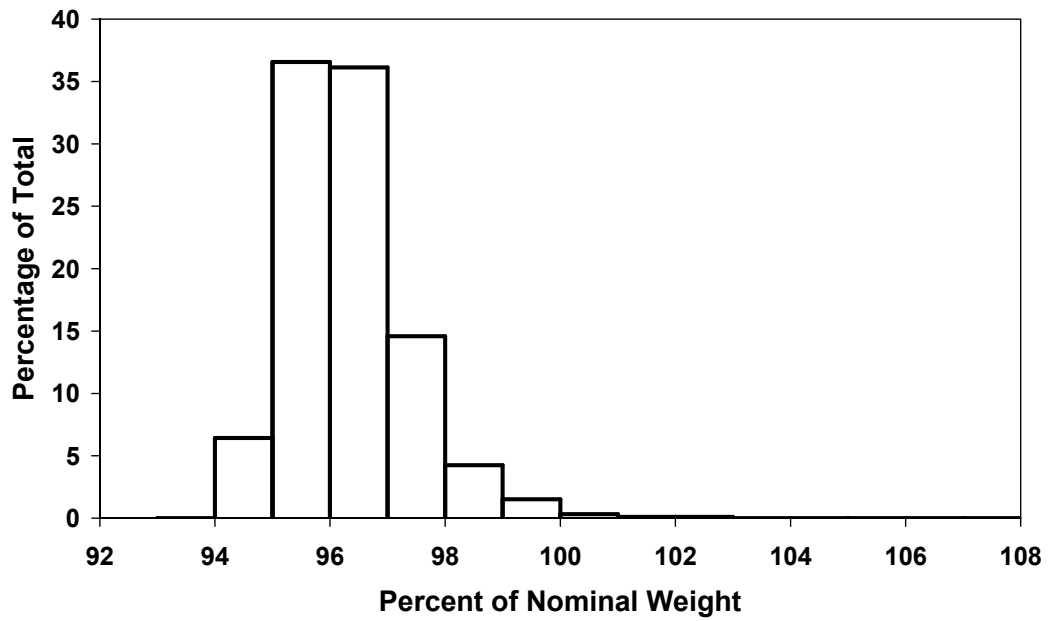


(b)

Figure 102: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 4 bars

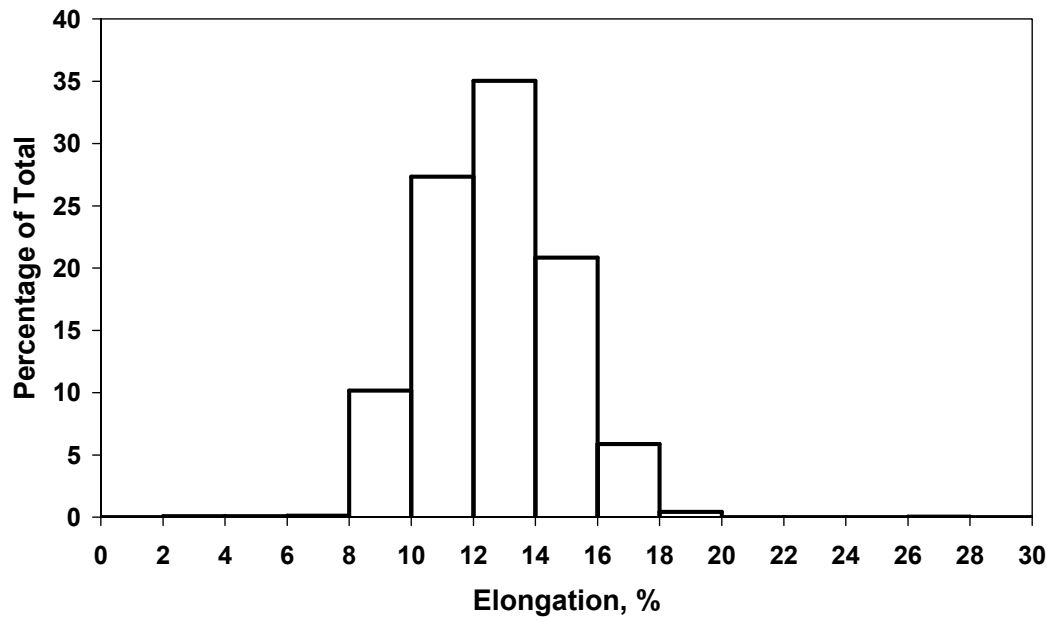


(a)

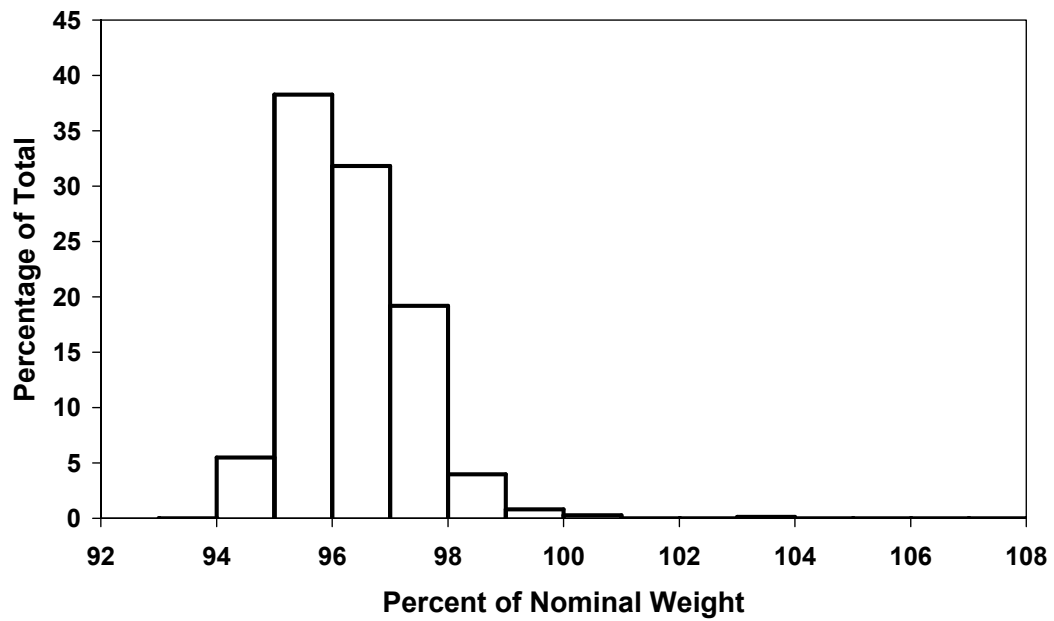


(b)

Figure 103: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 5 bars

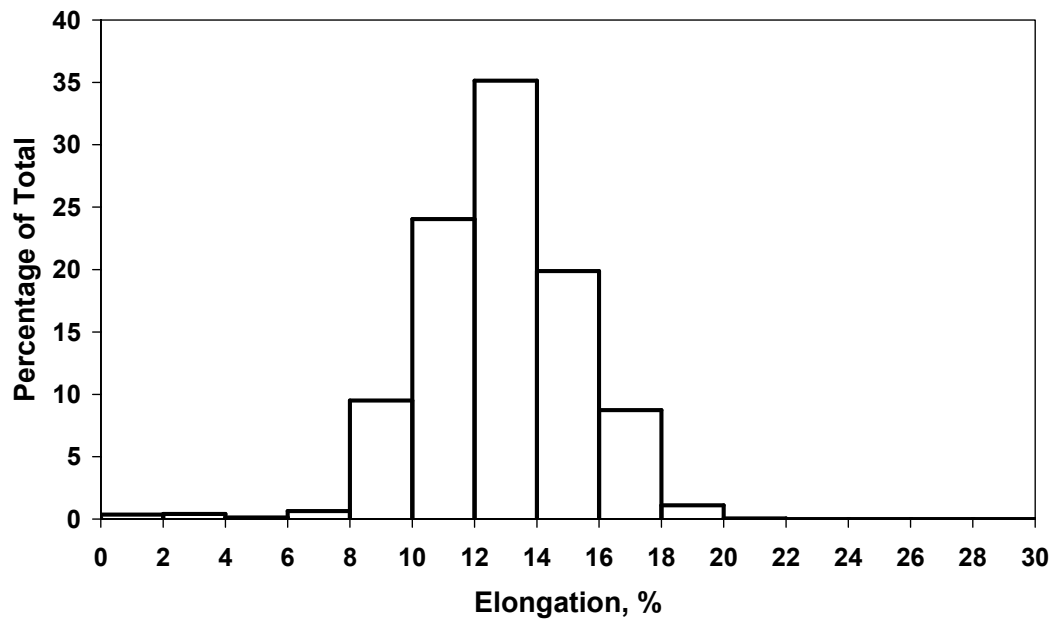


(a)

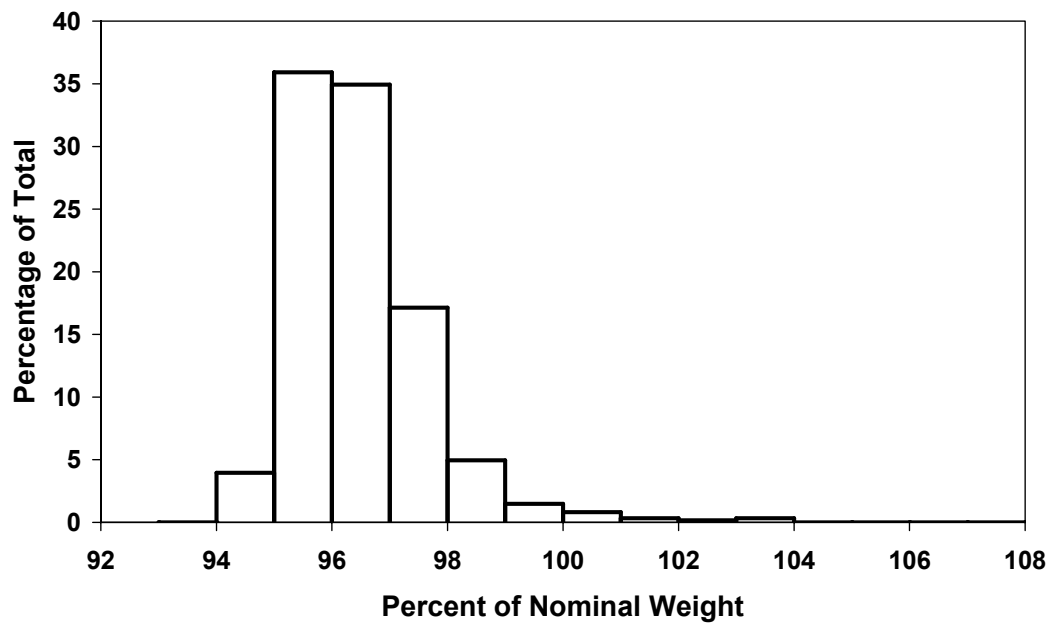


(b)

Figure 104: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 6 bars

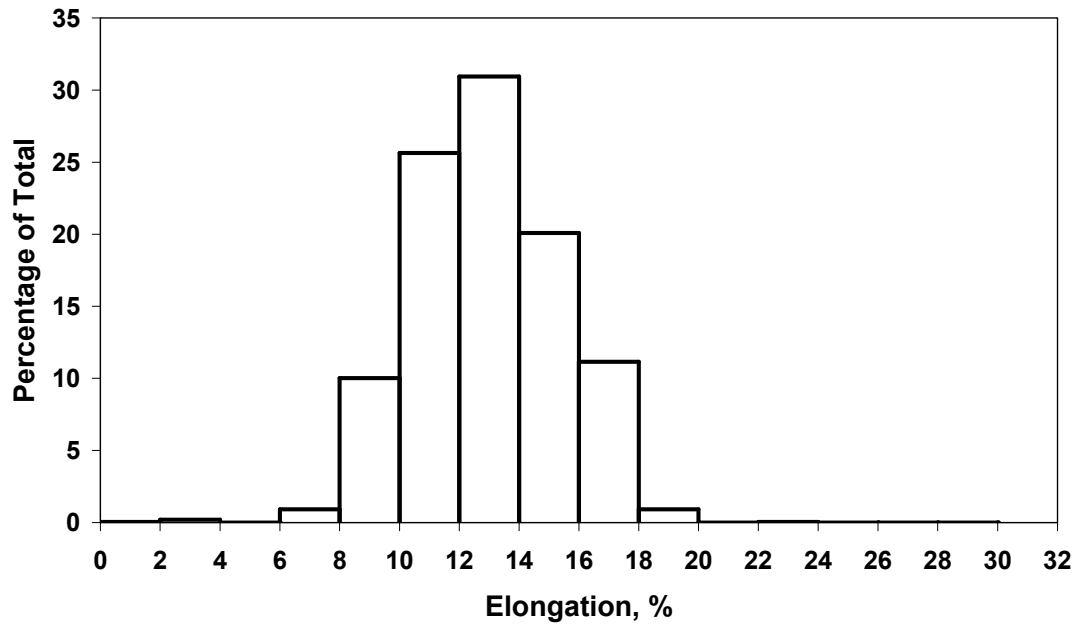


(a)

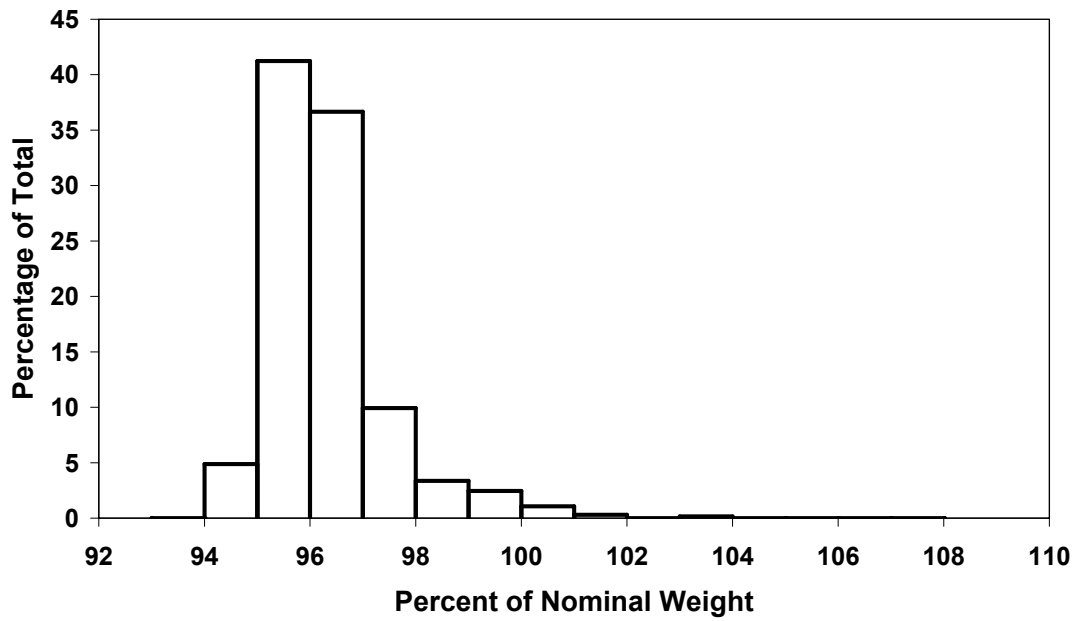


(b)

Figure 105: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 7 bars

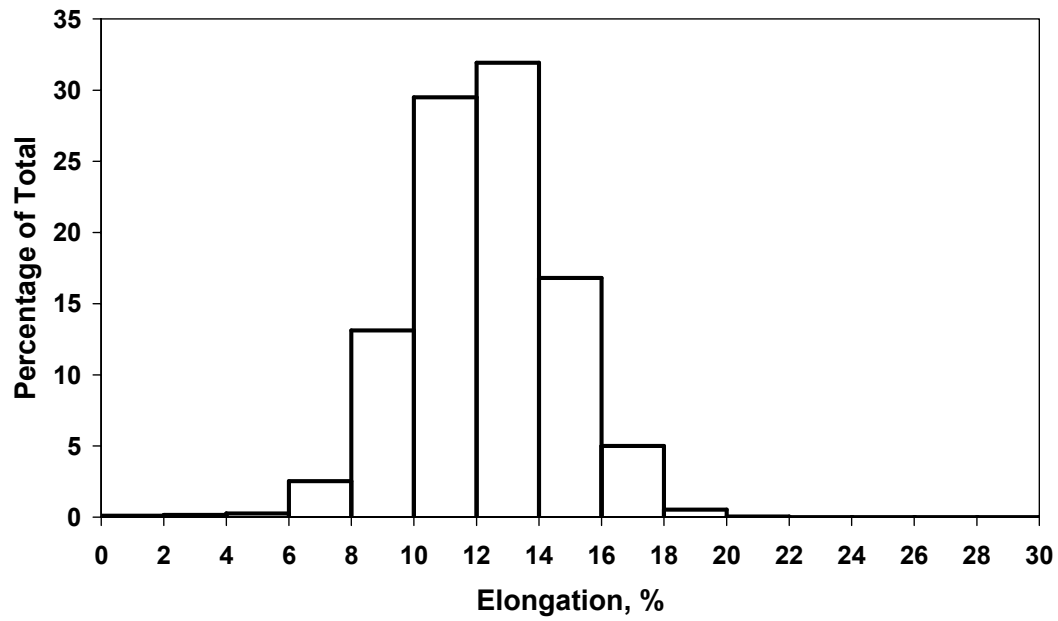


(a)

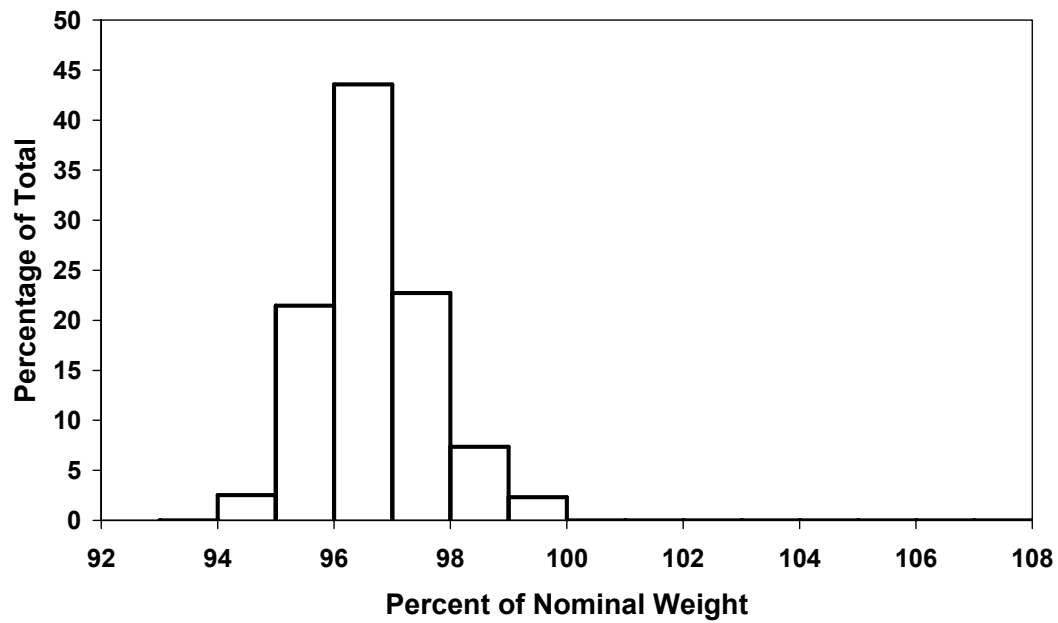


(b)

Figure 106: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 8 bars

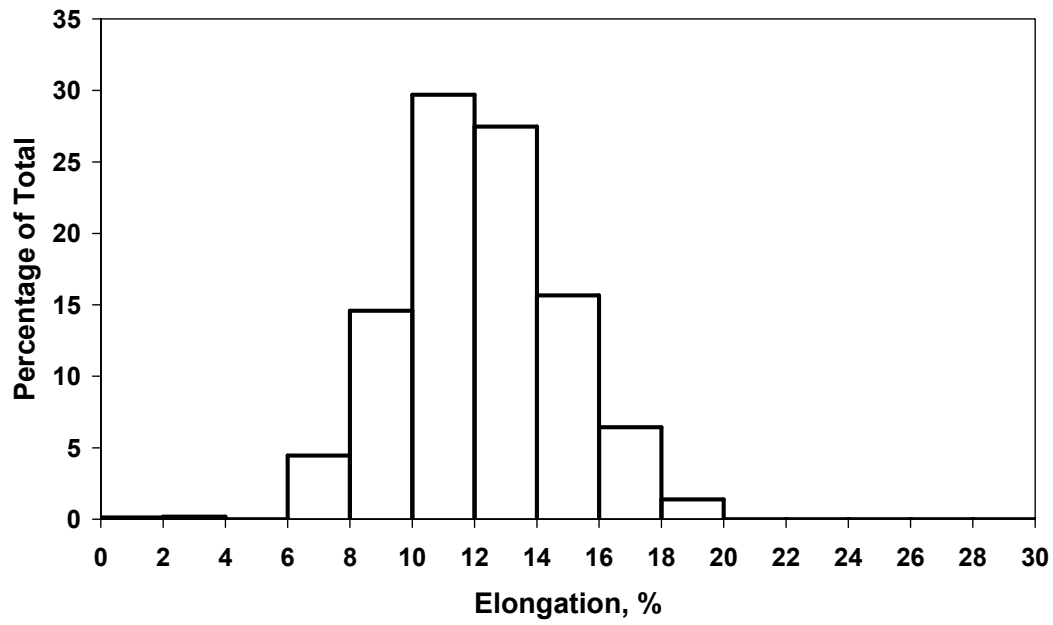


(a)

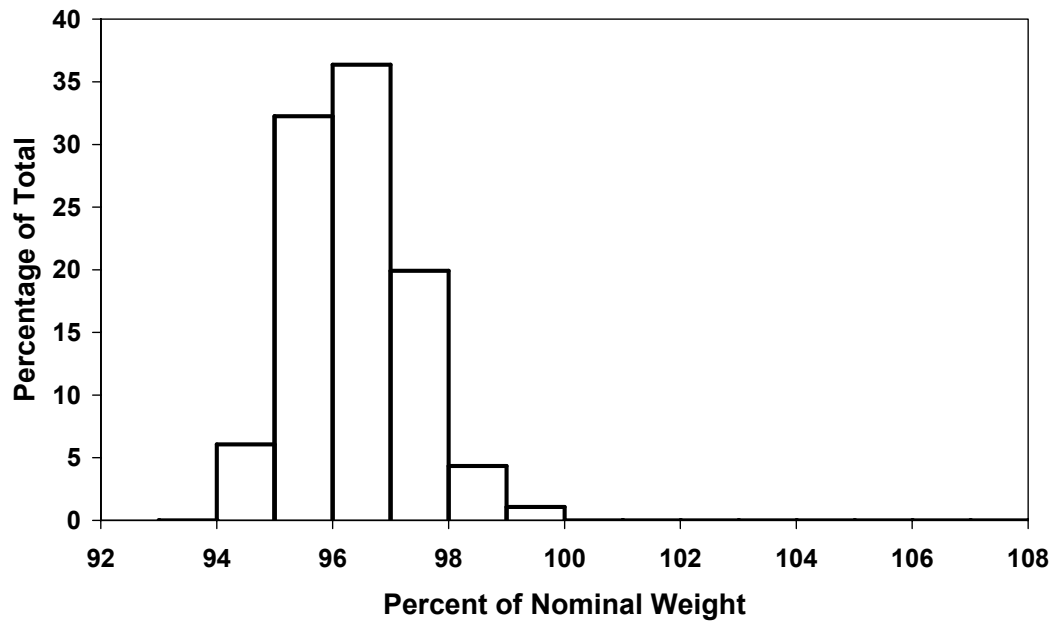


(b)

Figure 107: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 9 bars

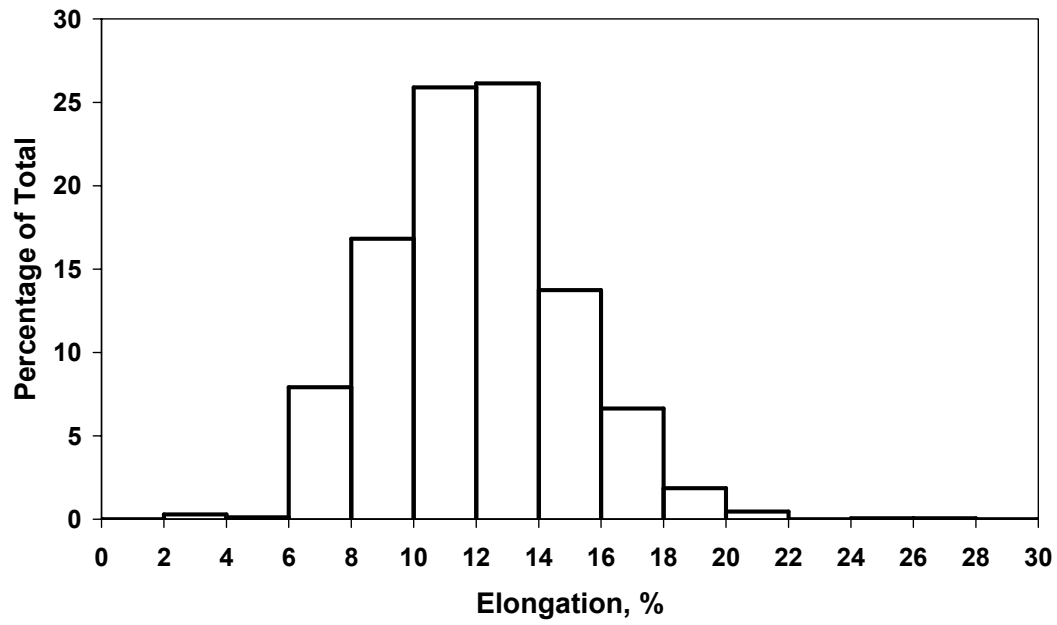


(a)

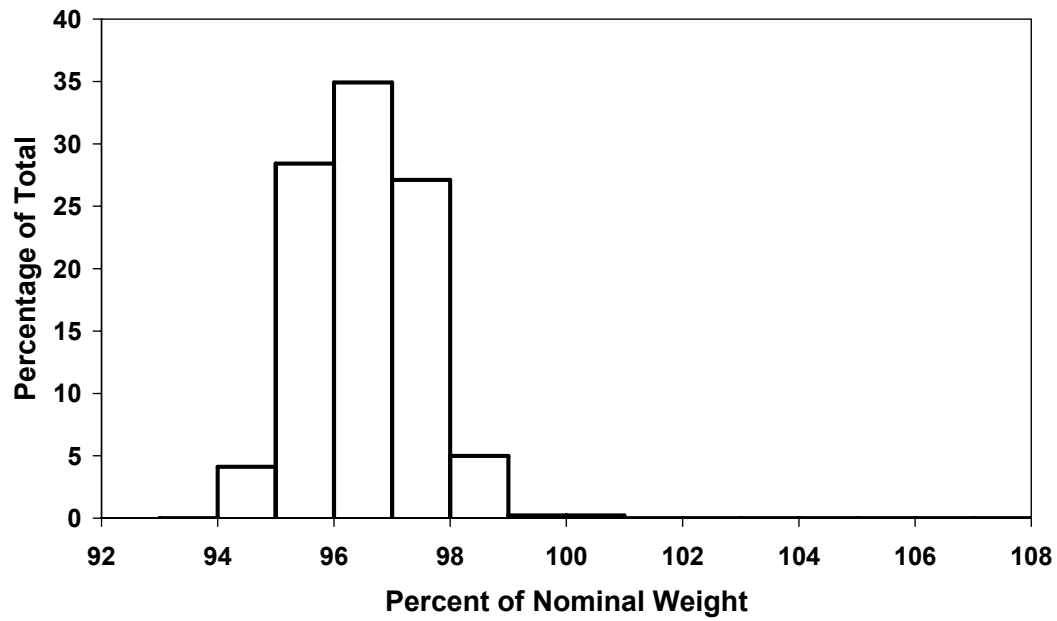


(b)

Figure 108: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 10 bars

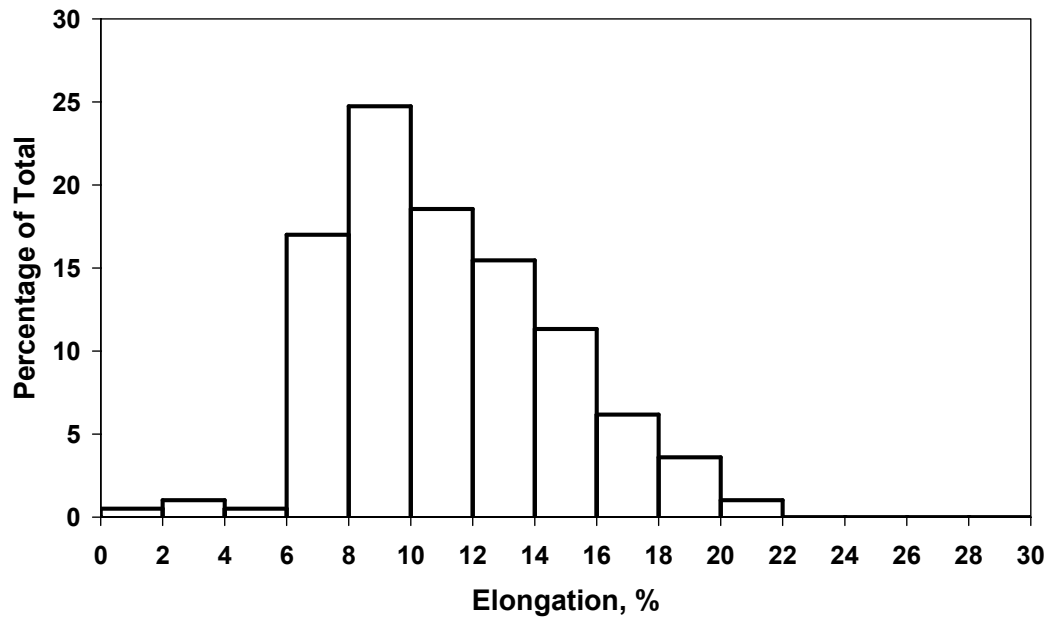


(a)

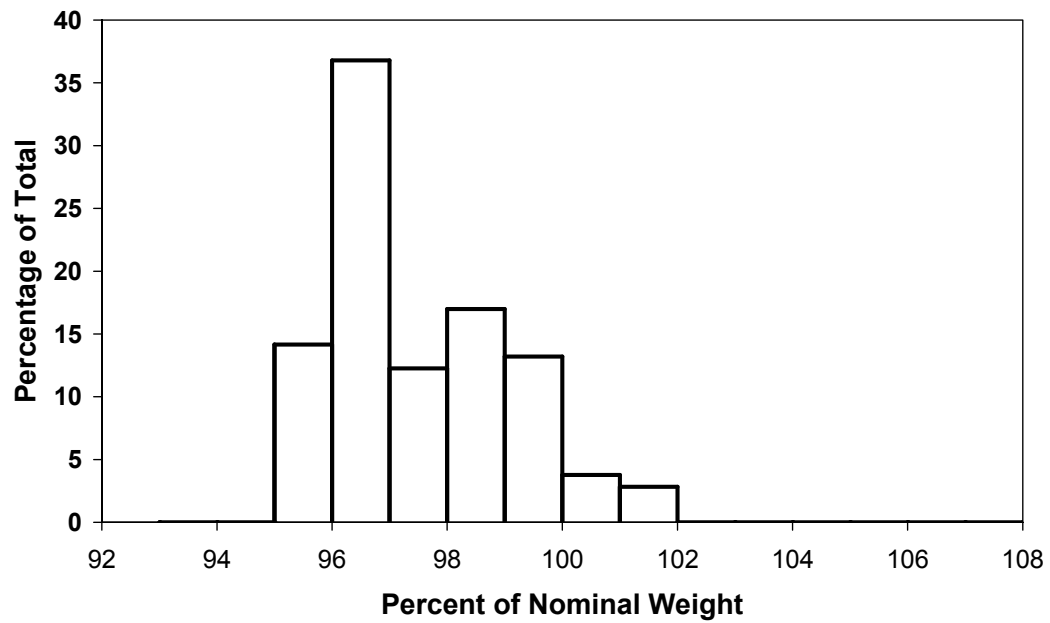


(b)

Figure 109: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 11 bars

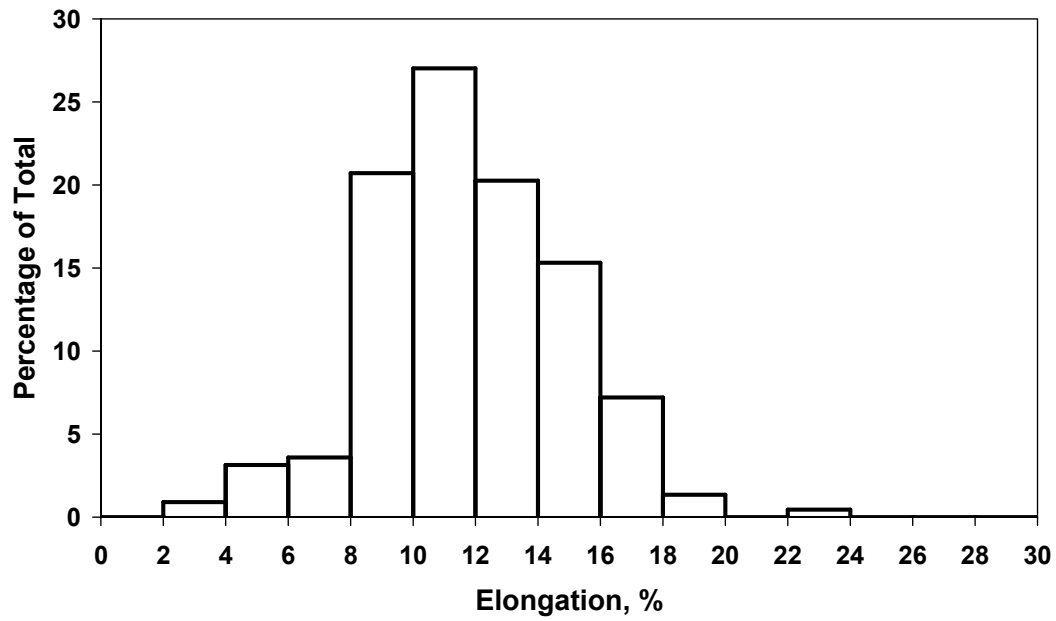


(a)

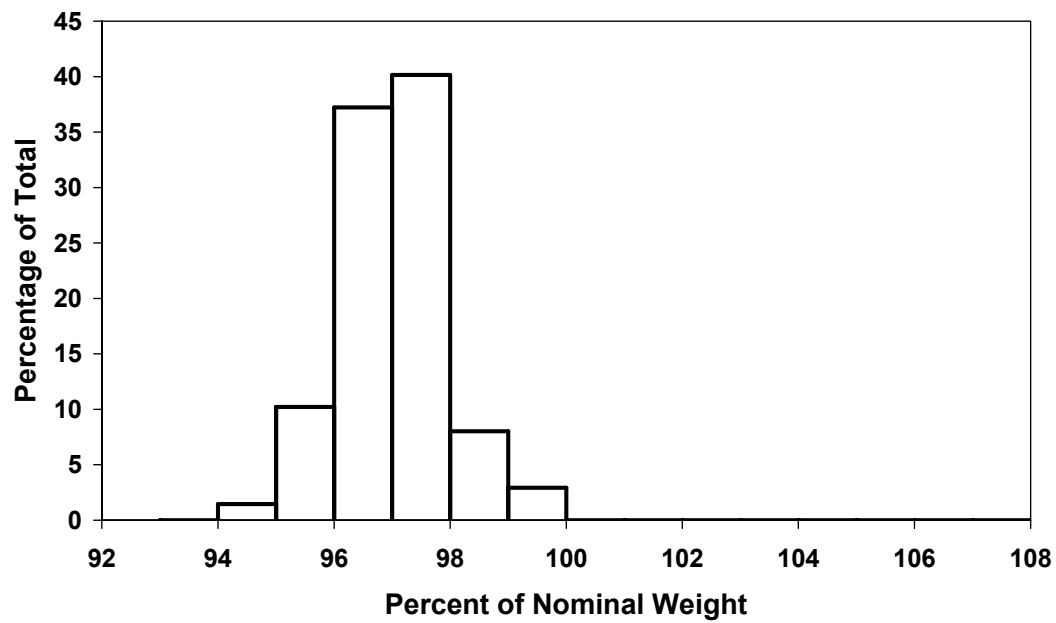


(b)

Figure 110: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 14 bars

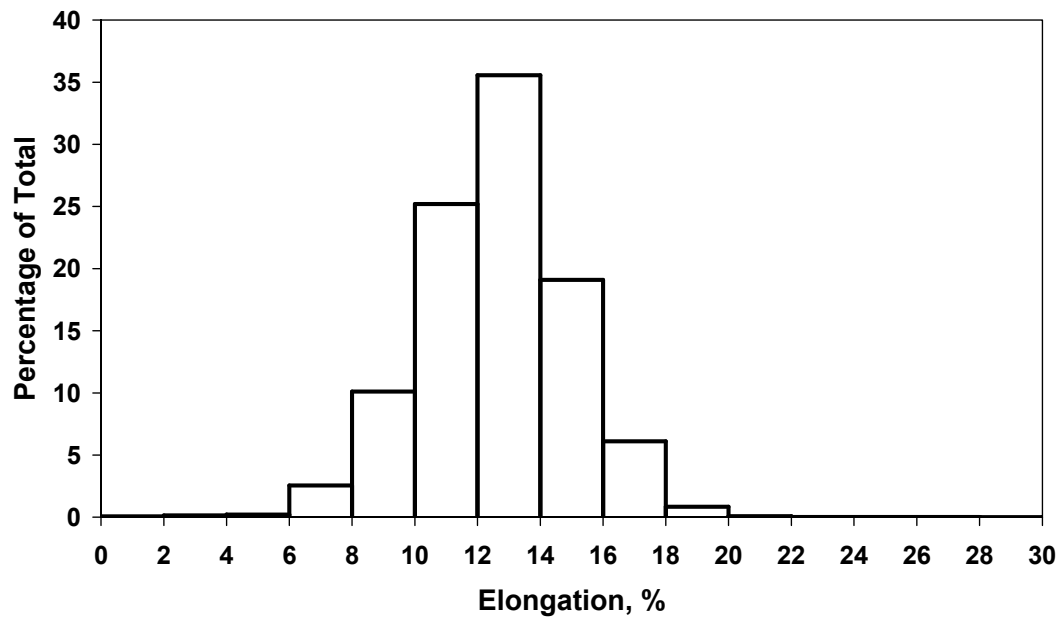


(a)

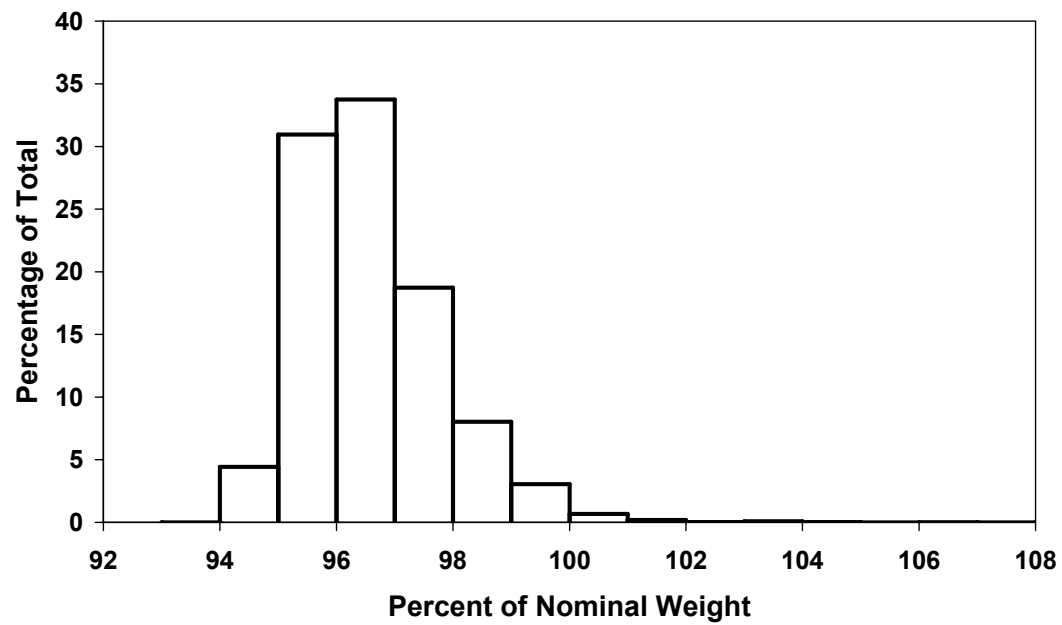


(b)

Figure 111: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 60 No. 18 bars



(a)



(b)

Figure 112: (a) Elongation and (b) Percent of Nominal Weight for All A 615 Grade 60 bars

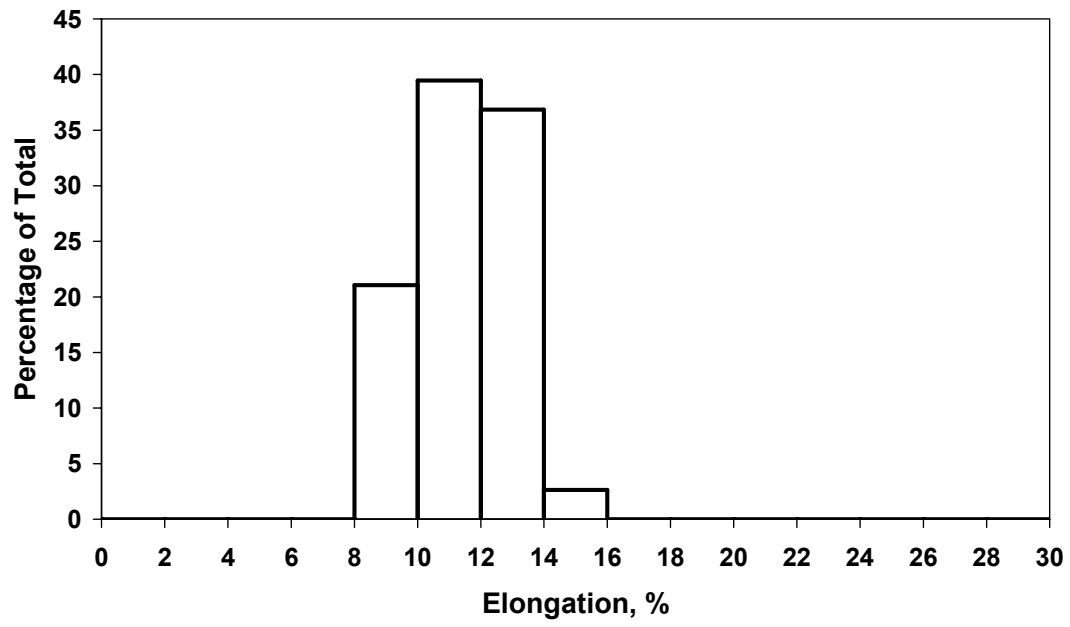


Figure 113: Elongation for A 615 Grade 75 No. 4 bars

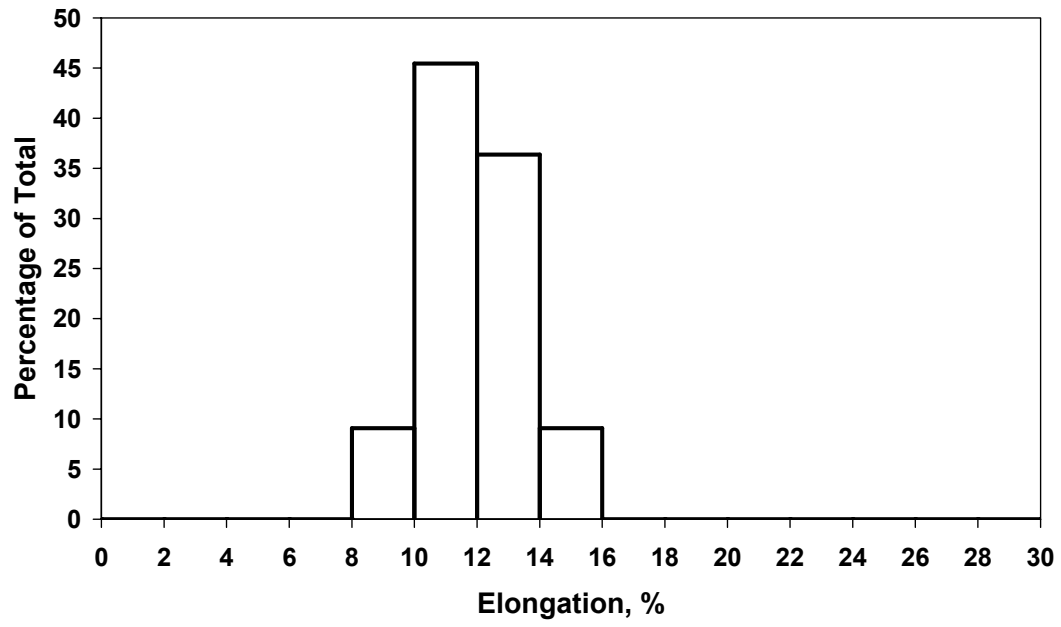


Figure 114: Elongation for A 615 Grade 75 No. 5 bars

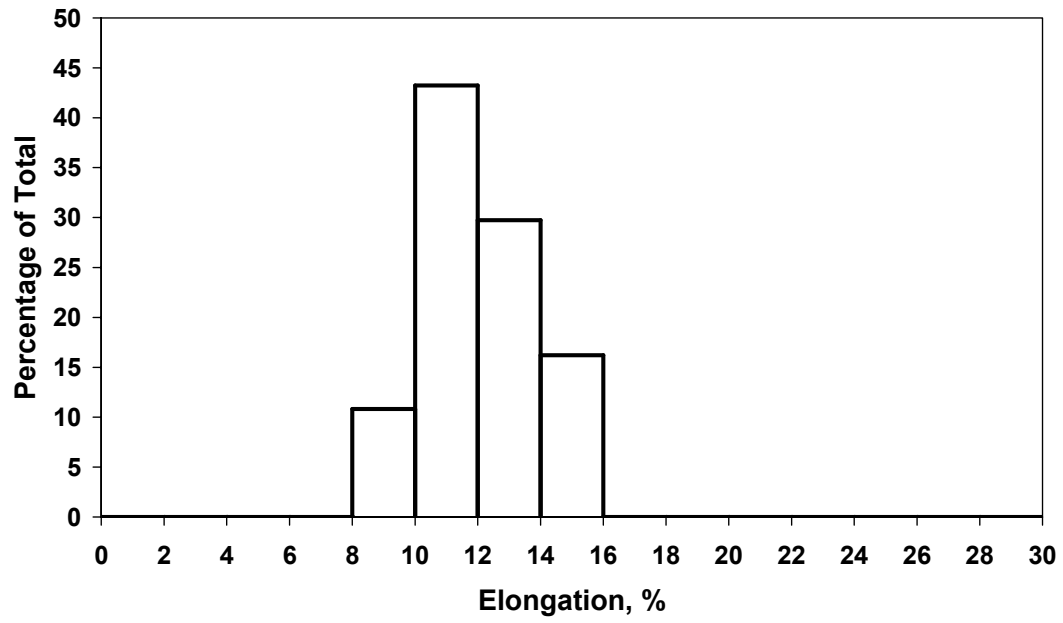


Figure 115: Elongation for A 615 Grade 75 No. 6 bars

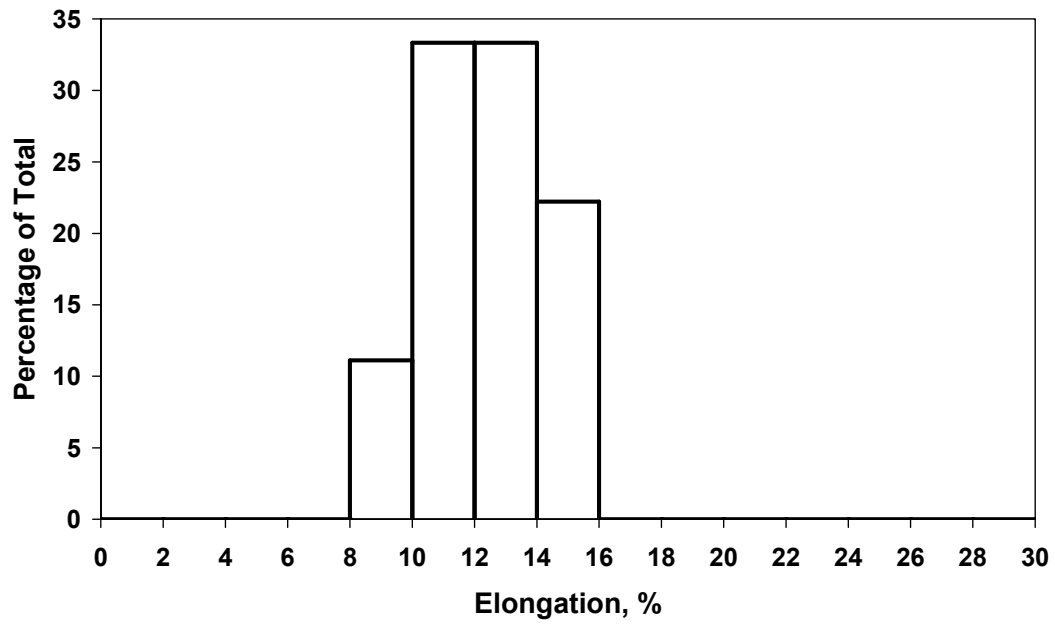


Figure 116: Elongation for A 615 Grade 75 No. 7 bars

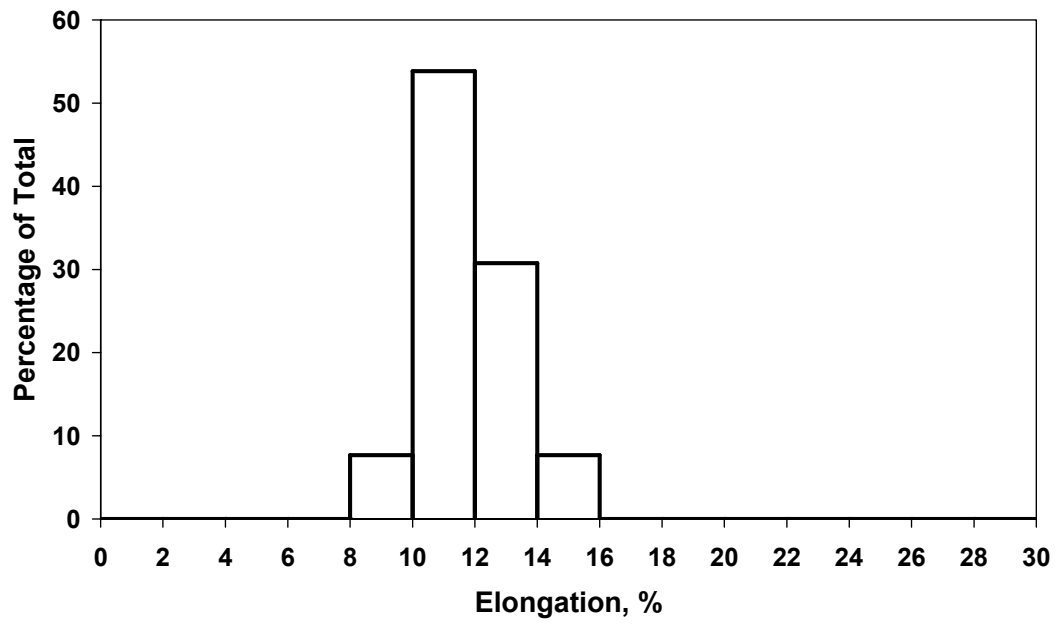


Figure 117: Elongation for A 615 Grade 75 No. 8 bars

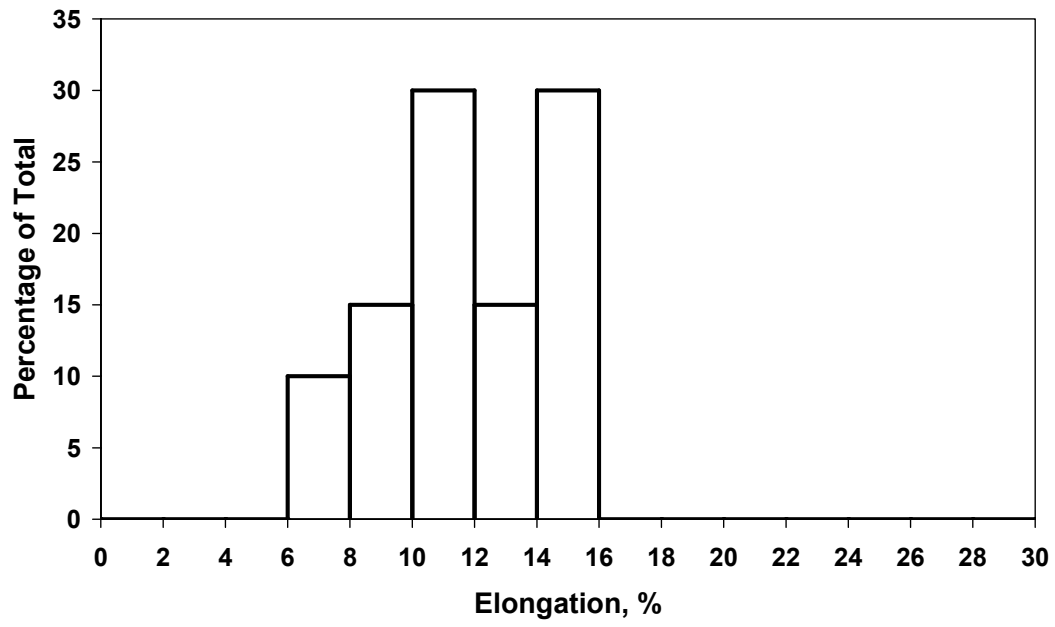


Figure 118: Elongation for A 615 Grade 75 No. 9 bars

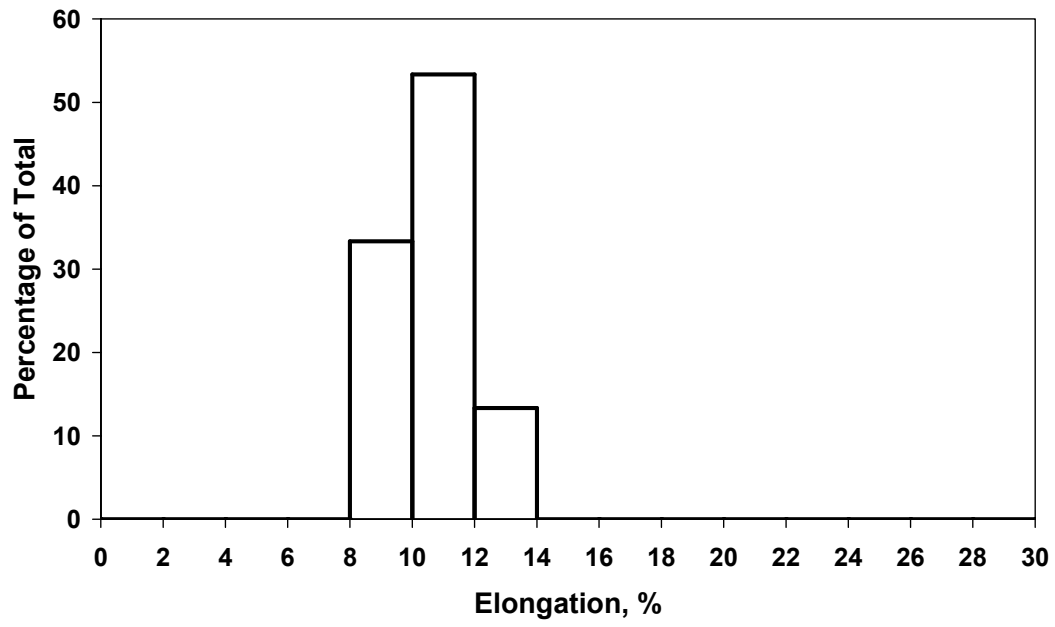


Figure 119: Elongation for A 615 Grade 75 No. 10 bars

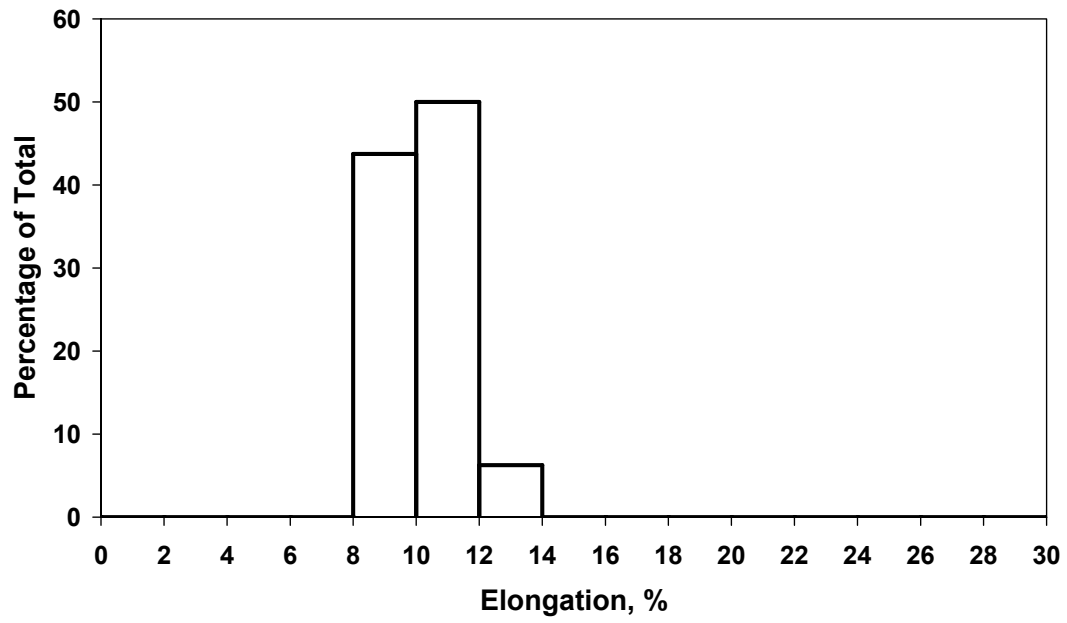
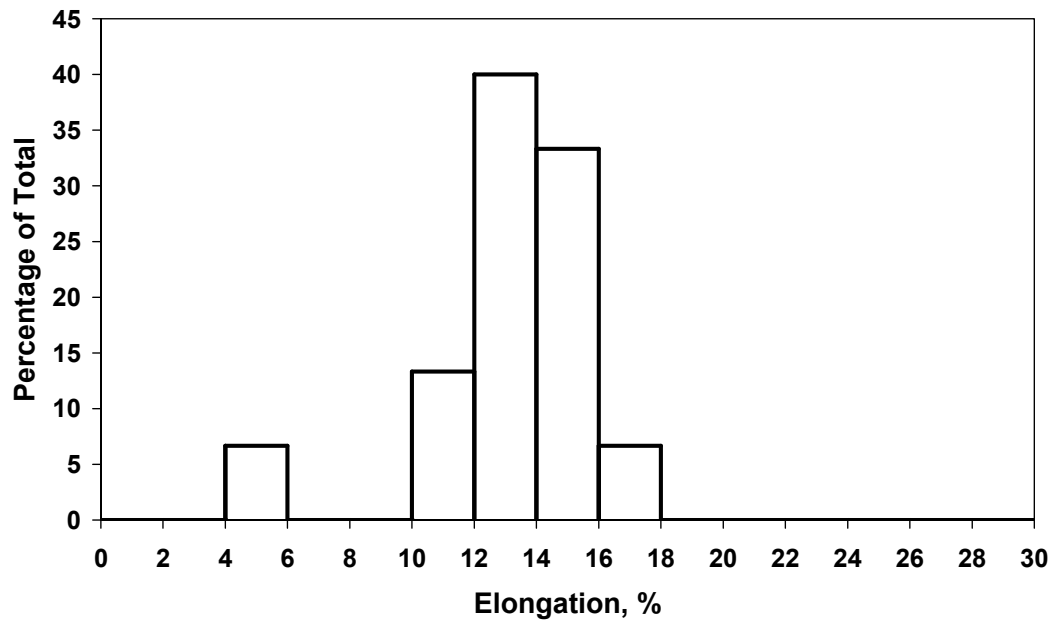
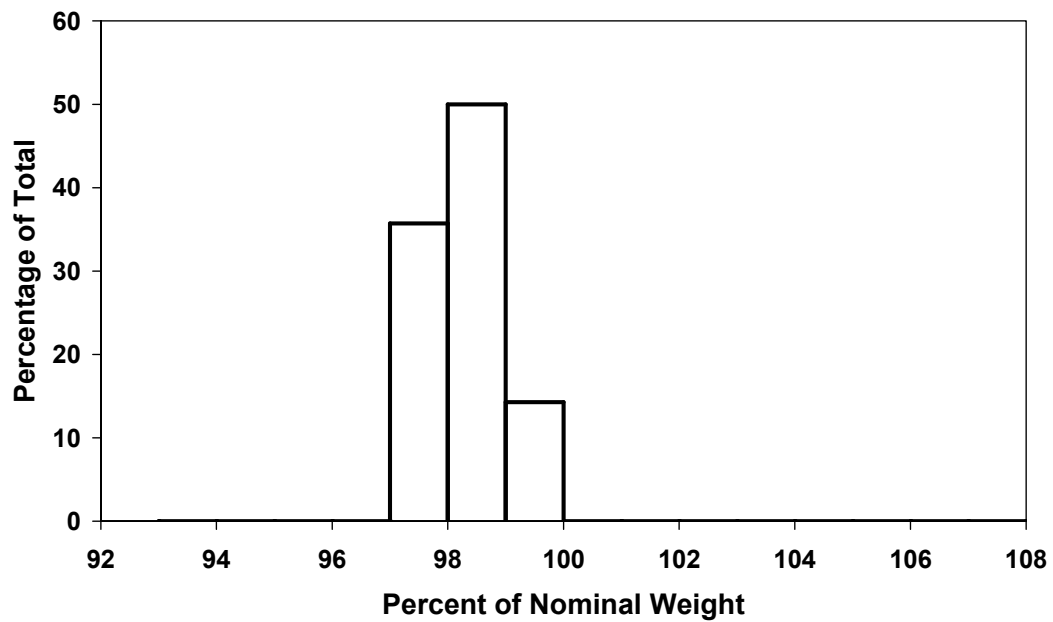


Figure 120: Elongation for A 615 Grade 75 No. 11 bars



(a)



(b)

Figure 121: (a) Elongation and (b) Percent of Nominal Weight for A 615 Grade 75 No. 14 bars

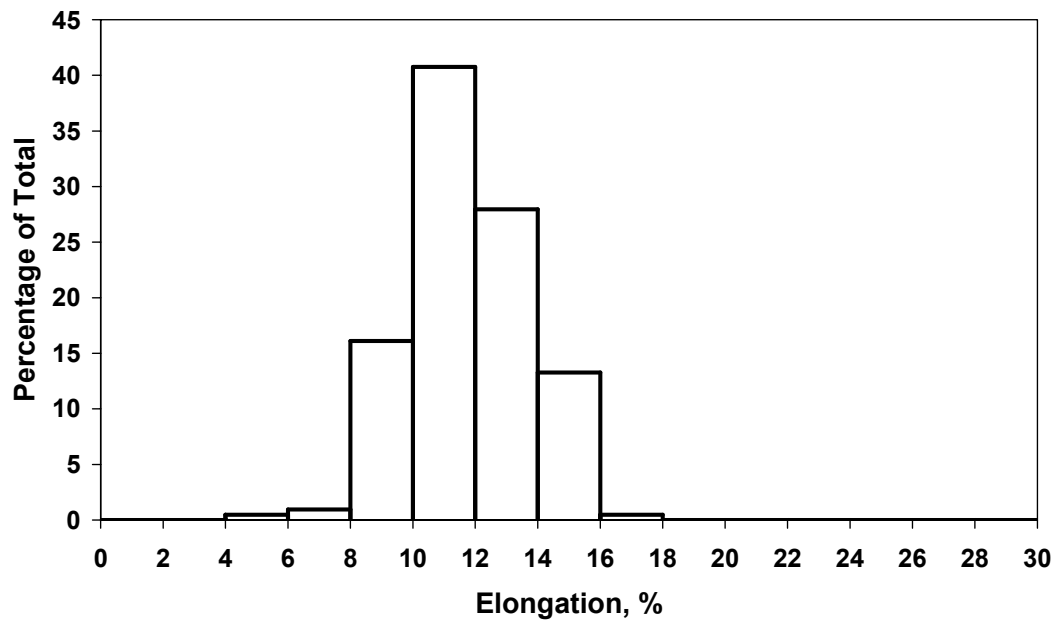
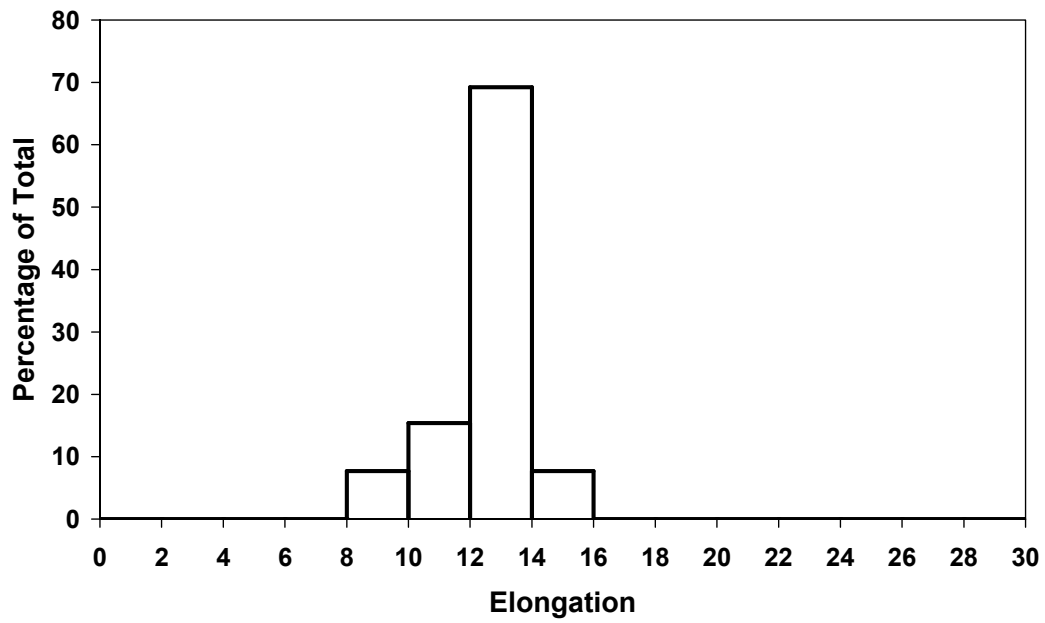
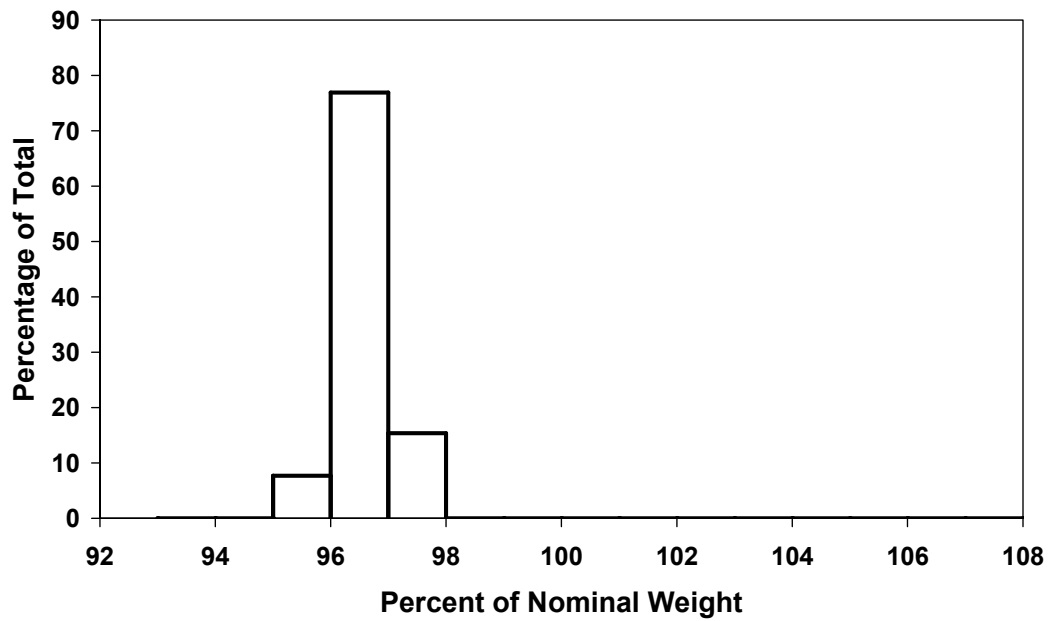


Figure 122: Elongation for All A 615 Grade 75 bars

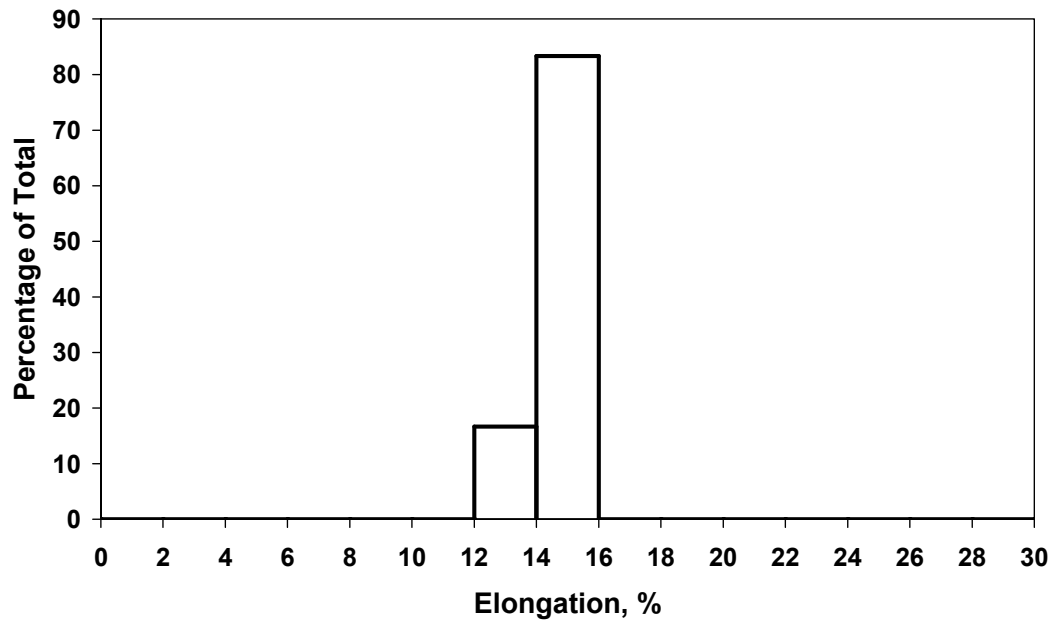


(a)

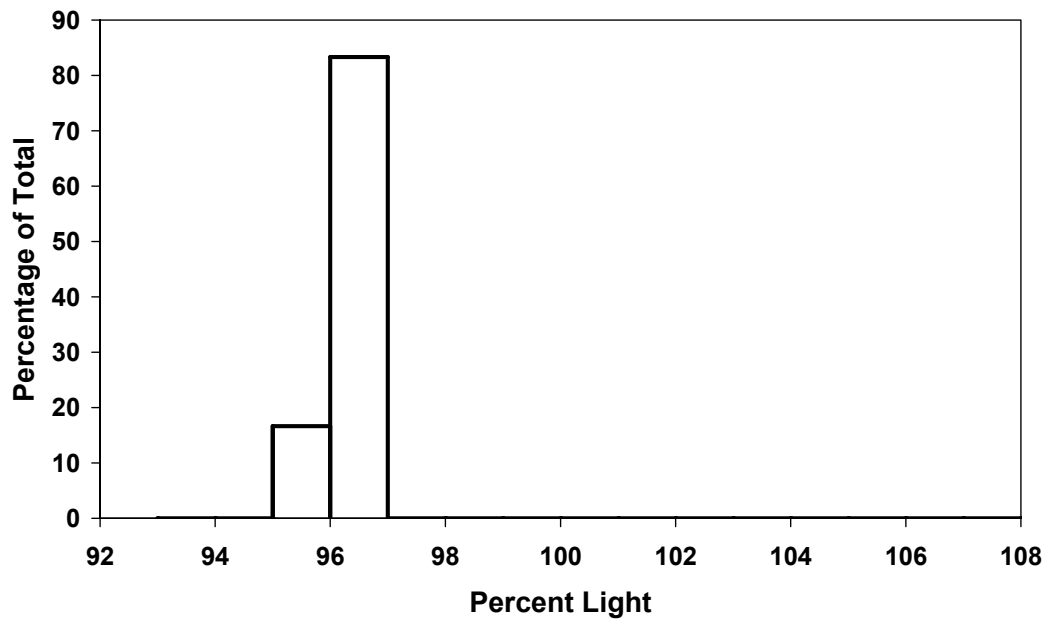


(b)

Figure 123: (a) Elongation and (b) Percent of Nominal Weight for A 616 Grade 60 No. 8 bars

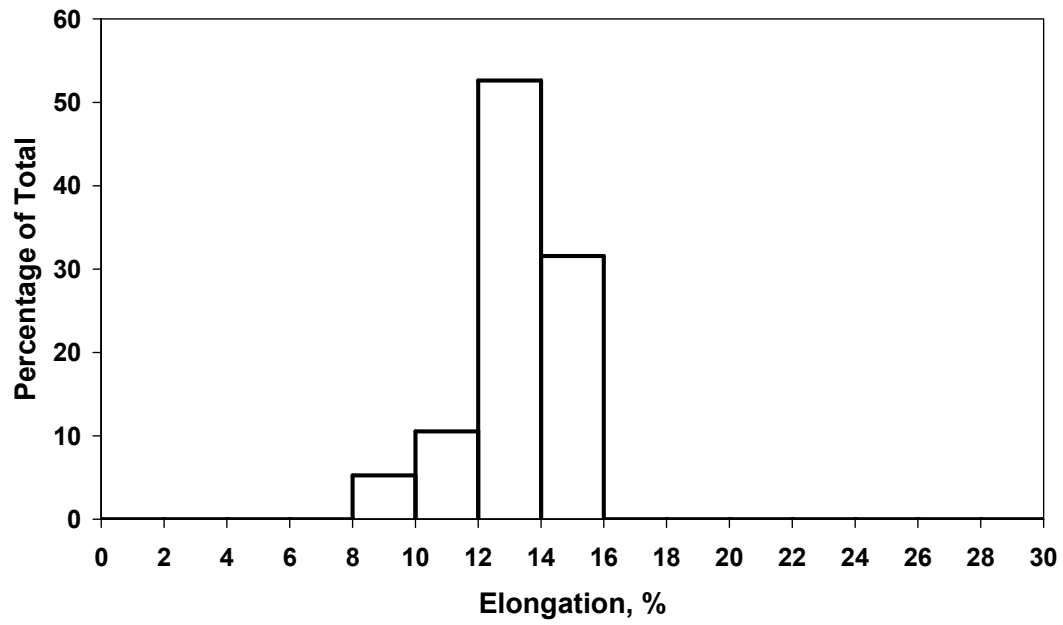


(a)

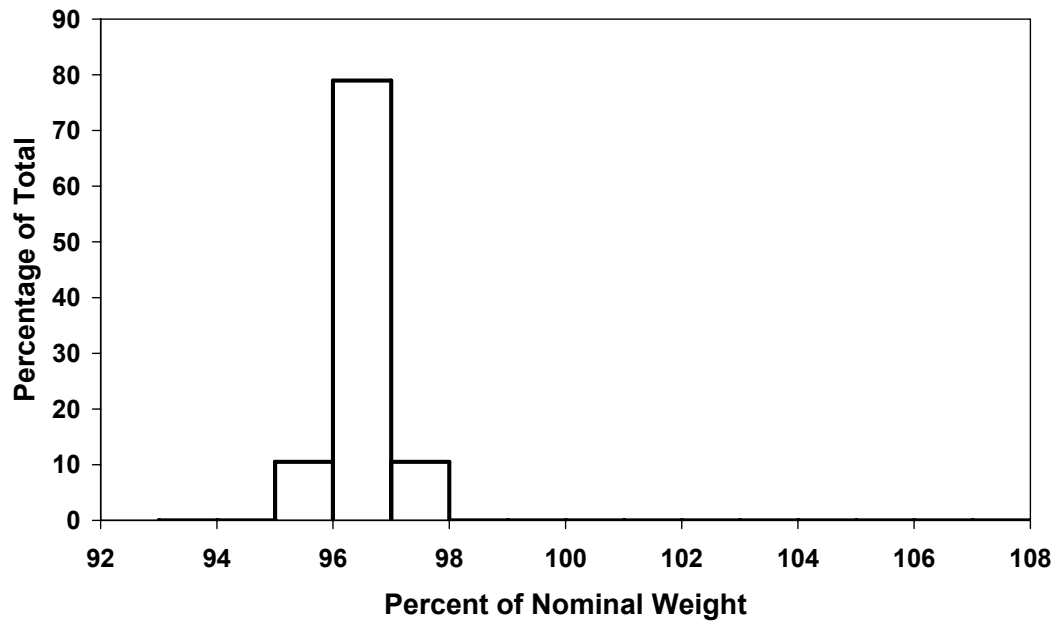


(b)

Figure 124: (a) Elongation and (b) Percent of Nominal Weight for A 616 Grade 60 No. 10 bars



(a)



(b)

Figure 125: (a) Elongation and (b) Percent of Nominal Weight for All A 616 Grade 60 bars

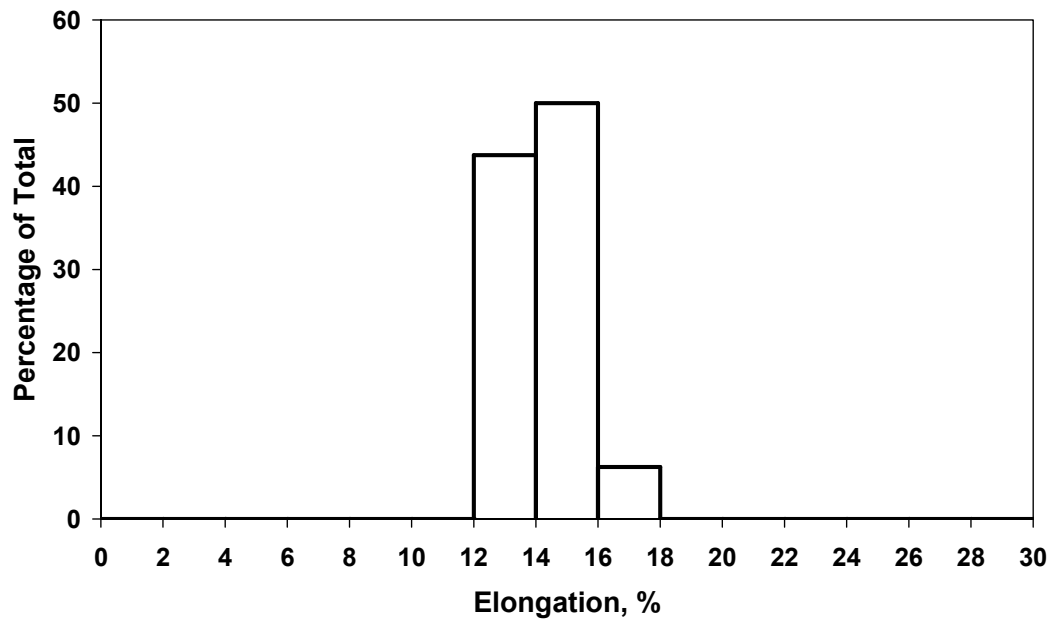
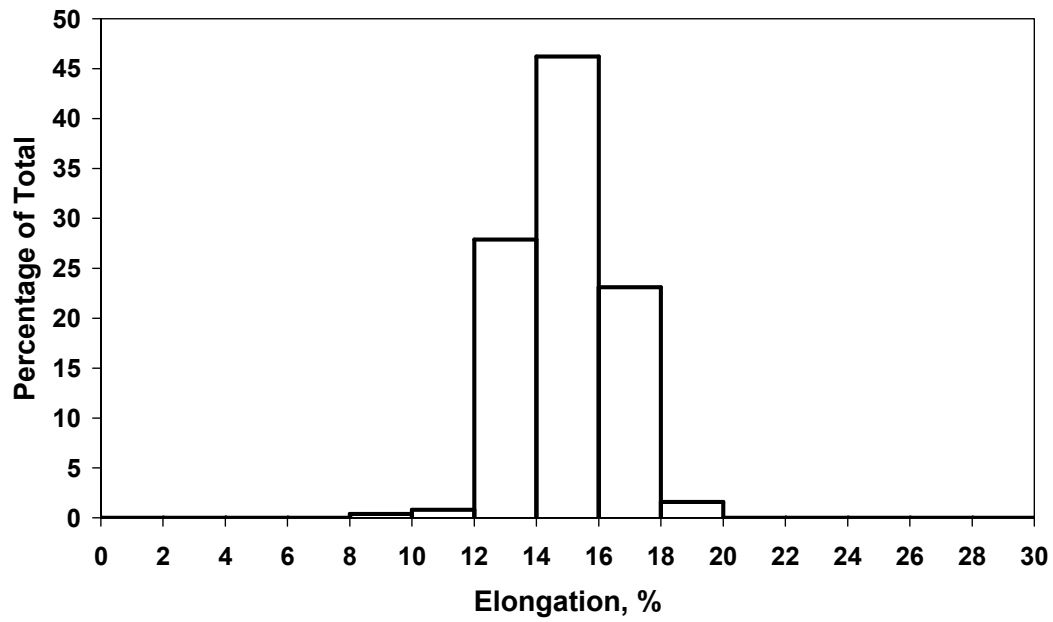
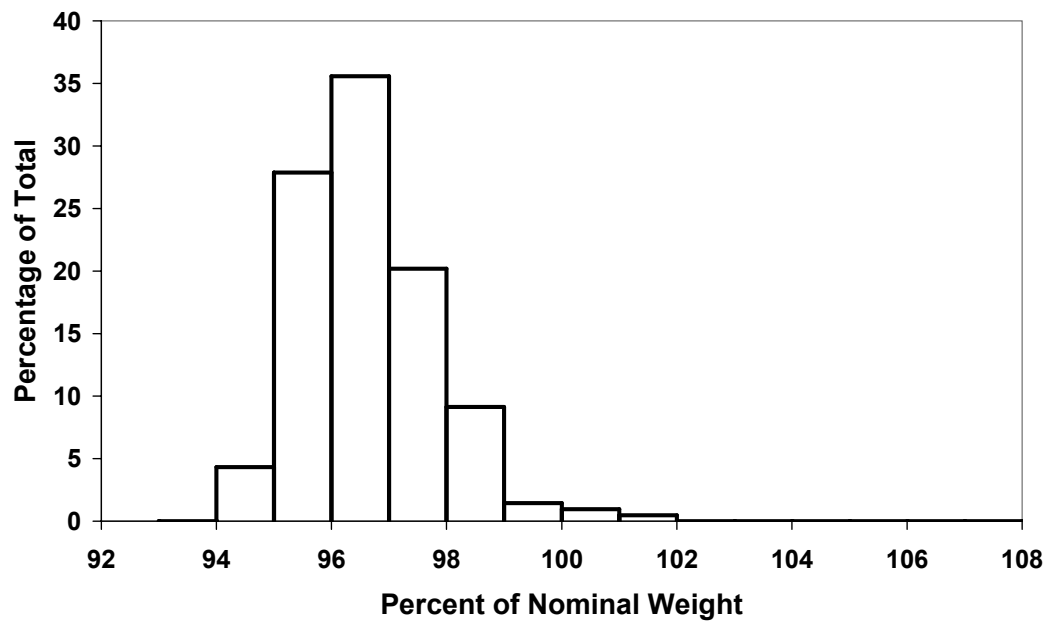


Figure 126: Elongation for A 706 Grade 60 No. 3 bars

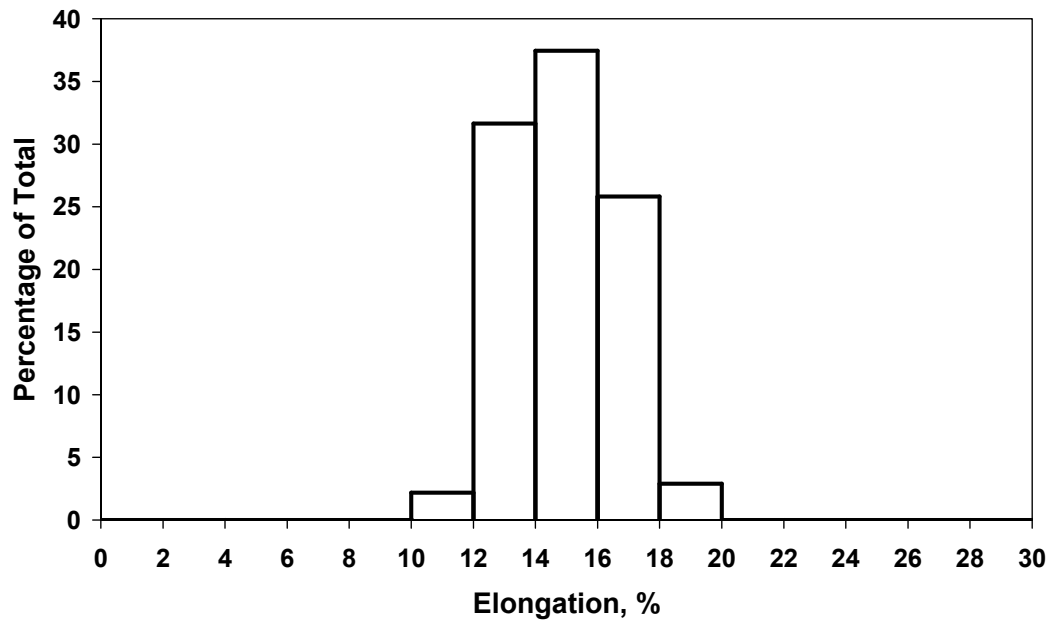


(a)

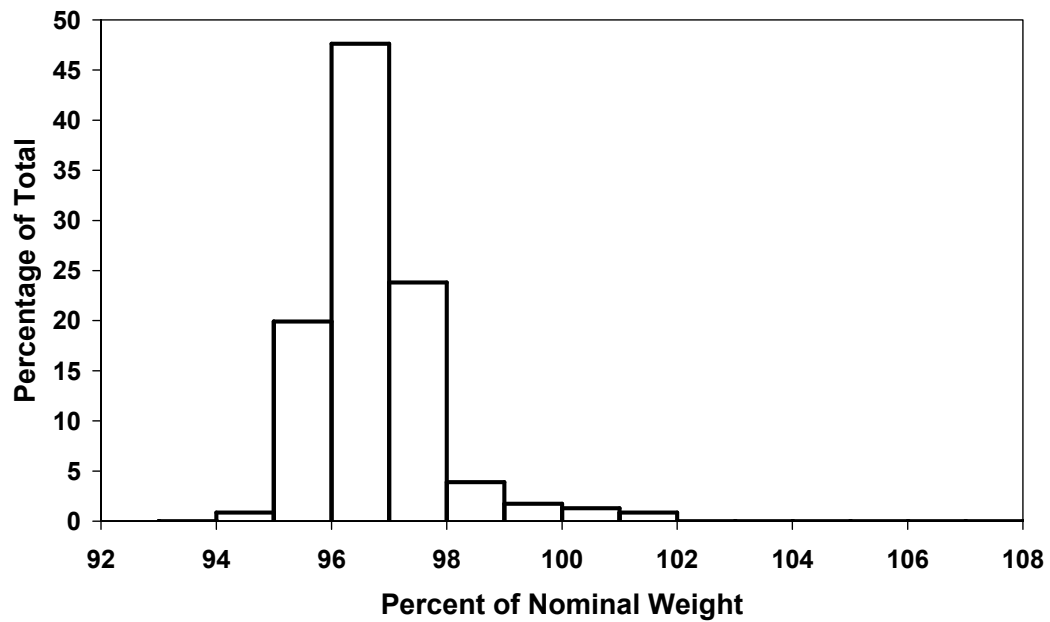


(b)

Figure 127: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 4 bars

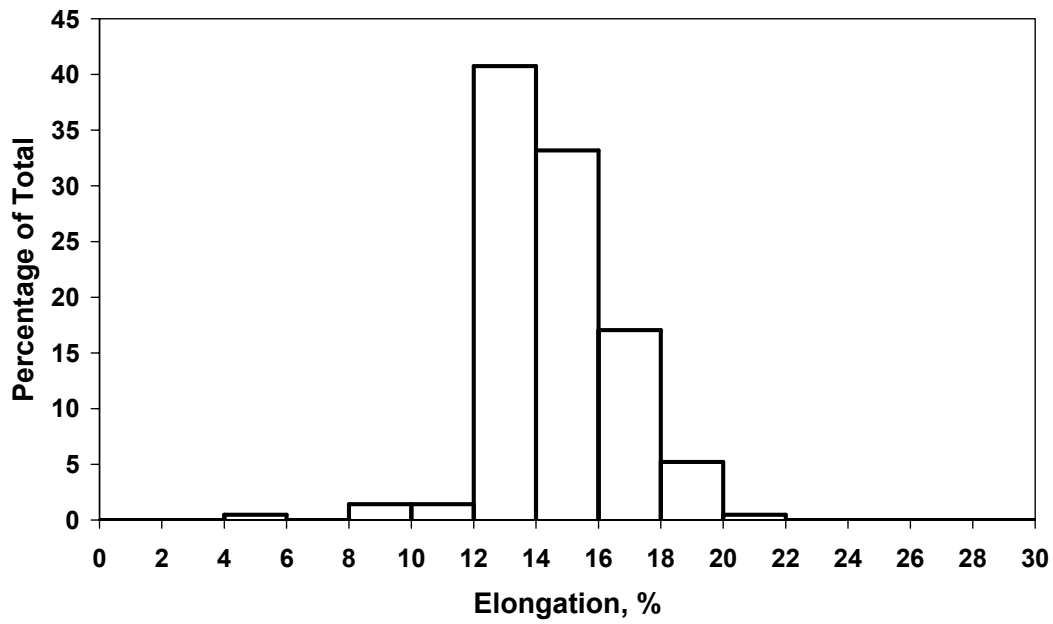


(a)

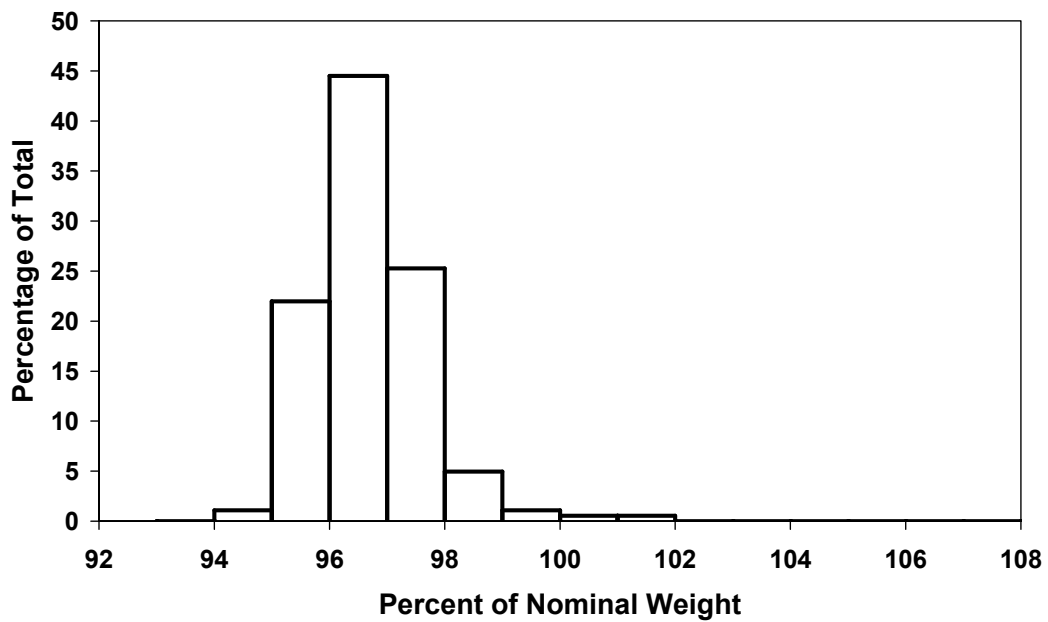


(b)

Figure 128: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 5 bars

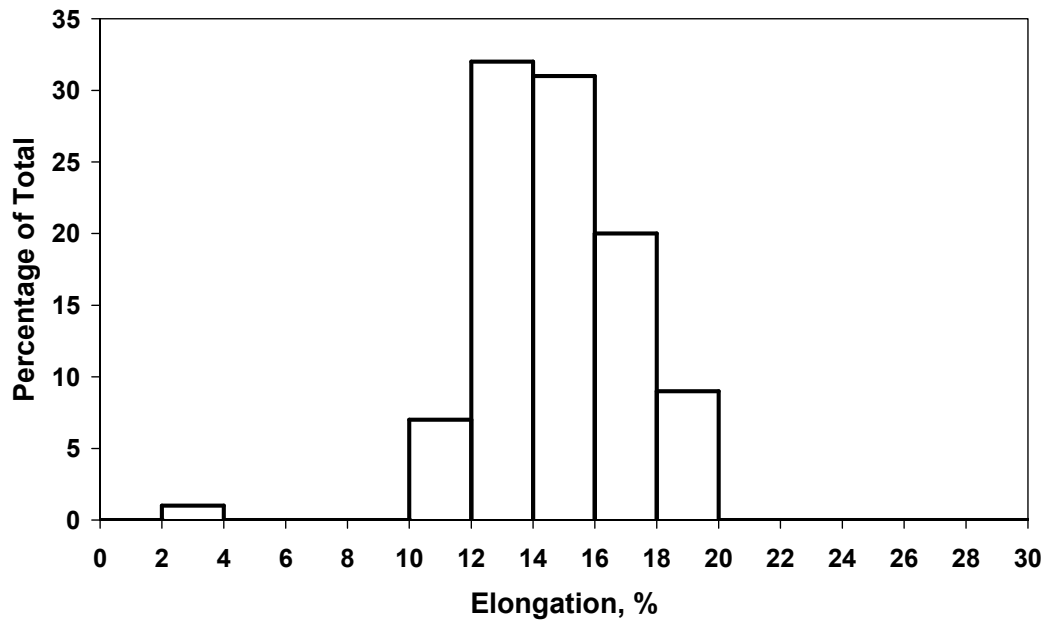


(a)

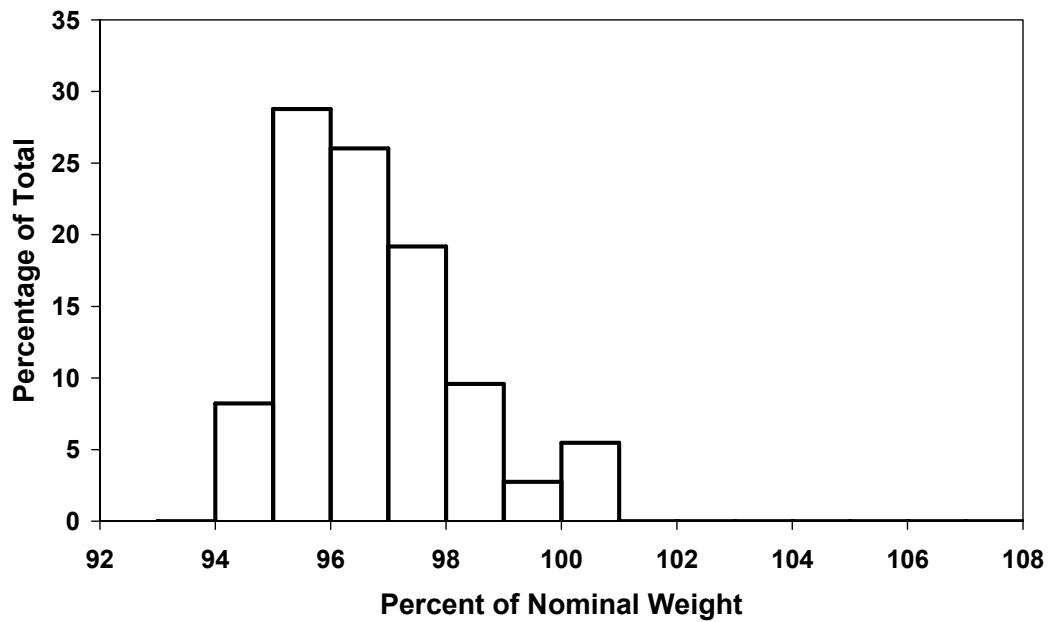


(b)

Figure 129: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 6 bars

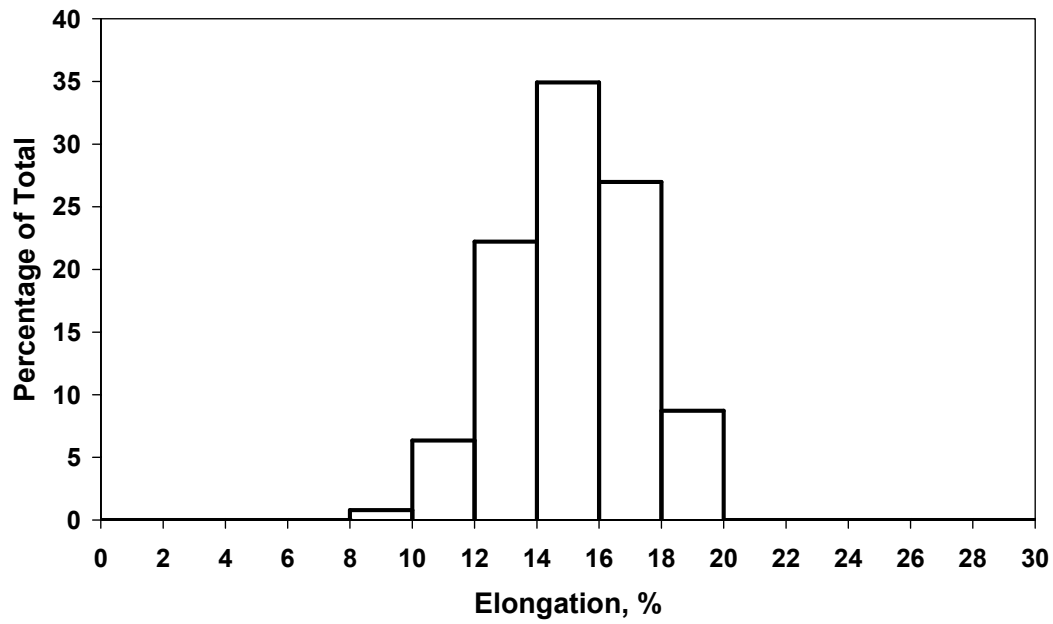


(a)

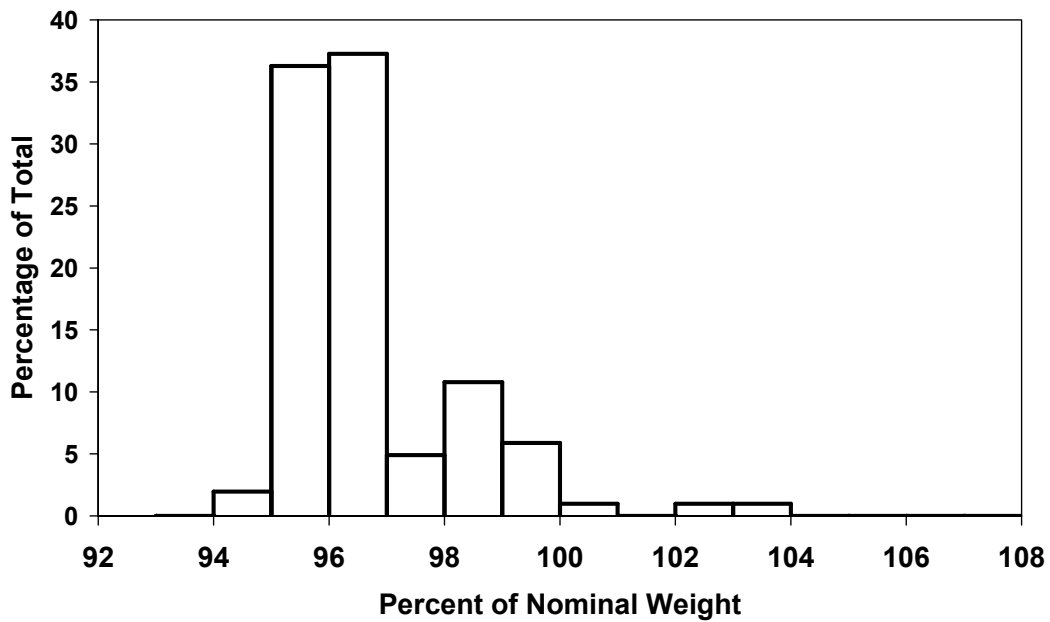


(b)

Figure 130: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 7 bars

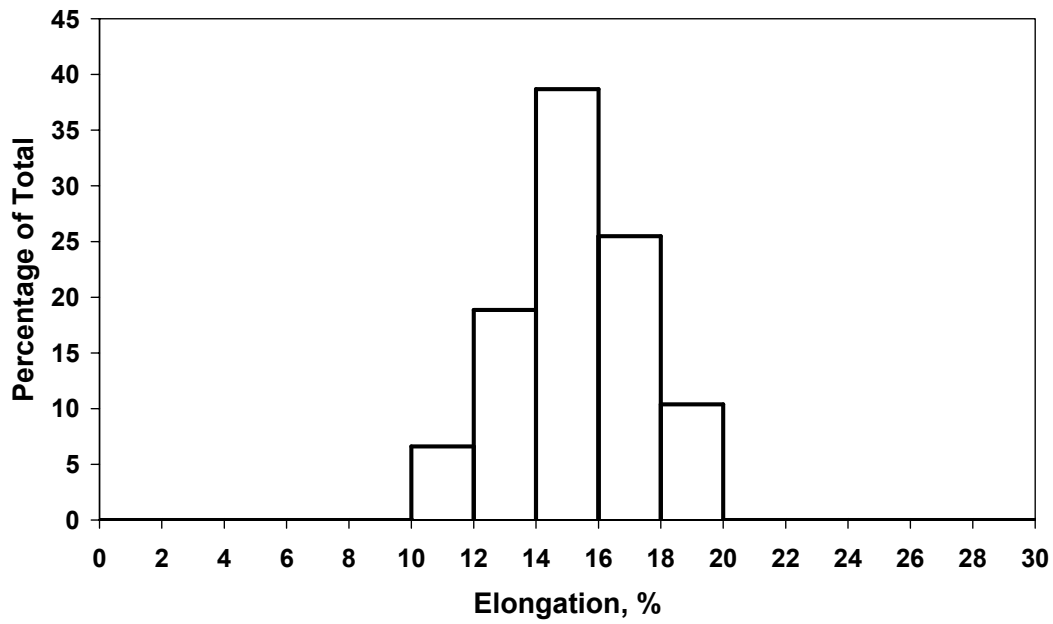


(a)

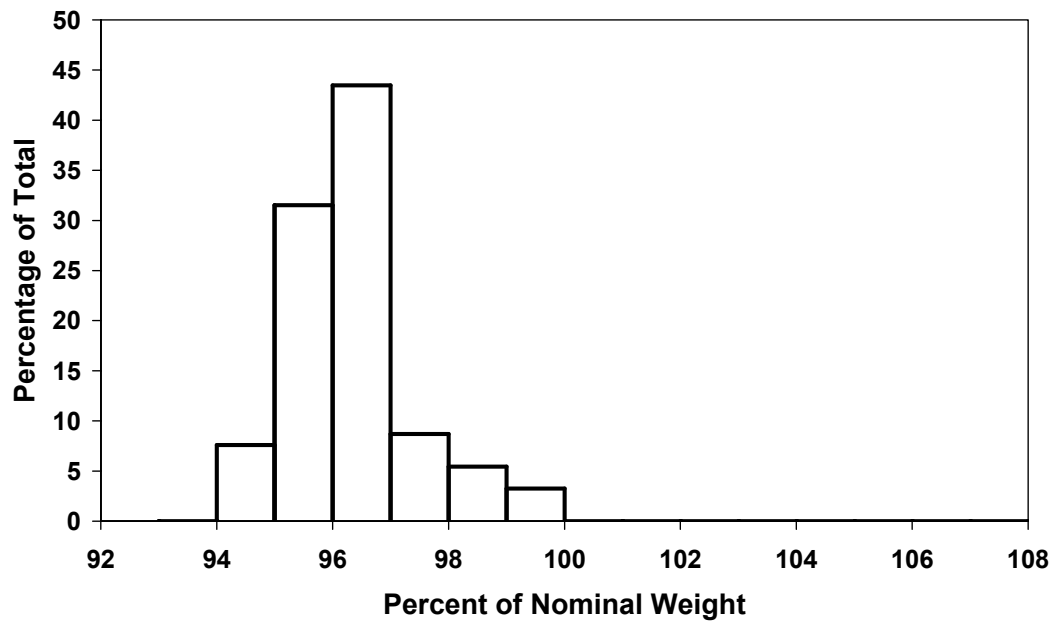


(b)

Figure 131: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 8 bars

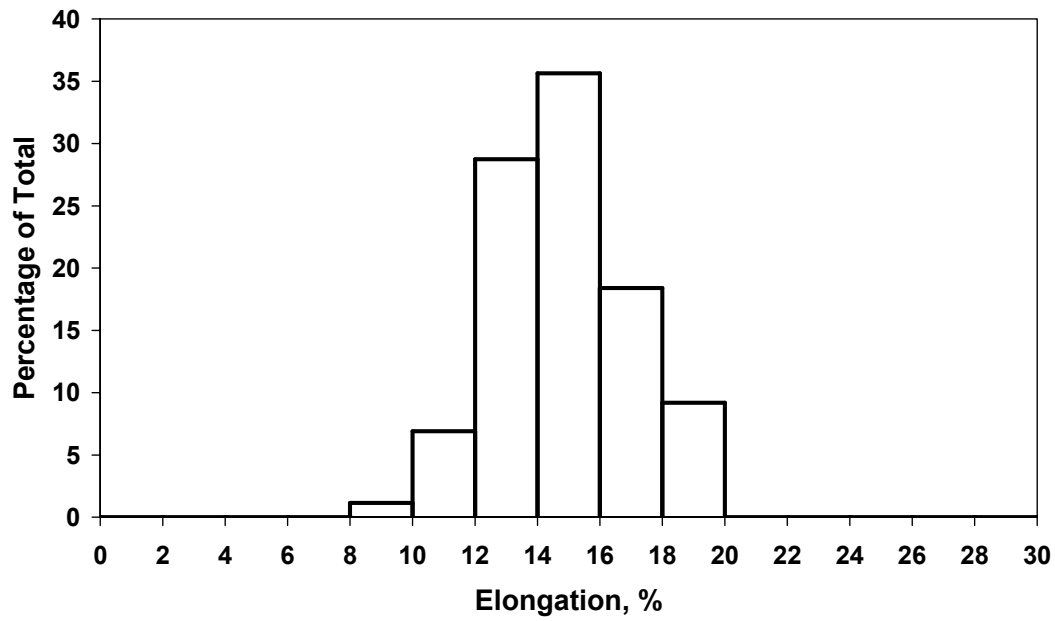


(a)

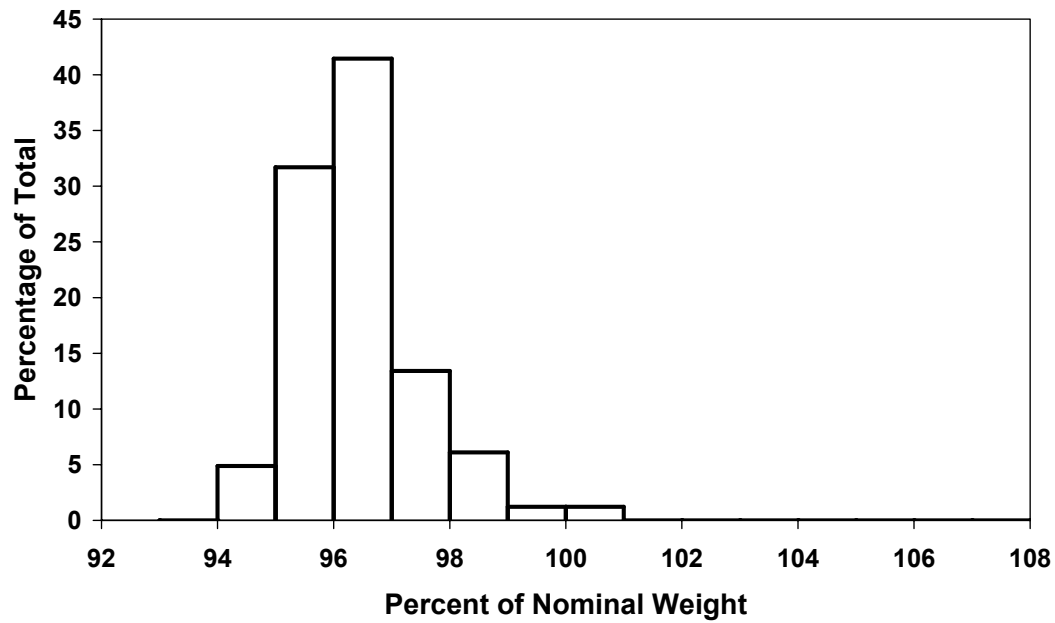


(b)

Figure 132: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 9 bars

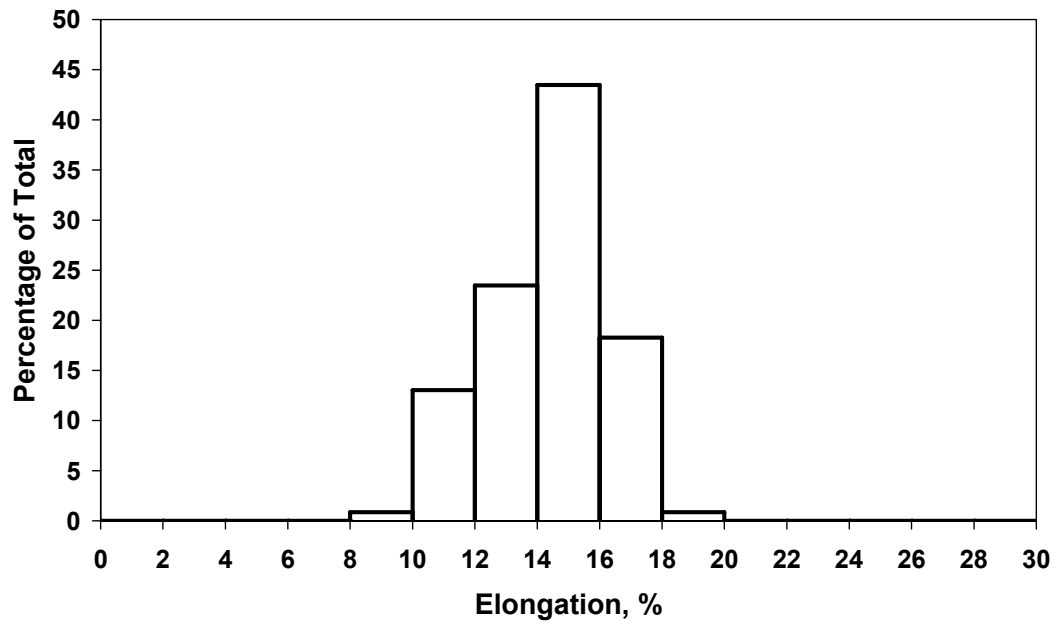


(a)

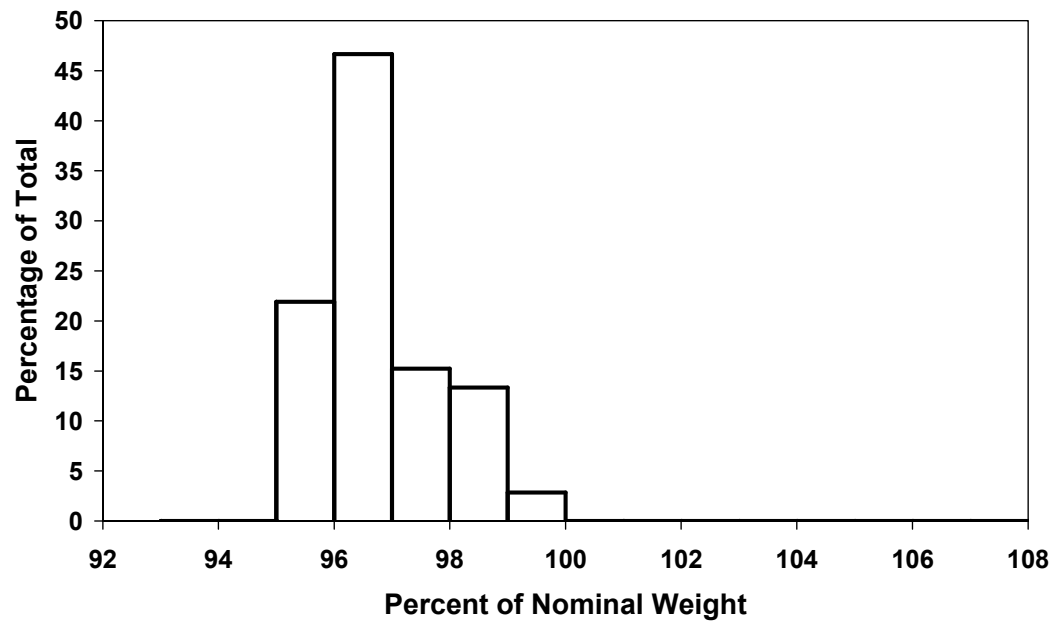


(b)

Figure 133: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 10 bars

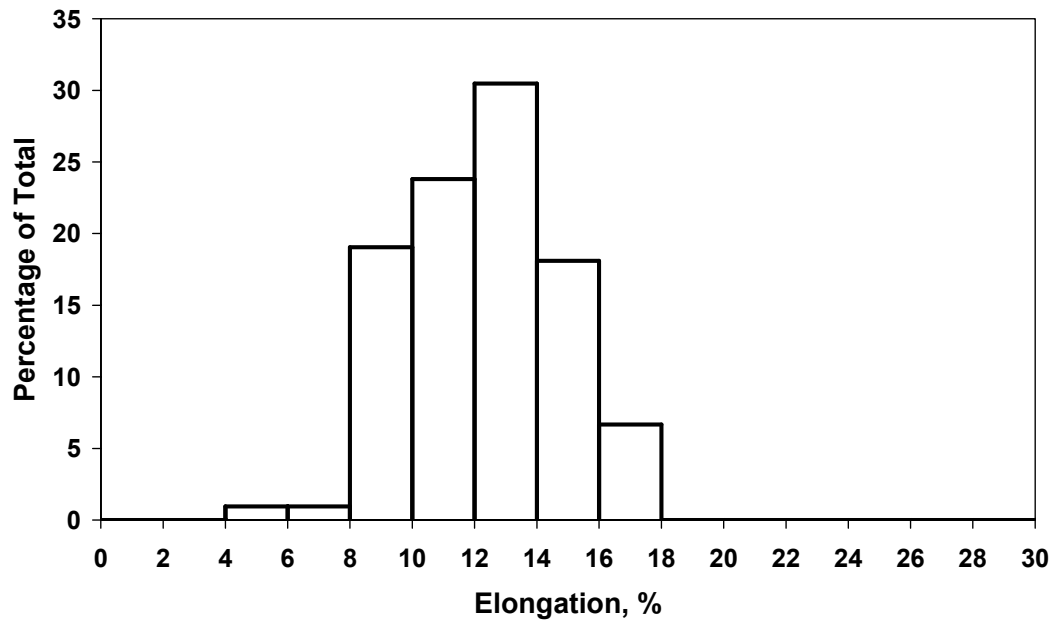


(a)

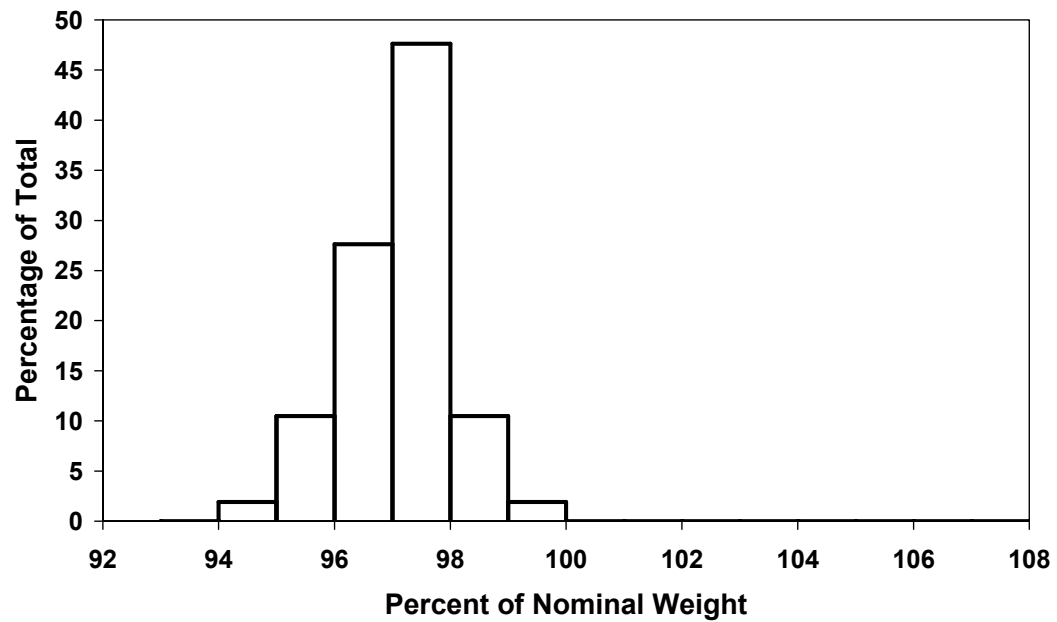


(b)

Figure 134: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 11 bars

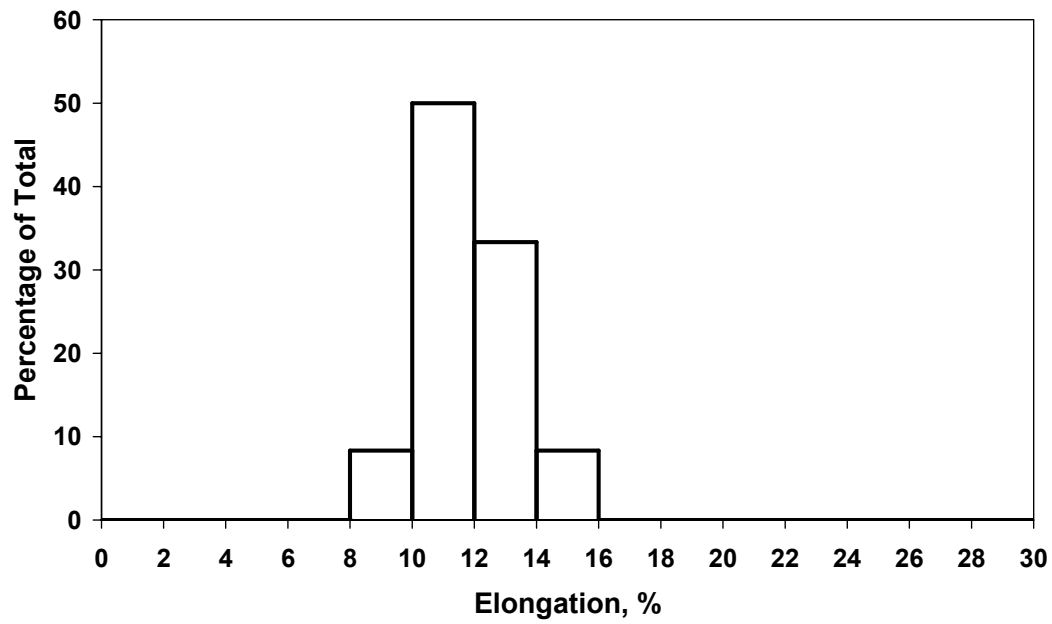


(a)

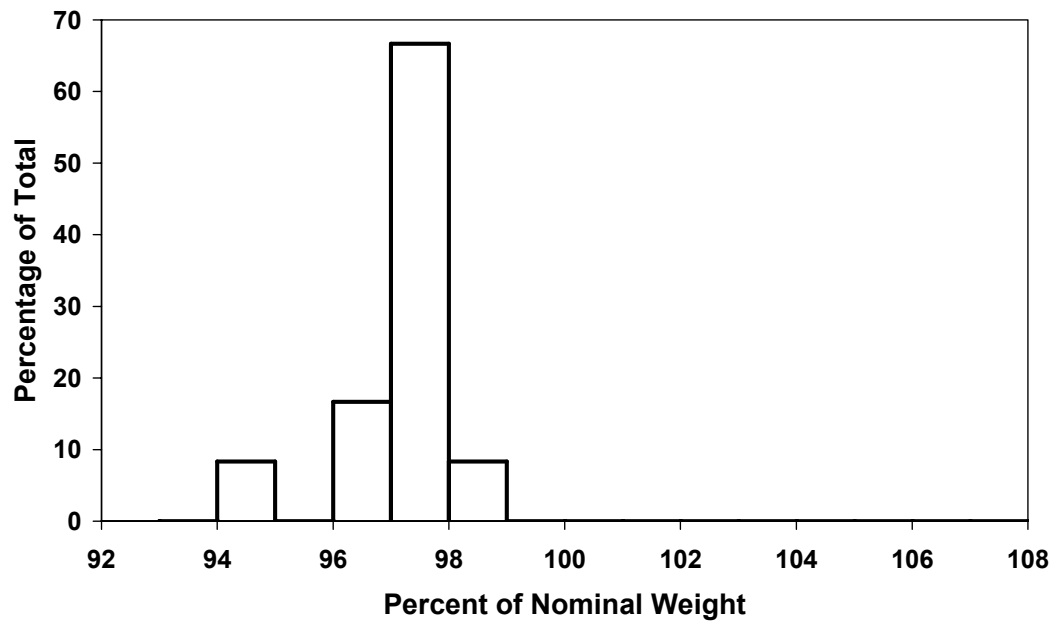


(b)

Figure 135: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 14 bars

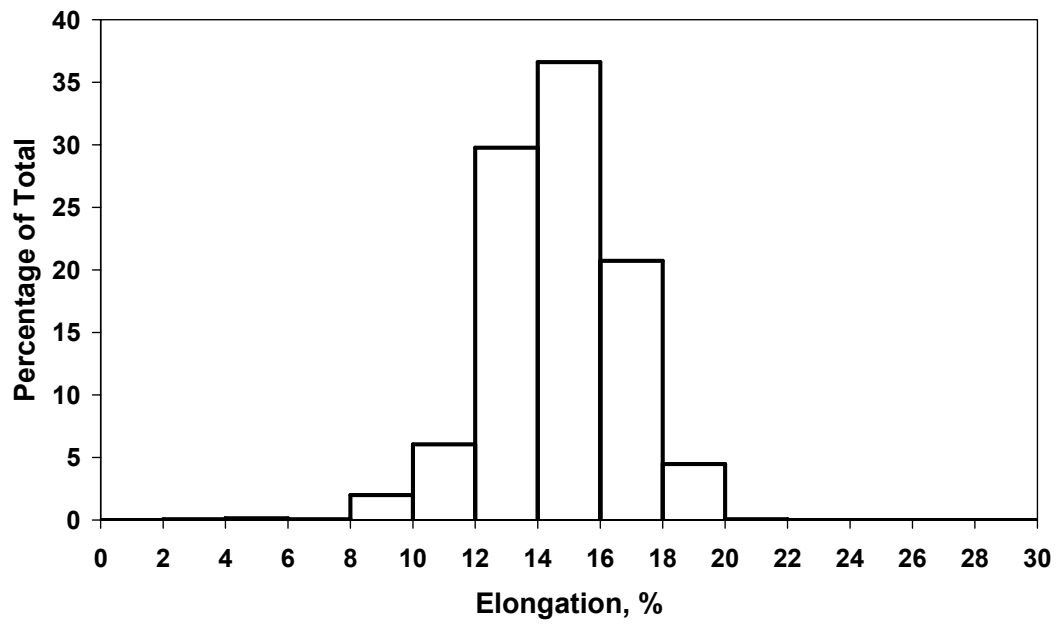


(a)

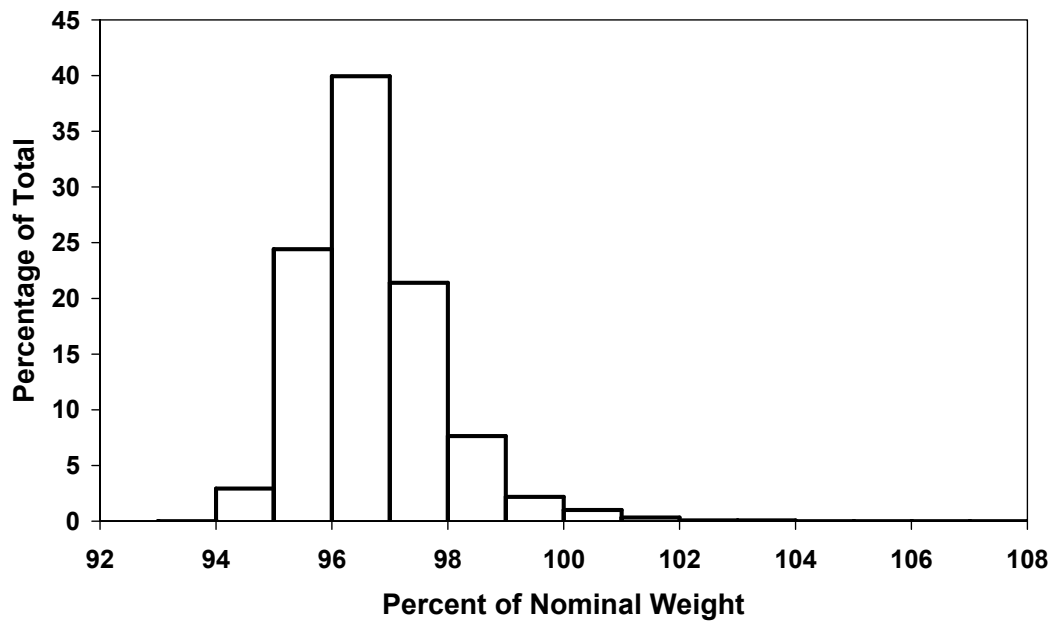


(b)

Figure 136: (a) Elongation and (b) Percent of Nominal Weight for A 706 Grade 60 No. 18 bars



(a)



(b)

Figure 137: (a) Elongation and (b) Percent of Nominal Weight for All A 706 Grade 60 bars